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***Abstract*— We propose a modified layered structure for *multiple-input multiple-output* (MIMO) systems, where the layer detection order is fixed and the data rate for each layer is allocated based on the detection order and channel statistics. With Gaussian approximation of layer capacities, we derive the optimum data rate allocation and the amount of backoff from mean layer capacity is proportional to the standard deviation of the layer capacity. The minimum overall outage probability of a layered system is uniquely determined by the normalized capacity margin. We then investigate how to select the total information rate to maximize effective throughput. Simulation results show significant performance improvement with the proposed algorithm, and the performance gap between layered structure and the channel capacity diminishes with increasing ergodicity within each codeword.**

I. INTRODUCTION

Ever since the revelation of the huge capacity of MIMO systems was shown by information theoretic analysis [1] [2], great effort has been made by numerous researchers to design a practical system that approaches this attractive capacity.

In space-time coding, a single data stream is encoded and mapped to signals to be transmitted via all transmit antennas. In [3], *bit interleaved coded modulation* (BICM) with list sphere decoding and iterative channel decoding has been shown to successfully approach the capacity of MIMO channels for low and medium rate transmission and moderate number of transmit antennas. For the case of a large number of transmit antennas or high order modulation, the performance gap becomes large, due to the limit of list size used in sphere decoding.

In layered systems, such as *vertical Bell Labs layered space-time structure* (V-BLAST) proposed by Foschini [4] [5], the input data stream is demultiplexed, independently coded using 1-dimensional coding, and sent via different transmit antennas simultaneously. The received signal from each substream is separated by nulling according to *zero-forcing* (ZF) or *minimum mean square-error* (MMSE) criterion and *successive interference cancellation* (SIC).

The simplicity of layer processing makes the layered structure a very promising candidate for systems with many transmit antennas and higher order modulation. However, in the original layered system, the input data is evenly divided into substreams and all layers have the same code rate. Due to the loss of signal energy and degree of freedom by nulling, the channel quality for the layers to be first detected frequently can not support reliable transmission with the given layer data rate and those layers are more error-prone. To remedy this problem, optimum detection ordering has been proposed [6]. The idea is to always select the layer with the best channel quality among the remaining layers to be detected first. The drawback of detection ordering is the high computational complexity, especially in frequency-selective channels, such as wide-band *orthogonal-frequency-division-multiplexing* (OFDM) systems, where the channel responses are different for different subcarriers. Another issue is that the channel quality difference between different layers tends to decrease with increasing frequency-selectivity, making the gain of using optimum detection order less significant. Therefore, the original V-BLAST system can only achieve a portion of the system capacity, even with optimal detection ordering.

Based on the observation above, we suggest that fixing the detection order while adapting the data rate for each layer depending on the detection order is a more promising solution for frequency-selective channels. Intuitively, the data rate should increase for later layers corresponding to the increasing channel quality for later detection stages. An interesting fact has been discovered that if we properly select the rate for each layer, the sum of capacities of all layers (with perfect SIC) is exactly the instantaneous open-loop capacity [7] [8].

To achieve the open-loop capacity, instantaneous rate feedback is needed. However, for channels with high frequency selectivity or enough time variation, the variation of layer capacities tend to decrease, as suggested by the law of large numbers. Thus, we can approach the open-loop channel capacity by statistically determining the rate for each layer with small penalty. Our approach is to minimize the overall layer outage probability given the total information rate.

The paper is organized as follows. The system model is briefly introduced in Section II, and the optimum rate allocation is derived in Section III. Section IV investigates the total rate selection for maximum effective throughput. Simulation results are given in Section V and the paper is concluded in Section VI.

II. SYSTEM MODEL

In a flat-fading MIMO system with N_t transmit and N_r receive antennas ($N_t \leq N_r$), the relationship between transmitted and received signals can be expressed as

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n},$$

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where $\mathbf{r}$ is a $N_r \times 1$ received signal vector, $\mathbf{s}$ is a $N_t \times 1$ transmitted signal vector, and $\mathbf{H}$ is a $N_r \times N_t$ channel matrix. The $N_r \times 1$ noise vector $\mathbf{n}$ has entries being independent and identically distributed (i.i.d.) zero-mean circular complex Gaussian random variables with variance N_0. Assume each transmit antenna has the same transmit power, the instantaneous open-loop channel capacity is then [1]

$$C(\mathbf{H},\ SNR) = \log_2 \det \left(\mathbf{I}_{N_r} + \frac{SNR}{N_t} \mathbf{H}\mathbf{H}^H \right),$$

where $\mathbf{I}_{N_r}$ is a $N_r \times N_r$ identity matrix and SNR is the signal-to-noise ratio.

III. Rate Allocation for Layered Systems

In the proposed system, the detection order is fixed and the data rate for each layer is adjusted according to the channel quality and detection order of that particular layer. Without loss of generality assume the order of detection is from transmit antenna 1 to N_t. The instantaneous information rate to be allocated to transmit antenna l is [7] [8] [9]

$$\begin{aligned} C_l &= \log_2 \det \left(\mathbf{I}_{N_r} + \frac{SNR}{N_t} \mathbf{H}_{(l-1)} \mathbf{H}_{(l-1)}^H \right) \\ &\quad - \log_2 \det \left(\mathbf{I}_{N_r} + \frac{SNR}{N_t} \mathbf{H}_{(l)} \mathbf{H}_{(l)}^H \right), \end{aligned}$$

where $\mathbf{H}_{(l)} = [\mathbf{h}_{l+1}\ \mathbf{h}_{l+2}\ \cdots\ \mathbf{h}_{N_t}]$, and $\mathbf{h}_l$ is the lth column of $\mathbf{H}$. It is obvious that $\mathbf{H}_{(0)} = \mathbf{H}$.

It has been observed by various researchers that the distribution of the capacity of a MIMO channel, Rayleigh or Ricean, can be accurately approximated by a Gaussian distribution at medium and high SNR's [10] [11]. Thus the instantaneous capacity of each layer is also Gaussian distributed, denoted as

$$C_l \sim N\left(\eta_l, \sigma_l^2\right),$$

where η_l and σ_l^2 are the mean and variance of the capacity of layer l, respectively.

In order to properly select data rates for different layers, we choose to minimize P_{out}, the outage probability of a layered system, which is the probability when no layers have information rates greater than the respective layer capacities, i.e.,

$$1 - P_{out} = \prod_{l=1}^{N_t} \int_{u_l}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_l} e^{-\frac{(t-\eta_l)^2}{2\sigma_l^2}}\, dt,$$

subject to the constraint that the total information rate is fixed,

$$\sum_{l=1}^{N_t} u_l = C_T.$$

Let

$$u_l = x_l + \eta_l.$$

By setting up the equivalent Lagrangean, we try to find stationaries points, i.e.,

$$\begin{aligned} J &= \ln \left(\prod_{l=1}^{N_t} \int_{x_l}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_l} e^{-\frac{t^2}{2\sigma_l^2}}\, dt \right) \\ &\quad - \lambda \left(\sum_{l=1}^{N_t} x_l + \sum_{l=1}^{N_t} \eta_l - C_T \right). \end{aligned}$$

We can easily verify that the stationary point satisfy

$$-\frac{\frac{1}{\sqrt{2\pi}\sigma_l} e^{-\frac{x_l^2}{2\sigma_l^2}}}{\frac{1}{\sqrt{2\pi}\sigma_l} \int_{x_l}^{\infty} e^{-\frac{t^2}{2\sigma_l^2}}\, dt} = \lambda,\ l = 1,\ 2,\ \cdots,\ N_t.$$

Since we are only interested in the case when the input data rate is set such that reliable transmission is guaranteed most of the time, we assume that the outage probability given the total information rate is small, or equivalently, $x_l < 0$, and $|x_l| \gg \sigma_l$. Thus,

$$\frac{1}{\sqrt{2\pi}\sigma_l} \int_{x_l}^{\infty} e^{-\frac{t^2}{2\sigma_l^2}}\, dt \approx 1,$$

and

$$\frac{x_l^2}{\sigma_l^2} + \ln \sigma_l \approx \frac{x_l^2}{\sigma_l^2},$$

with proper rescaling, since σ_l's have magnitudes of the same order.

We then have

$$x_l^* \approx \frac{\sigma_l}{\sum_{m=1}^{N_t} \sigma_m} \left(C_T - \sum_{m=1}^{N_t} \eta_m \right),$$

and the optimum rate for layer l is

$$u_l^* \approx \eta_l + \frac{\sigma_l}{\sum_{m=1}^{N_t} \sigma_m} \left(C_T - \sum_{m=1}^{N_t} \eta_m \right). \tag{1}$$

Therefore, the outage probability for each layer is

$$\begin{aligned} P_l^* &= \int_{-\infty}^{x_l^*} \frac{1}{\sqrt{2\pi}\sigma_l} e^{-\frac{t^2}{2\sigma_l^2}}\, dt \\ &= \int_{-\infty}^{\frac{C_T - \sum_{m=1}^{N_t} \eta_m}{\sum_{m=1}^{N_t} \sigma_m}} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\, dt, \end{aligned}$$

which is the same for all layers. Thus, the optimum data rate allocation is that the data rate for each layer is backed off from the mean capacity of the layer, and the amount of backoff is proportional to the standard deviation of the layer capacity, which agrees with our intuition that more backoff is needed for the more fluctuant channel. In addition, the minimum overall layer outage probability is achieved when each layer has the same layer outage probability. Define the normalized capacity margin as

$$\varphi \triangleq \frac{\sum_{m=1}^{N_t} \eta_m - C_T}{\sum_{m=1}^{N_t} \sigma_m}.$$

The optimum overall layer outage probability is then

$$\begin{aligned} P_{out}^* &= 1-\prod_{l=1}^{N_t}(1-P_l^*) \\ &= 1-\left(\int_{-\varphi}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt\right)^{N_t}, \end{aligned}$$

which states the interesting fact that the minimum overall layer outage probability of a layered system is uniquely determined by the normalized capacity margin.

Using the asymptotic expansion [12],

$$\int_x^{\infty} e^{-\frac{t^2}{2}}dt \approx \frac{e^{-\frac{x^2}{2}}}{x}\left(1-\frac{1}{x^2}+\frac{1\cdot 3}{(x^2)^2}\cdots\right),\ x\gg 0, \quad (2)$$

then

$$P_{out}^* \approx \frac{N_t}{\sqrt{2\pi}\varphi}e^{-\varphi^2/2}.$$

Similarly, we can derive the asymptotic outage probability of the MIMO channel with the same overall information rate C_T as

$$P_{ch} \approx \frac{1}{\sqrt{2\pi}\varphi_{ch}}e^{-\varphi_{ch}^2/2},$$

where

$$\varphi_{ch} = \frac{\eta_{ch}-C_T}{\sigma_{ch}},$$

η_{ch} is the ergodic MIMO channel capacity, and σ_{ch}^2 is the variance of the MIMO channel capacity. Note that

$$\eta_{ch} = \sum_{l=1}^{N_t}\eta_l,$$

and

$$\sigma_{ch} \le \sum_{l=1}^{N_t}\sigma_l,$$

since

$$E\left\{\left(\sum_l v_l\right)^2\right\} \le \left(\sum_l\sqrt{E\left\{v_l^2\right\}}\right)^2,$$

for any set of random variables $\{v_l\text{'s}\}$.

Thus,

$$\varphi_{ch} \ge \varphi,$$

and

$$P_{out} \ge P_{out}^* \approx \frac{N_t}{\sqrt{2\pi}\varphi}e^{-\varphi^2/2} \ge N_t\frac{1}{\sqrt{2\pi}\varphi_{ch}}e^{-\varphi_{ch}^2/2} \approx N_t P_{ch},$$

which implies that with the same overall information rate, the asymptotic outage probability of layered structure is at least N_t times that of the MIMO channel. However, the low complexity of layered structure makes it still a good candidate for high-speed wireless systems.

IV. Rate Selection for Maximum Effective Throughput

In the previous section, we have introduced our algorithm of optimum statistical data rate allocation for a given total data rate. In practice, we want to use the channel to transmit as much information as possible. If the total data rate is too low, the total successfully transmitted information is limited by the total amount of information during each transmission even though each transmission is correctly detected. On the other hand, if the total data rate is set too high, almost every transmission is lost because of the high outage probability. Therefore, we expect a best total data rate at each SNR that maximizes the effective throughput. In this section, we will specify the condition that the best total data rate satisfies with Gaussian approximation of layer capacities.

$$C_{eff} = C_T*(1-P_{out}) = C_T\left(\int_{\frac{C_T-\sum_{m=1}^{N_t}\eta_m}{\sum_{m=1}^{N_t}\sigma_m}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt\right)^{N_t}.$$

$$\frac{\partial}{\partial C_{eff}} = \frac{\partial}{\partial C_T}\left\{C_T\left(\int_{\frac{C_T-\sum_{m=1}^{N_t}\eta_m}{\sum_{m=1}^{N_t}\sigma_m}}^{\infty}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt\right)^{N_t}\right\} = 0.$$

It is easy to verify that the stationary point satisfies

$$\int_{\frac{C_T-\sum_{m=1}^{N_t}\eta_m}{\sum_{m=1}^{N_t}\sigma_m}}^{\infty}e^{-\frac{t^2}{2}}dt - \frac{N_t}{\sum_{m=1}^{N_t}\sigma_m}e^{-\frac{\left(\frac{C_T-\sum_{m=1}^{N_t}\eta_m}{\sum_{m=1}^{N_t}\sigma_m}\right)^2}{2}} = 0, \quad (3)$$

which is not easy to solve. Instead, we try to derive an asymptotic solution as $\sum_{m=1}^{N_t}\sigma_m$ tends to zero. Using the same expansion as in Equation (2), Equation (3) becomes

$$\begin{aligned} &\int_{\frac{C_T-\sum_{m=1}^{N_t}\eta_m}{\sum_{m=1}^{N_t}\sigma_m}}^{\infty}e^{-\frac{t^2}{2}}dt - \frac{N_t}{\sum_{m=1}^{N_t}\sigma_m}e^{-\frac{\left(\frac{C_T-\sum_{m=1}^{N_t}\eta_m}{\sum_{m=1}^{N_t}\sigma_m}\right)^2}{2}} \\ \approx&\sqrt{2\pi}+e^{-\frac{\left(\frac{C_T-\sum_{m=1}^{N_t}\eta_m}{\sum_{m=1}^{N_t}\sigma_m}\right)^2}{2}}\left\{\frac{\sum_{m=1}^{N_t}\sigma_m}{C_T-\sum_{m=1}^{N_t}\eta_m}-\frac{N_t}{\sum_{m=1}^{N_t}\sigma_m}\right\} \\ \approx&\sqrt{2\pi}-e^{-\frac{\left(\frac{C_T-\sum_{m=1}^{N_t}\eta_m}{\sum_{m=1}^{N_t}\sigma_m}\right)^2}{2}}\frac{N_t}{\sum_{m=1}^{N_t}\sigma_m} \\ =&0. \end{aligned}$$

Therefore,

$$C_T^* \approx \sum_{m=1}^{N_t}\eta_m - \sum_{m=1}^{N_t}\sigma_m\sqrt{2\ln\frac{N_t}{\sqrt{2\pi}\sum_{m=1}^{N_t}\sigma_m}}. \quad (4)$$

To summarize, the rate adaption of the proposed system is in two steps. First, the mean and standard deviation of layer capacities are used to determine the total information rate of the system by Equation (4). Then the data rate of each layer is determined by Equation (1), depending on the channel statistics and detection order of the layer. Note that some SNR margin may be necessary due to the fact that the coding in practical systems is not capacity-approaching.

V. Simulation Results

First, we use the outage probability to evaluate the performance gap between layered structure and the MIMO channel. We compare the outage probability of a 2x2 OFDM system with layered structure with the channel outage probability, which may serve as lower bounds of *word-error-rate* (WER) of practical systems. The OFDM symbol structure conforms to the IEEE 802.11a PHY standard in the 5 GHz band. IEEE 802.11 TGn channel models 'B' and 'D' generated by the Matlab program available at [13] are used. Figures 1 and 2 give the outage probability comparison for channels 'B' and 'D', respectively. Channel 'B' has a shorter delay spread than channel 'D', thus has less frequency-selectivity. The outage probabilities of equal-rate V-BLAST systems are also shown. The total information rate per subcarrier is 9 bits, and the rate allocation is optimized for SNR=24 dB.

It can be seen that for an outage probability of 1%, the gap between outage probability of the channel and that of the proposed system reduces from 3.6 dB for channel 'B' to 1.5 dB for channel 'D'. It can be explained by the fact that increased frequency-selectivity reduces the variation of each layer capacity. The SNR improvement for an outage probability of 1% is 6.3dB and 3.6dB for channel 'B' and 'D', respectively. From Figure 2, it is obvious that the outage probability with optimum rate allocation levels off at higher SNR's. The reason is that the rate allocation is optimized for a particular SNR, and the optimum rates are not the same for all SNR's. As SNR varies, some of the layers become a bottleneck.

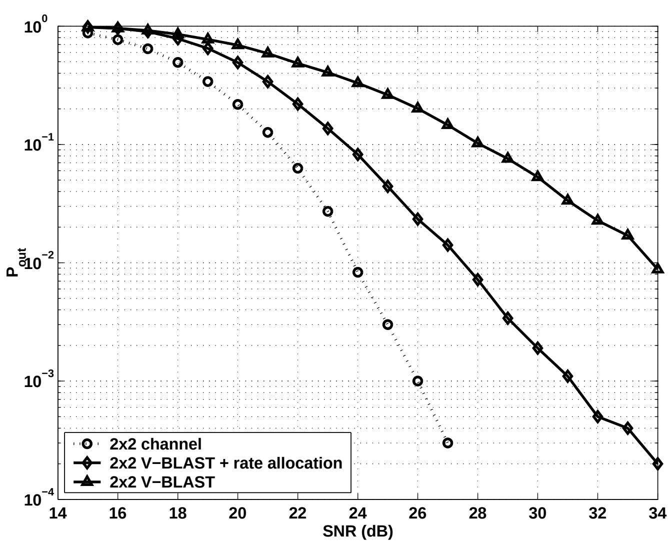

Fig. 1. Outage probability of a 2x2 layered system with statistical rate allocation, channel model 'B'.

Next, to demonstrate the performance of rate allocation, simulation is carried out for a 2x2 layered system with statistical rate allocation. Comparison is made between the proposed system and the conventional system with equal rate allocation, given the same or similar total input information rate. Perfect channel estimation and synchronization is assumed. 1000B data in total is transmitted in each packet and 10,000 packets are sent for each SNR. A 2x2 conventional layered system with equal rate allocation is provided for comparison. In the conventional system, each substeam is coded using rate 3/4 industry standard convolutional coding, interleaved, and modulated using 64QAM constellation. Two substreams are then sent simultaneously, resulting in a total data rate of 108 Mbps.

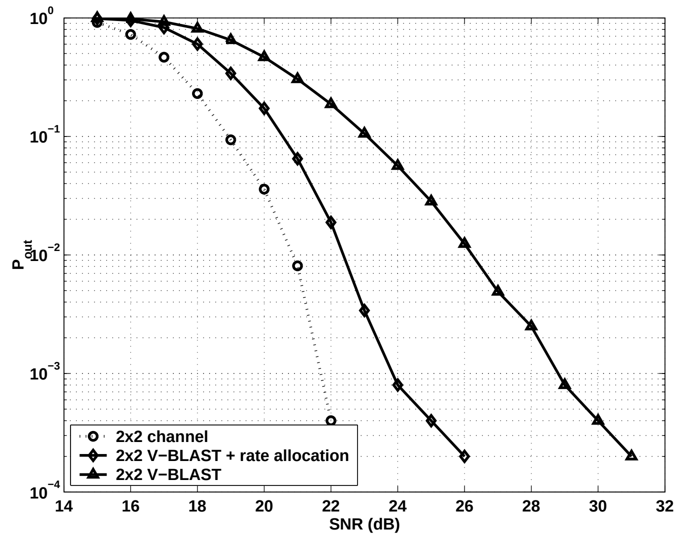

Fig. 2. Outage probability of a 2x2 layered system with statistical rate allocation, channel model 'D'.

Due to the limited number of supported data rates in a practical system, only BPSK, QPSK, 16QAM, and 64QAM with convolutional coding of code rate 1/2, 2/3, 3/4, and 7/8 may be used for transmission of each layer. Therefore, the optimum data rates has to be quantized to the closest supported transmission modes. The nominal total information rate as input to the rate allocation algorithm is set such that the resultant total information rate after rate allocation is as close to that of the conventional system as possible to make the comparison reasonable. The rate allocation is optimized for the lowest SNR such that the optimum overall layer outage probability is below 1%. The optimized rates are then shown in Table I and Table II, for channel 'B' and 'D', respectively.

TABLE I
Transmission modes of different layers in a 2x2 system with statistical rate allocation, channel model 'B'.

Detection order	Modulation	Code rate	Layer data rate (Mbps)
1	16QAM	3/4	36
2	64QAM	7/8	63

Therefore, the total data rate is 99 Mbps and 111 Mbps,

TABLE II

TRANSMISSION MODES OF DIFFERENT LAYERS IN A 2X2 SYSTEM WITH STATISTICAL RATE ALLOCATION, CHANNEL MODEL 'D'.

Detection order	Modulation	Code rate	Layer data rate (Mbps)
1	64QAM	2/3	48
2	64QAM	7/8	63

respectively, as compared to the 108 Mbps of the conventional layered system.

In Figure 3, the WER's of the proposed system and the conventional layered system are shown. For a WER of 10%, the proposed system provides an SNR gain of about 5 dB and 3 dB for channel model 'B' and 'D', respectively.

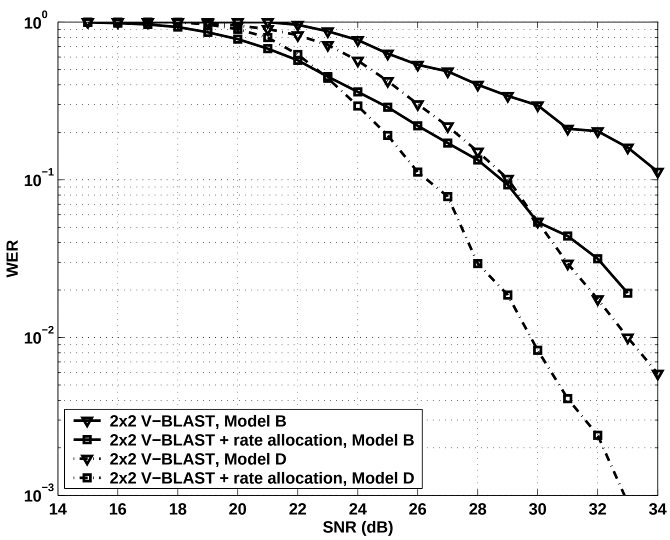

Fig. 3. WER of a 2x2 layered system with statistical rate allocation, 1000B packet, channel model 'B' and 'D'.

The effective throughput of the same system is shown in Figure 4. It is seen that with the same SNR, the proposed system yields a maximum increase of about 38 Mbps and 32 Mbps in effective throughput, for channel 'B' and 'D', respectively, while maintaining similar peak data rate.

VI. CONCLUSION

In this paper, we propose a modified layered structure, where the detection order is fixed and the data rate for each layer is determined by the detection order and channel statistics. With Gaussian approximation of layer capacities, optimum data rate allocation algorithm and total information rate selection for maximum effective throughput are derived. Simulation results show significant performance improvement over the original V-BLAST structure and the system performance improves with increasing ergodicity within each codeword, making it promising for frequency-selective channels.

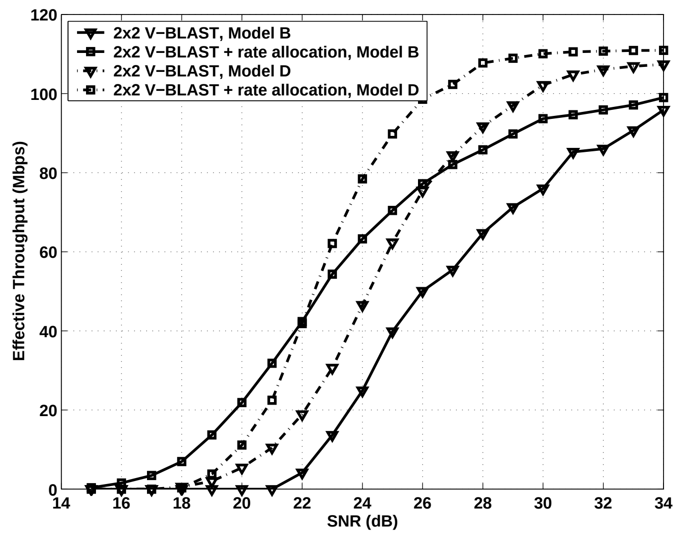

Fig. 4. Effective throughput of a 2x2 layered system with statistical rate allocation, 1000B packet, channel model 'B' and 'D'.

REFERENCES

[1] E. Telatar, "Capacity of multi-antenna Gaussian channels," *European Transactions on Telecommunications*, vol. 10, pp. 585–595, Nov-Dec 1999.

[2] G. J. Foschini and M. J. Gans, "On the limits of wireless communications in a fading environment when using multiple antennas," *Wireless Personal Commun.*, vol. 6, pp. 315–335, Mar. 1998.

[3] B. M. Hochwald and S. ten Brink, "Achieving near-capacity on a multiple-antenna channel," *IEEE Trans. Commun.*, vol. 51, pp. 389–399, Mar. 2003.

[4] P. W. Wolniansky, G. J. Foschini, G. D. Golden, and R. A. Valenzuela, "V-BLAST: An architecture for realizing very high data rates over the rich-scattering wireless channel," in *Proc. URSI Int. Symp. Signals, Systems, and Electronics*, Pisa, Italy, Oct. 1998, pp. 295–300.

[5] H. E. Gamal and J. A. R. Hammons, "A new approach to layered space-time coding and signal processing," *IEEE Trans. Inform. Theory*, vol. 47, pp. 2321–2334, Sept. 2001.

[6] G. D. Golden, C. J. Foschini, R. A. Valenzuela, and P. W. Wolniansky, "Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture," *IEEE Electr. Lett.*, vol. 35, pp. 14–16, Jan. 1999.

[7] M. K. Varanasi and T. Guess, "Optimum decision feedback multiuser equalization with successive decoding achieves the total capacity of the Gaussian multiple-access channel," in *IEEE Asilomar Conference on Signals, Systems, and Computers*, Nov. 1997, pp. 1405–1409.

[8] S. T. Chung, A. Lozano, and H. Huang, "Approaching eigenmode BLAST channel capacity using V-BLAST with rate and power feedback," in *Proc. VTC 2001*, Oct. 2001, pp. 915–919.

[9] A. Lozano, "Capacity-approaching rate function for layered multi-antenna architectures," *IEEE Trans. Wireless Commun.*, vol. 2, pp. 616–620, July 2003.

[10] P. J. Smith and M. Shafi, "On a Gaussian approximation to the capacity of wireless MIMO systems," in *Proc. ICC 2002*, Apr. 2002, pp. 406–410.

[11] M. A. Kamath, B. L. Hughes, and X. Yu, "Gaussian approximations for the capacity of MIMO Rayleigh fading channels," in *IEEE Asilomar Conference on Signals, Systems, and Computers*, Nov. 2002, pp. 614–618.

[12] J. G. Proakis, *Digital Communications*. McGraw Hill, 1989.

[13] L. Schumacher, "Matlab implementation of the IEEE 802.11 HTSG channel model special committee proposal." [Online]. Available: http://www.info.fundp.ac.be/∼lsc/Research/IEEE_80211_HTSG_CMSC/distribution_terms.html