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Publication History:

1. First printing, TR-2003-143, January 2004

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201 Broadway, Cambridge MA 02139

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Index Terms—TDOA, RSS, TOA, location, sensor networks

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Amer Catovic, *Bechtel Telecommunications and Industrial (acatovic@bechtel.com)*
Zafer Sahinoglu, *Mitsubishi Electric Research Laboratories (zafer@merl.com)*

***Abstract*— In short range communications, the use of RSS measurements in conjunction with TOA or TDOA leads to enhanced estimation accuracy with respect to the case where only TOA or TDOA are used. In this article, we derive the Cramer-Rao Bound (CRB) for location estimation accuracy of two different hybrid schemes: TOA/RSS and TDOA/RSS. For short ranges, the hybrid schemes offer improved accuracy, particularly in the proximity of the reference devices.**

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I. INTRODUCTION

THERE is a wide range of wireless sensor network (WSN) applications requiring knowledge of sensor locations, from indoor user tracking to environmental and structural monitoring. Location estimation schemes used in long-range communications, such as wireless cellular networks (WCN), include time of arrival (TOA), time difference of arrival (TDOA), and received signal strength (RSS) [1]. Although RSS measurements are easily available, since mobile terminals (MT) constantly monitor the strength of the neighboring base stations' pilot signals for handoff purposes [2],[3], the RSS technique has been circumvented in WCNs, because of its dependency on the distance of the located device to the reference devices (i.e. base stations). In WSNs, on the other hand, the distances between mobile sensor nodes (SN) and the neighboring reference devices are by an order of magnitude smaller than in WCNs. For example, in the emerging ZigBee standards [3], the transmission range of reduced function devices (RFD) and full function devices (FFD) are set to be 30 meters. At this range, RSS-based location estimation performs better. Moreover, since WSNs are characterized by high device density, the likelihood of a sensor device being in very close proximity of a reference device is substantial. Hence, exploiting RSS in short-range communications could provide low-cost improvement of the location accuracy.

In this paper, two novel hybrid location estimation schemes, based on the combination of RSS measurements and either TOA or TDOA measurements, denoted as TOA/RSS and TDOA/RSS, respectively, are studied. In [5], the CRB of location estimation techniques in non-line-of-sight environment is examined. In [6], the CRB for only TOA and only RSS-based location estimation is computed. Here, these results are extended to the case of the TOA/RSS and TDOA/RSS hybrid schemes. The new results indicate that in short range communications, the use of RSS measurements in conjunction with TOA or TDOA leads to two enhancements with respect to the case where only TOA or TDOA are used; namely: improved overall location estimation accuracy and significantly improved behavior in the proximity of the reference devices.

II. THE CRB OF THE TOA/RSS HYBRID SCHEME

Consider a sensor device whose location is being estimated, denoted as SN-0, and m reference devices denoted with indexes $1\ldots m$, which are within the range of SN-0. Each reference device i is assumed to be capable of simultaneously performing both TOA, t_i, and RSS, r_i, observations of the signals transmitted by SN-0. The ability to perform TOA observations implies full network synchronization. Therefore, the TOA/RSS scheme is applicable to synchronized WSNs only. This requirement can be relaxed by using TDOA/RSS scheme, which is treated in the next section.

The observations are organized into the observation-vector $X = [X_T, X_R] = [[t_1,\ldots,t_m],[r_1,\ldots,r_m]]$. Assume that the actual location coordinate of SN-0 is $\theta_0 = [x_0, y_0]$. The location estimation problem then consists in finding the estimate of the coordinate, $\hat{\theta}_0$, given the coordinate vector of the reference devices $\theta = [\theta_1,\ldots,\theta_m]$. The t_i observations are commonly modeled as normal random variables $f_{t_i/\theta_0,\theta_i} \sim N(d_{i,0}/c, \sigma_T^2)$, where $d_{i,0}$ is the separation of SN-0 and reference device i, c is the speed of radio-wave propagation and σ_T is the parameter describing the joint nuisance of the multipath nature of the propagation channel and the measurement error. The r_i measurements are log-normal random variables $f_{r_i(dB)/\theta_0,\theta_i} \sim N(P_0(dB), \sigma_{sh}^2)$, with $P_0(dB) = P_t(dB) - 10n_p \log_{10}(d_{0,i})$, where $P_0(dB)$ and $P_t(dB)$ are the decibel values of the mean received power and the mean transmitted power of SN-0 respectively, n_p is the propagation exponent, and σ_{sh}^2 is the variance of the lognormal shadowing.

The CRB of an unbiased estimator $\hat{\theta}_0$ is $\text{cov}(\hat{\theta}_0) \geq I(\theta_0)^{-1}$, where $I(\theta_0)$ is the Fisher information matrix (FIM), defined as:

$$I(\theta_0) = -E\nabla_{\theta_0}(\nabla_{\theta_0} l(X|\theta_0,\theta)) \quad (1)$$

and $l(X|\theta_0,\theta)$ is the logarithm of the joint conditional probability density function:

$$l(X|\theta_0,\theta) = \sum_{i=1}^{m} \log f_{t_i|\theta_0,\theta_i}(t_i|\theta_0,\theta_i) + \sum_{i=1}^{m} \log f_{r_i/\theta_0,\theta_i}(r_i|\theta_0,\theta_i) \quad (2)$$

It can be shown that (1) develops into:

$$I(\theta_0) = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{xy} & I_{yy} \end{bmatrix} = \begin{bmatrix} I_{T_{xx}} + I_{R_{xx}} & I_{T_{xy}} + I_{R_{xy}} \\ I_{T_{xy}} + I_{R_{xy}} & I_{T_{yy}} + I_{R_{yy}} \end{bmatrix}$$
$$= \begin{bmatrix} I_{T_{xx}} & I_{T_{xy}} \\ I_{T_{xy}} & I_{T_{yy}} \end{bmatrix} + \begin{bmatrix} I_{R_{xx}} & I_{R_{xy}} \\ I_{R_{xy}} & I_{R_{yy}} \end{bmatrix} = I_T(\theta_0) + I_R(\theta_0) \quad (3)$$

where

$$I_{xx} = -\sum_{i=1}^{m} E\left[\frac{\partial^2 \log f_{t_i|\theta_0,\theta_i}(t_i|\theta_0,\theta_i)}{\partial x_0^2}\right] - \sum_{i=1}^{m} E\left[\frac{\partial^2 \log f_{r_i|\theta_0,\theta_i}(r_i|\theta_0,\theta_i)}{\partial x_0^2}\right] = I_{T_{xx}} + I_{R_{xx}} \quad (4)$$

Equivalent definitions are valid for I_{yy} and I_{xy}. The CRB on the variance of the TOA/RSS location estimation is:

$$\sigma_{CRB}^2 = \min_{\hat{\theta}_0} E[(\hat{x}_0 - x_0)^2 + (\hat{y}_0 - y_0)^2]$$
$$= \min tr\{\operatorname{cov}(\hat{\theta}_0)\} = tr\{I(\theta_0)^{-1}\} = \frac{I_{xx} + I_{yy}}{I_{xx}I_{yy} - I_{xy}^2} \quad (5)$$

The derivatives in (4) were computed in [6]. Substituting them back into (4) and then into (5) yields the final expression for the CRB of the TOA/RSS scheme:

$$\sigma_{CRB}^2 = \frac{\dfrac{m}{c^2\sigma_T^2} + b\sum_{i=1}^{m} d_{i,0}^{-2}}{\dfrac{1}{(c^2\sigma_T^2)^2}\sum_{i=1}^{m}\sum_{\substack{j=1\\i<j}}^{m} A_{i,j}^2 + \dfrac{b}{c^2\sigma_T^2}\sum_{i=1}^{m}\sum_{\substack{j=1\\i<j}}^{m} \dfrac{A_{i,j}^2}{d_{j,0}^2} + b^2\sum_{i=1}^{m}\sum_{\substack{j=1\\i<j}}^{m}\left(\dfrac{A_{i,j}}{d_{i,0}d_{j,0}}\right)^2} \quad (6)$$

where $b = \left(\frac{10n_p}{\sigma_{sh}\log 10}\right)^2$, $A_{i,j} = \frac{d_{0\times i,j}d_{i,j}}{d_{i,0}d_{j,0}}$ is a unitless parameter, representing the geometric conditioning [6] of devices i and j with respect to SN-0, and $d_{0xi,j}$ is the length of the shortest distance between SN-0 and the line connecting devices i and j. The $A_{i,j}$ in fact represents the area of the parallelogram defined by the devices SN-0, i and j, normalized by $d_{i,0} \cdot d_{j,0}$. The first term in the denominator represents the contribution of the TOA measurements to the CRB, and it depends only on the number of reference devices and their geometric conditioning with respect to SN-0, regardless of their separation. This is also clear from the first term in the numerator. The third term in the denominator represents the contribution of the RSS observations.

Due to $(d_{i,0} \cdot d_{j,0})^2$ in the denominator of this term, the contribution of the RSS measurements to the location accuracy is determined by the separation of SN-0 from the reference devices, as expected. Reducing this separation (e.g, by increasing the node density), increases the RSS contribution.

II. THE CRB OF THE TDOA/RSS HYBRID SCHEME

It may be difficult to provide global synchronization in a WSN, due to the requirements related to highly precise central clock in sensor devices and low-network installation cost. The lack of synchronization can be mitigated by replacing TOA measurements with TDOA, whereby instead of t_i observations, observations $T_i = t_i - t_1,\ i = 2,\ldots,m$ are used, in order to filter out the unknown clock offsets between the clock of SN-0 and the reference devices' clocks. The pdf of T_i is $f_{T_i/\theta_0,\theta_i} \sim N((d_{i,0} - d_{1,0})/c, 2\sigma_T^2)$. The observation vector now becomes $X = [X_T, X_R] = [[T_2,\ldots,T_m],[r_1,\ldots,r_m]]$. Effectively, in order to cope with the lack of synchronization, the TDOA sacrifices one observation and doubles the variance of the remaining m-1 observations. The CRB of the TDOA/RSS estimation can again be computed using (1) through (5), whereas (2) now becomes:

$$l(X|\theta_0,\theta) = \sum_{i=2}^{m} \log f_{T_i|\theta_0,\theta_i}(T_i|\theta_0,\theta_i) + \sum_{i=1}^{m} \log f_{r_i/\theta_0,\theta_i}(r_i|\theta_0,\theta_i) \quad (7)$$

and (4) becomes:

$$I_{xx} = -\sum_{i=2}^{m} E\left[\frac{\partial^2 \log f_{T_i|\theta_0,\theta_i}(T_i|\theta_0,\theta_i)}{\partial x_0^2}\right] - \sum_{i=1}^{m} E\left[\frac{\partial^2 \log f_{r_i|\theta_0,\theta_i}(r_i|\theta_0,\theta_i)}{\partial x_0^2}\right] = I_{T_{xx}} + I_{R_{xx}} \quad (8)$$

The derivatives in (7) were computed in [7]. The resulting CRB of the TDOA/RSS scheme can then be obtained from (8) and (5). After some simplification, one gets:

$$\sigma_{CRB}^2 = \frac{\dfrac{(m-1)}{c^2\sigma_T^2} - \dfrac{1}{2c^2\sigma_T^2}\sum_{i=2}^{m}\cos\gamma_{1,i} + b\sum_{i=1}^{m} d_{i,0}^{-2}}{\left(\begin{array}{l}\dfrac{1}{(2c^2\sigma_T^2)^2}\sum_{i=2}^{m}\sum_{\substack{j=2\\i<j}}^{m}(A_{1,i} + A_{1,j} - A_{i,j})^2 \\ + \dfrac{b}{2c^2\sigma_T^2}\sum_{i=2}^{m}\left[\begin{array}{c} A_{1,i}^2(d_{1,0}^{-2} + d_{i,0}^{-2}) + \\ \sum_{\substack{j=2\\i\neq j}}^{m}\left(\dfrac{A_{1,i} - A_{i,j}}{d_{i,0}}\right)^2 \end{array}\right] + b^2\sum_{i=1}^{m}\sum_{\substack{j=1\\i<j}}^{m}\left(\dfrac{A_{i,j}}{d_{i,0}d_{j,0}}\right)^2\end{array}\right)} \quad (9)$$

where $\gamma_{1,i}$ is the angle between $d_{0,1}$ and $d_{0,i}$.

III. COMPARISON AND DISCUSSION

For CRB computations, the channel measurement results given in [6] are used as reference, yielding $\sigma_{dB}/n_P = 2$ and $c\,\sigma_T = 1.8$. Four reference devices are first placed in the corners of an 18m by 18m square, and the CRB for every coordinate in that region for the TOA (Fig.1) and TOA/RSS (Fig.2) cases is computed. Due to the desired symmetry of the plot, in the TDOA/RSS case (Fig. 3), a circular area is used, but the radius of the circle is set to $18/\sqrt{\pi} \approx 10$ m, in order to maintain the same size of the area as those in the TOA and TOA/RSS cases. The reference device 1 is now placed in the center of the circle, while the other three devices are placed equidistantly around the circle.

The comparison between Fig.1 and Fig.2 reveals that TOA/RSS has better overall location accuracy than TOA. In particular, the use of RSS measurements mitigates the difficulties in locating the devices in the proximity of the reference devices, which are inherent to the time of arrival-based schemes. The accuracy of the TDOA/RSS scheme is inferior to TOA/RSS, as expected, due to the sacrifice of one TOA measurement. However, it also exhibits smoothened accuracy around the reference devices.

Fig.4 shows the average CRB of the three schemes over the entire area shown in Fig.1-3, for varying sizes of the area. The *x*-axis shows the radius *x* of the circular region for TDOA/RSS scheme, while for TOA and TOA/RSS scheme the average CRB is computed over the square region whose side is equal to $x\sqrt{\pi}$, so that the corresponding area sizes are identical in all three cases. The benefits of the use of RSS measurements in conjunction with TOA and TDOA, are evident for communication ranges below 30m, which are characteristic for WSNs. For longer ranges, TOA/RSS and TDOA/RSS perform essentially the same as TOA and TDOA, respectively.

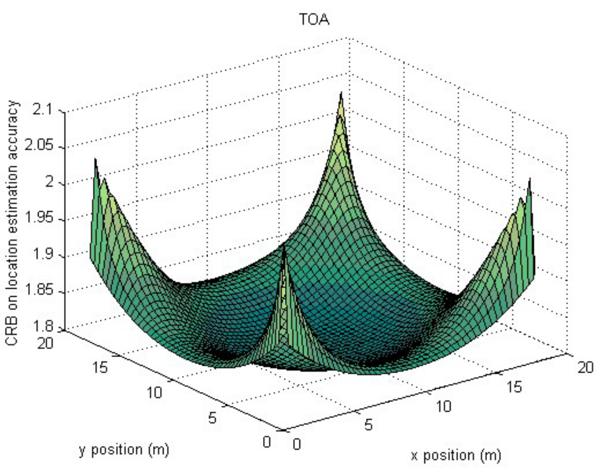

Figure 1 The CRB of the TOA scheme estimation with four reference devices in the four corners ($\sigma_{dB} / n_P = 2$ and $c\sigma_T = 1.8$).

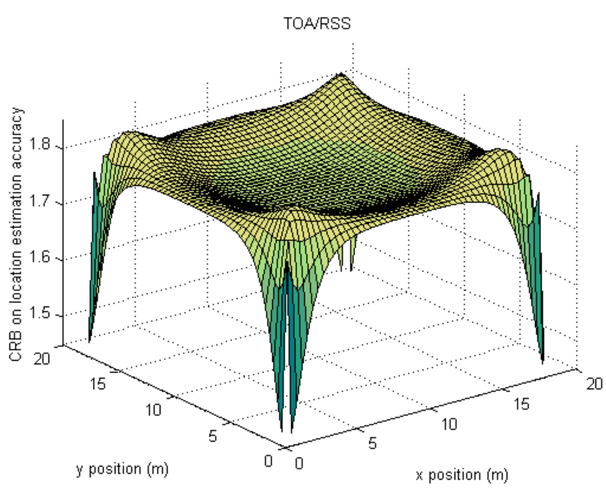

Figure 2 The CRB of the TOA/RSS scheme with four reference devices in the four corners ($\sigma_{dB} / n_P = 2$ and $c\sigma_T = 1.8$).

IV. Conclusion

In this paper, the closed-form expressions for the CRB for two hybrid location estimation schemes, TDOA/RSS and TOA/RSS, are computed. The results indicate that the two schemes provide improved location accuracy with respect to TOA and TDOA location estimation schemes for networks with devices having communications ranges of 30m or less, such as WSNs. In addition, they remove the inherent difficulties of TOA and TDOA schemes with respect to locating devices in the close proximity of the reference devices.

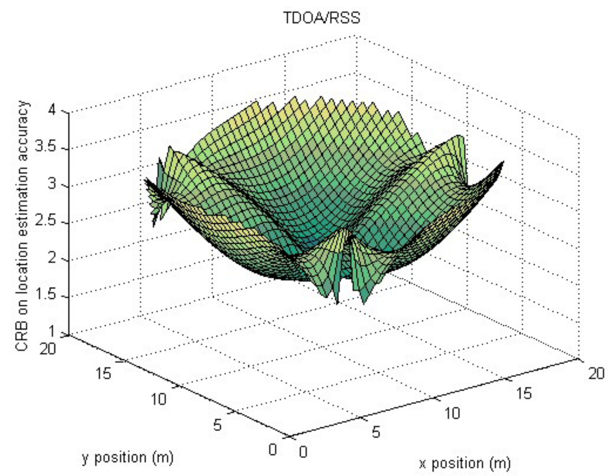

Figure 3 The CRB of the TDOA/RSS scheme with four reference devices, one in the center of the circular region ($\sigma_{dB} / n_P = 2$ and $c\sigma_T = 1.8$).

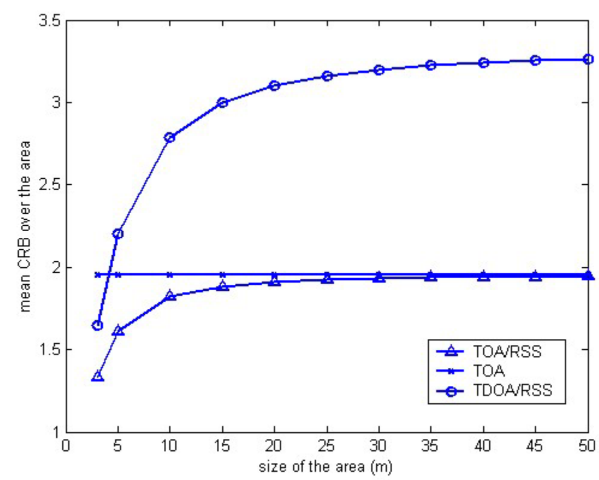

Figure 4 Mean CRB of the TOA, TOA/RSS and TDOA/RSS schemes ($\sigma_{dB} / n_P = 2$ and $c\sigma_T = 1.8$).

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