MITSUBISHI ELECTRIC RESEARCH LABORATORIES
http://www.merl.com

Estimation of Performance Loss Due to Delay in Channel Feedback in MIMO Systems

Jianxuan Du, Ye (Geoffrey) Li, Daqing Gu, Andreas F. Molisch and Jinyun Zhang

TR2004-118 October 2004

Abstract

Latency of available channel state information (CSI) at the transmitter in time-varying channels greatly affects the performance of multiple-input multiple-output (MIMO) systems. We have derived a simple algorithm to calculate an approximation of the expected performance loss based on the most recent channel feedback. The proposed algorithm can be used to determine the maximum tolerable channel feedback delay for each particular channel realization.

IEEE Vehicular Technology Confereance (VTC) 2004 - Fall

IEEE Vehicular Technology Conference (VTC) 2004 - Fall

Estimation of Performance Loss Due to Delay in Channel Feedback in MIMO Systems

Jianxuan Du, Ye (Geoffrey) Li
School of Electrical and Computer Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0250
Email: {jxdu, liye}@ece.gatech.edu

Daqing Gu, Andreas F. Molisch†, and Jinyun Zhang
MERL Mitsubishi Electric Research Laboratories
201 Broadway
Cambridge, MA 02139
Email: {dgu, molisch, jzhang}@merl.com

***Abstract*— Latency of available *channel state information* (CSI) at the transmitter in time-varying channels greatly affects the performance of *multiple-input multiple-output* (MIMO) systems. We have derived a simple algorithm to calculate an approximation of the expected performance loss based on the most recent channel feedback. The proposed algorithm can be used to determine the maximum tolerable channel feedback delay for each particular channel realization.**

***Index Terms*— MIMO, latent CSI, CSI feedback.**

I. Introduction

The employment of multiple transmit and receive antennas has been shown to greatly increase the spectral efficiency of wireless communication systems [1] [2]. When CSI is known at the transmitter in a MIMO system, *singular value decomposition* (SVD) transmission with water-filling can be used to achieve the closed-loop capacity [1]. However, channel estimation error and CSI feedback delay in time-varying channels cause CSI ambiguity at the transmitter. The loss of orthogonality between virtual channels formed by the SVD causes mutual interference and significantly degrades system performance.

This paper proposes an efficient algorithm to compute the approximate expected capacity loss for each instantaneous channel realization based on the channel autocorrelation. By approximating a weighted sum of a number of chi-square random variables with another chi-square random variable with different degrees of freedom that has the same first two moments, we are able to derive a simple closed form expression for the expected capacity with the current channel matrix and power allocation. And the result can be used to determine how often CSI needs to be fed back to the transmitter.

The rest of the paper is organized as follows. The system model and SVD transmission are briefly introduced in Section II. The approximation of expected capacity is described in Section III. Extension of the result to frequency-selective channels is discussed in Section IV. Then, in Section V, numerical results are given to prove the accuracy of the approximation. The paper is concluded in Section VI.

II. MIMO Systems with SVD Technique

For a MIMO system with N_t transmit and N_r receive antennas, the received signal at each receive antenna is the superposition of distorted signals from N_t transmit antennas.

$$y_k\left(t\right) = \sum_{k=1} h_{kl}\left(t\right) x_l\left(t\right) + n_k\left(t\right),\ k = 1,\ 2,\ \ldots,\ Nr,$$

where $h_{kl}\left(t\right)$ is the channel gain corresponding to transmit antenna l and receive antenna k at time t and is assumed to be independent and identically distributed (i.i.d.) complex Gaussian random variable and have the same autocorrelation function $r\left(\Delta t\right)$. For a discussion of this model, see [3]. $n_k\left(t\right)$ is additive white complex Gaussian noise at receive antenna k and is assumed to be zero-mean with variance N_0 and independent for different t's and k's.

The channel state information at time t can be represented by a matrix defined as

$$\mathbf{H}\left(t\right) = \begin{pmatrix} h_{11}\left(t\right) & h_{12}\left(t\right) & \ldots & h_{1N_t}\left(t\right) \\ h_{21}\left(t\right) & h_{22}\left(t\right) & \ldots & h_{2N_t}\left(t\right) \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_r1}\left(t\right) & h_{N_r2}\left(t\right) & \ldots & h_{N_rN_t}\left(t\right) \end{pmatrix}.$$

With channel state information at the transmitter, linear pre-processing at the transmitter and post-processing at the receiver can be used to decouple the MIMO channel into a number of parallel *single-input single-output* (SISO) channels. Let the SVD of $\mathbf{H}\left(t\right)$ be $\mathbf{H}\left(t\right) = \mathbf{U}\left(t\right)\Sigma\left(t\right)\mathbf{V}^H\left(t\right)$, then the received signal using SVD technique becomes

$$\begin{aligned} \mathbf{y}\left(t\right) &= \mathbf{U}^H\left(t\right)\left[\mathbf{H}\left(t\right)\mathbf{V}\left(t\right)\mathbf{s}\left(t\right) + \mathbf{n}\left(t\right)\right] \\ &= \Sigma\left(t\right)\mathbf{s}\left(t\right) + \tilde{\mathbf{n}}\left(t\right), \end{aligned}$$

where $\Sigma\left(t\right)$ is a diagonal matrix. Optimum power allocation using water-filling according to the diagonal elements of $\Sigma\left(t\right)$ is given in [4].

III. Approximation of Expected Capacity with CSI Error Due to Channel variation

Due to the time-varying value of the channel and insufficient frequency of feedback, the channel state information at the time of transmission is different from that available at the transmitter. Here we assume that accurate CSI, $\mathbf{H}\left(t\right)$, is fed back to the transmitter. The channel matrix at time t' is

$$\mathbf{H}\left(t'\right) = \hat{\mathbf{H}}\left(t'\right) + \Delta\mathbf{H}\left(t'\right),$$

where $\hat{\mathbf{H}}\left(t'\right)$ is the predicted channel response at time t' from $\mathbf{H}\left(t\right)$, and $\Delta\mathbf{H}\left(t'\right)$ is the prediction error matrix. Let $\hat{\mathbf{U}}\left(t'\right)\hat{\Sigma}\left(t'\right)\hat{\mathbf{V}}^H\left(t'\right)$ be the SVD of $\hat{\mathbf{H}}\left(t'\right)$. Then the received

†Also at Department of Electroscience, Lund University, Lund, Sweden.

signal vector at the receiver after the SVD processing [1] becomes

$$\begin{aligned}\tilde{\mathbf{y}}(t') &= \hat{\mathbf{U}}^H(t')\left[\mathbf{H}(t')\hat{\mathbf{V}}(t')\mathbf{s}(t')+\mathbf{n}(t')\right] \\ &= \left(\hat{\Sigma}(t')+\Delta\Sigma(t')\right)\mathbf{s}(t')+\tilde{\mathbf{n}}(t'), \end{aligned} \tag{1}$$

where

$$\begin{aligned}\Delta\Sigma(t') &= \hat{\mathbf{U}}^H(t')\,\Delta\mathbf{H}(t')\,\hat{\mathbf{V}}(t') \\ &= \begin{pmatrix} \delta_{11}(t,\ t') & \delta_{12}(t,\ t') & \dots & \delta_{1N_t}(t,\ t') \\ \delta_{21}(t,\ t') & \delta_{22}(t,\ t') & \dots & \delta_{2N_t}(t,\ t') \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{N_r1}(t,\ t') & \delta_{N_r2}(t,\ t') & \dots & \delta_{N_rN_t}(t,\ t') \end{pmatrix}. \end{aligned}$$

From Equation (1), $\Delta\Sigma(t')$ causes mutual interference between supposedly decoupled virtual channels. $\tilde{\mathbf{n}}(t') = \hat{\mathbf{U}}^H(t')\,\mathbf{n}(t')$ is the noise vector after unitary transformation, which has the same distribution as $\mathbf{n}(t')$ since i.i.d. Gaussian distribution is invariant to unitary transformation. For simplicity, we assume $N_t = N_r = M$ and extension to systems with different numbers of transmit and receive antennas is straightforward.

The joint distribution of channel parameters at times t and t' is characterized by the channel autocorrelation function $r(\Delta t)\,|_{\Delta t=t'-t}$ and it is easy to verify that the $\delta_{kl}(t,\ t')$'s for *minimum mean-square error* (MMSE) prediction are i.i.d. complex Gaussian [5] that satisfy

$$\delta_{kl}(t,\ t') \sim CN\left(0,\eta^2\right),$$

where $\eta^2 = r(0)-|r(t'-t)|^2/r(0)$, and $CN(m,\eta^2)$ denotes a complex Gaussian random variable with mean m and variance η^2. '$\sim$' means both sides have the same distribution. From now on, we drop the time index for simplicity.

Now we compute the channel capacity with CSI ambiguity due to channel variation. Note that we assume complete decoupling of all virtual channels, thus no joint decoding is used and the capacity is just the sum of all those channels in the presence of mutual interference. Thus the capacity averaged over all possible value of $\Delta\Sigma(t')$ is

$$\begin{aligned} & E_{\delta_{lk}}\left\{\sum_{l=1}^{M}\log_2\left(1+\frac{A_l^2|\alpha_l+\delta_{ll}|^2}{\sum_{k\neq l}A_k^2|\delta_{lk}|^2+N_0}\right)\right\} \\ =\ & E_{\delta_{lk}}\left\{\sum_{l=1}^{M}\log_2\left(A_l^2|\alpha_l+\delta_{ll}|^2+\sum_{k\neq l}A_k^2|\delta_{lk}|^2+N_0\right)\right\} \\ & -E_{\delta_{lk}}\left\{\sum_{k\neq l}A_k^2|\delta_{lk}|^2+N_0\right\}, \end{aligned} \tag{2}$$

where $E_x\{\cdot\}$ denotes expectation (or ensemble average) with respect to random variable x, and α_l's are the diagonal elements of $\hat{\Sigma}(t')$. A_l is the amplitude of signal sent using the virtual channel corresponding to α_l and is determined by water-filling from α_l's [4].

There is no known closed form expression for the average capacity in Equation (2) and direct evaluation requires numerical integration. Instead, we try to derive an approximation of the average capacity. Note that $A_l^2|\alpha_l+\delta_{ll}|^2+\sum_{k\neq l}A_k^2|\delta_{lk}|^2+N_0$ is a weighted sum of chi-square random variables and we approximate the distribution by that of another chi-square random variable with different degrees of freedom as in [6] and [7]. It is obvious that

$$|\alpha_l+\delta_{ll}|^2 \sim \frac{\eta^2}{2}\chi^2\left(2,\frac{2|\alpha_l|^2}{\eta^2}\right),$$

$\chi^2(m,\ q)$ represents a noncentral chi-square random variable with m degrees of freedom and noncentrality parameter q. In particular, $\chi^2(m,\ 0)$ is simply written as $\chi^2(m)$. From [6] and [7], we can make the following approximation,

$$\frac{\eta^2A_l^2}{2}\chi^2\left(2,\frac{2|\alpha_l|^2}{\eta^2}\right)+\sum_{k\neq l}\frac{\eta^2A_k^2}{2}\chi^2(2)+N_0 \approx \beta_l\chi^2(d_l),$$

where β_l and d_l are chosen such that both sides have the same mean and variance, i.e.,

$$A_l^2|\alpha_l|^2+\eta^2\sum_{k=1}^{M}A_k^2+N_0=\beta_ld_l$$

and

$$2A_l^4\eta^2|\alpha_l|^2+\eta^4\sum_{k=1}^{M}A_k^4=2\beta_l^2d_l.$$

Then

$$\beta_l=\frac{2A_l^4\eta^2|\alpha_l|^2+\eta^4\sum_{k=1}^{M}A_k^4}{2\left(A_l^2|\alpha_l|^2+\eta^2\sum_{k=1}^{M}A_k^2+N_0\right)}, \tag{3}$$

and

$$d_l=\frac{2\left(A_l^2|\alpha_l|^2+\eta^2\sum_{k=1}^{M}A_k^2+N_0\right)^2}{2A_l^4\eta^2|\alpha_l|^2+\eta^4\sum_{k=1}^{M}A_k^4}. \tag{4}$$

Thus

$$\begin{aligned} & E_{\delta_{lk}}\left\{\log_2\left(A_l^2|\alpha_l+\delta_{ll}|^2+\sum_{k\neq l}A_k^2|\delta_{lk}|^2+N_0\right)\right\} \\ \approx\ & \log_2(\beta_l)+E_{\chi^2(d_l)}\left\{\log_2\left(\chi^2(d_l)\right)\right\} \\ =\ & \log_2(\beta_l)+\int_0^\infty\frac{1}{2^{d_l/2}\Gamma(d_l/2)}\log_2(u)\,u^{d_l/2-1}e^{-u/2}du \\ =\ & \log_2(\beta_l)+\frac{1}{2^{d_l/2}\Gamma(d_l/2)}\frac{\Gamma(d_l/2)\left[\psi(d_l/2)+\ln 2\right]}{(1/2)^{d_l/2}\ln 2} \\ =\ & \log_2(\beta_l)+\frac{\psi(d_l/2)}{\ln 2}+1, \end{aligned} \tag{5}$$

where $\Gamma(x)=\int_0^\infty u^{x-1}e^{-u}du$ is the gamma function and $\psi(x)=\frac{d}{dx}\ln\Gamma(x)$ is known as digamma function or psi function [8]. Similarly, we find β_l' and d_l' for $\sum_{k\neq l}A_k^2|\delta_{lk}|^2+N_0$. Thus,

$$\beta_l'=\frac{\eta^4\sum_{k\neq l}A_k^4}{2\left(\eta^2\sum_{k\neq l}A_k^2+N_0\right)},$$

and

$$d'_l = \frac{2\left(\eta^2 \sum_{k\neq l} A_k^2 + N_0\right)^2}{\eta^4 \sum_{k\neq l} A_k^4}.$$

Therefore, the approximate average capacity is

$$E_{\delta_{lk}}\left\{\sum_{l=1}^{M} \log_2\left(1 + \frac{A_l^2 \left|\alpha_l + \delta_{ll}\right|^2}{\sum_{k\neq l} A_k^2 \left|\delta_{lk}\right|^2 + N_0}\right)\right\}$$
$$\approx \sum_{l=1}^{M}\left[\log_2 \frac{\beta_l}{\beta'_l} + \frac{\psi\left(d_l/2\right) - \psi\left(d'_l/2\right)}{\ln 2}\right]. \quad (6)$$

From Equation (6), the maximum tolerable channel feedback delay can be determined by finding the maximum Δt_{max} such that for all $\Delta t \leq \Delta t_{max}$, the expected capacity is greater than certain threshold.

IV. Extension to Frequency-selective Fading MIMO Systems

Now we consider systems with frequency-selective fading. The channel response at frequency f becomes

$$h_{kl}\left(t, f\right) = \sum_{m} \alpha_{klm}\left(t\right) e^{-j2\pi f \tau_l},$$

where $\alpha_{klm}\left(t\right)$'s are wide-sense stationary narrow band complex Gaussian process, which are independent for different paths and different k's and l's. Also assume $\alpha_{klm}\left(t\right)$'s have the same normalized correlation function and

$$\sum_{m} E\left\{\left|\alpha_{klm}\left(t\right)\right|^2\right\} = r\left(0\right).$$

Thus the channel response varies with frequency, so does the mutual interference between virtual channels in Equation (1). Similarly, we now denote the power spectral density of the mutual interference as $\delta_{kl}\left(t, t', f\right)$ at frequency *f*. It is easy to verify that $\delta_{kl}\left(t, t', f\right)$'s have the same distribution and are correlated across frequency. However, due to the linearity of expectation, the correlation does not affect the total average capacity when added up over frequency. Therefore, we can obtain the approximate average capacity derived in the previous section for each subcarrier, and then get the sum total to estimate the overall capacity loss, i.e.,

$$E_{\delta_{lk}(f)}\left\{\int \sum_{l=1}^{M} \log_2\left(1 + \frac{A_l^2\left(f\right)\left|\alpha_l\left(f\right) + \delta_{ll}\left(f\right)\right|^2}{\sum_{k\neq l} A_k^2\left(f\right)\left|\delta_{lk}\left(f\right)\right|^2 + N_0}\right) df\right\}$$
$$\approx \int \sum_{l=1}^{M}\left[\log_2 \frac{\beta_l\left(f\right)}{\beta'_l\left(f\right)} + \frac{\psi\left(d_l\left(f\right)/2\right) - \psi\left(d'_l\left(f\right)/2\right)}{\ln 2}\right] df. \quad (7)$$

Here we apply Equation (3) and (4) to derive $\beta_l\left(f\right)$, $\beta'_l\left(f\right)$,$d_l\left(f\right)$, and $d'_l\left(f\right)$ at each frequency f. Note that in this case, $A_l\left(f\right)$'s, $\alpha_l\left(f\right)$'s, $\delta_l\left(f\right)$'s, and N_0 are power spectral densities.

This problem is in general hard to solve. As an approximation, we may simply divide the entire bandwidth into small bins and treat the frequency response within each bin as constant, as we do in *orthogonal frequency division multiplexing* (OFDM). Then Equation (7) becomes a finite sum.

V. Numerical Results

We use numerical examples to test the accuracy of the approximation. First, we consider a 4x4 flat-fading MIMO system at different *signal-to-noise ratio*s (SNR) with $\eta^2 = 0.1 r\left(0\right)$, which corresponds to $\Delta t f_d = 0.0726$ for Jakes' model [9]. Figure 1.a and 1.b give the relative error at SNR=10dB and 15dB, respectively, where the relative error is defined as the ratio of the approximation error to the actual expected capacity with channel error. In both cases, the relative error due to the approximation is less than 2%.

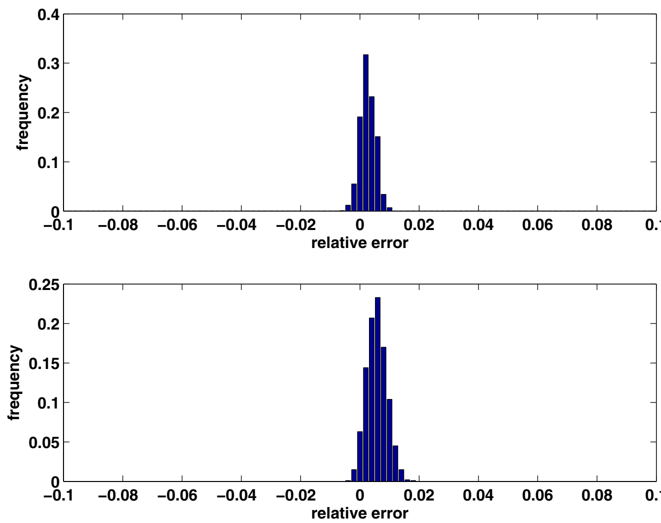

Fig. 1. Relative error of estimated capacity for a 4x4 flat-fading MIMO system with normalized Doppler frequency $\Delta t f_d = 0.0726$. (a) SNR=10dB, (b) SNR=15dB.

Then we compare the estimated maximum feedback delay with the actual value. The maximum delay is set to yield a capacity loss of at most 5% of that with accurate CSI. For the same system with the channel autocorrelation $r\left(\Delta t\right) = J_0\left(2\pi \Delta t f_d\right)$, Figure 2 shows the mean of the estimate using the approximate formula, and Figure 3 gives normalized mean-square error. The delay is expressed in terms of normalized Doppler frequency $\Delta t f_d$. From the figure, the proposed method yields fairly accurate estimate and the accuracy increases with higher SNR. It can also be observed that the average maximum delay decreases with increasing SNR, which matches our intuition that at high SNR's, the mutual interference caused by channel variation dominates the system performance.

Next a 4x4 OFDM system with 1.25 MHz system bandwidth divided into 128 subcarriers is considered. The length of the cyclic prefix is 20.2 μs, resulting in a total block duration of 225 μs. The channel has *Typical Urban* (TU) delay profile [10]. Global water-filling is performed on all subcarriers with the total transmit power fixed. All possible delays are integer multiples of the block duration, and when the Doppler frequency is 10Hz, the search for maximum delay is incremented by a normalized Doppler frequency of 0.0012 at each step. Jakes' model is used for generating the channel. From the

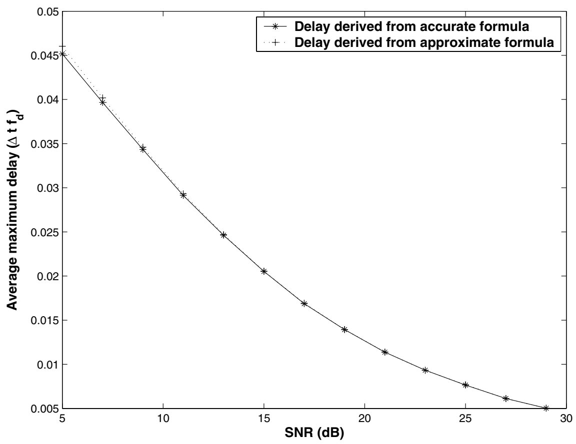

Fig. 2. Average maximum feedback delay for a 4x4 flat-fading MIMO system.

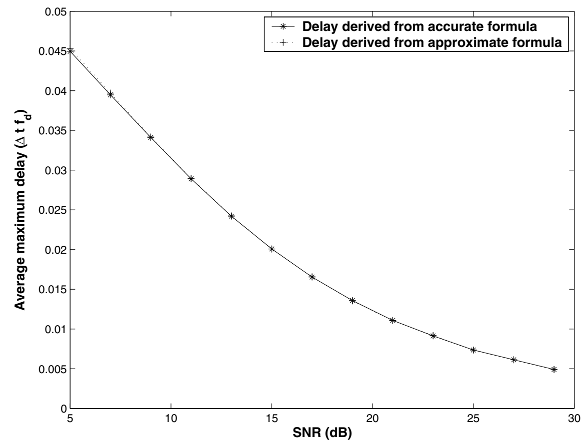

Fig. 4. Average maximum feedback delay for a 4x4 MIMO OFDM system with TU delay profile.

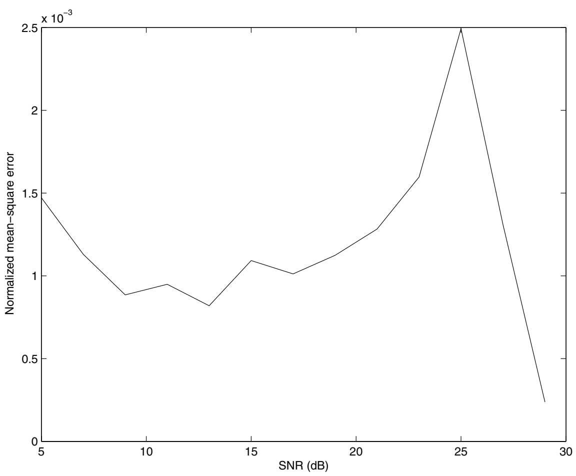

Fig. 3. Normalized mean-square error of estimated maximum feedback delay for a 4x4 flat-fading MIMO system.

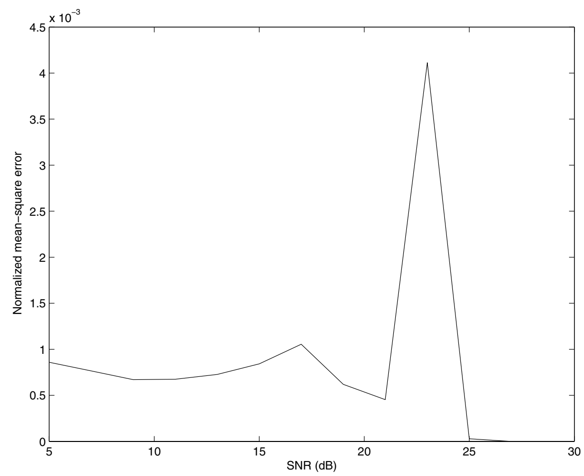

Fig. 5. Normalized mean-square error of estimated maximum feedback delay for a 4x4 MIMO OFDM system with TU delay profile.

mean and normalized mean-square error shown in Figure 4 and 5, we see again that the proposed method accurately gives the mean of the maximum delay and produces small estimation error.

VI. Conclusion

In this paper, we proposed an approximate algorithm to evaluate the performance loss in MIMO systems with CSI ambiguity due to channel variation. We assume that MMSE prediction of the channel information is used for SVD transmission and the channel variation is characterized by the time correlation. The proposed algorithm can be used to determine how often channel feedback should be in a given environment. The result is then generalized to frequency-selective channels. The effectiveness of the proposed method to estimate maximum channel feedback delay is shown by simulation.

References

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Transactions on Telecommunications*, vol. 10, pp. 585–595, Nov-Dec 1999.

[2] G. J. Foschini and M. J. Gans, "On the limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 315–335, Mar. 1998.

[3] A. F. Molisch and F. Tufvesson, "Multipath propagation models for broadband wireless systems," in *CRC Handbook of signal processing for wireless commmunications*, M. Ibnkahla, Ed., 2004, p. in press.

[4] T. M. Cover and J. A. Thomas, *Elements of Information Theory*. New York: Wiley, 1991.

[5] S. M. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*. Upper Saddle River, New Jersey: Prentice-Hall, Inc., 1993.

[6] Q. T. Zhang and D. P. Liu, "A simple capacity formula for correlated diveristy Rician fading channels," *IEEE Commun. Lett.*, vol. 6, pp. 481–483, Nov. 2002.

[7] J. P. Imhof, "Computing the distribution of quadratic forms in normal variables," *Biometrika*, vol. 48, pp. 419–426, 1961.

[8] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series, and Products*. London: Academic Press, 1994.

[9] W. C. Jakes, *Microwave Mobile Communications*. New York: Wiley, 1974.

[10] C. of the European Communities, *Digital land mobile radio communications - COST 207*. ECSC-EEC-EAEC, 1989.