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On the Temporal Evolution of Signal Subspaces in Vehicular MIMO Channels in the 5 GHz Band

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Abstract

We study the temporal evolution of the singular value decomposition of vehicular 4 X 4 MIMO channel measurements H(t)in the 5.2 GHz band for a selected single OFDM sub-carrier. First, we estimate the number of relevant singular values p(t), 0 less-than p(t) less-than 4 for each sampling time t by applying the minimum descriptive length (MDL) criterion. The MDL criterion is used here as an estimate for the time-variant dimension p(t) of the dominant "signal" subspace. Then we characterize the dynamics of the estimated dominant singular subspaces by the principal angles between the corresponding subspaces at times t and t + delta where delta is the channel sounder's MIMO channel acquisition period which defines the resolution in the Doppler domain. These principle angles are closely related to covariances and measures of dependency. When principal angles are evaluated for pairs of column spaces of matrices, the principal angles describe canonical correlations of a matrix pair. Ultimately, we wish to describe the evolution of the dominant singular vectors as trajectories in the (complex) Stiefel manifold Vp(t),4 and the corresponding evolution of the dominanant subspaces as trajectories in the (complex) Grassmann manifold Gp(t),4.

COST 2100 Meeting 2008

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On the Temporal Evolution of Signal Subspaces in Vehicular MIMO Channels in the 5 GHz Band

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Parts of this TD will be presented at the Joint Workshop on Coding and Communications (JWCC 2008) on October 26–28, 2008, [10].

Abstract

We study the temporal evolution of the singular value decomposition of vehicular 4×4 MIMO channel measurements $\boldsymbol{H}(t)$ in the 5.2 GHz band for a selected single OFDM sub-carrier. First, we estimate the number of relevant singular values $\hat{p}(t)$, $0 \leq \hat{p}(t) \leq 4$ for each sampling time t by applying the minimum descriptive length (MDL) criterion. The MDL criterion is used here as an estimator for the time-variant dimension $p(t)$ of the dominant "signal" subspace.

Then we characterize the dynamics of the estimated dominant singular subspaces by the *principal angles* between the corresponding subspaces at times t and $t + \Delta$ where Δ is the channel sounder's MIMO channel acquisition period which defines the resolution in the Doppler domain. These principal angles are closely related to covariances and measures of dependency. When principal angles are evaluated for pairs of column spaces of matrices, the principal angles describe canonical correlations of a matrix pair.

Ultimately, we wish to describe the evolution of the dominant singular vectors as trajectories in the (complex) Stiefel manifold $V_{p(t),4}$ and the corresponding evolution of the dominanant subspaces as trajectories in the (complex) Grassmann manifold $G_{p(t),4}$.

1 Introduction

Principle angles between subspaces have an important interpretation as covariances and measures of dependency of random vectors [1]. We wish to characterise the temporal evolution of the dominant MIMO channel's left and right singular subspaces by these principal angles. In this contribution, we estimate the principal angles between the dominant MIMO channel subspaces at time t and $t + \Delta$ from real-world time-variant vehicular measurements and discuss their evolution over time.

The dynamics of time-variant subspaces present a challenge for subspace tracking algorithms which are important for MIMO transmission over time-variant channels.

The underlying geometry has raised interest in the mathematical community. Smith explored the geometry of the Stiefel manifold in the context of subspace tracking and optimization problems [2]. Edelman, Arias, and Smith defined a taxonomy of optimization algorithms starting from Newton's method on Riemannian manifolds [3]. One can then restrict to a particular manifold such as the Stiefel or Grassmann manifolds. Abrudan et al. introduced Riemannian optimization algorithms on the Lie group of unitary matrices [4]. Such optimization algorithms inherently maintain a unitary constraint in each iteration.

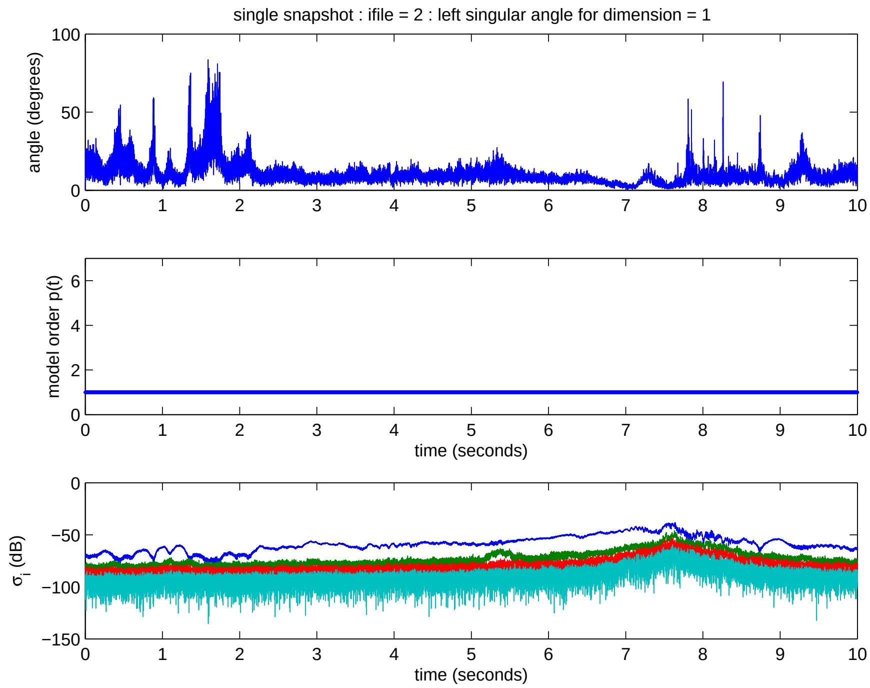

Figure 1: Highway opposite directions: Principle angle between left singular subspaces of dimension 1 at times t and $t + \Delta$ at 5.2 GHz.

2 Vehicular MIMO Measurement

2.1 Channel Sounder

The measurements were carried out with the RUSK LUND channel sounder. Measurement vehicles are two VW LT35 transporters. We have acquired vehicular 4×4 MIMO channel measurements at 5.2 GHz with 240 MHz of bandwidth. For characterising the time-variance of the MIMO channel, we chose an unusually short MIMO channel acquisition period $\Delta \approx 307.2\,\mu$s allowing for a maximum one-sided Doppler shift of 1625 Hz, i.e. a Doppler bandwidth of 3250 Hz. A detailed description of the measurement instrumentation and vehicular scenarios is documented in Refs. [5, 6, 7].

3 Results

Results for the first principle angle

$$\alpha(t) = \min_{\boldsymbol{u}_1 \in \mathcal{U}_t, \boldsymbol{u}_2 \in \mathcal{U}_{t+\Delta}} \arccos |\boldsymbol{u}_1^H \boldsymbol{u}_2| \qquad \text{subject to } |\boldsymbol{u}_1| = 1, |\boldsymbol{u}_2| = 1. \tag{1}$$

between the left singular subspaces $\mathcal{U}_t$ and $\mathcal{U}_{t+\Delta}$ are given in Figures 1, 2, and 3.

4 Outlook

In a later investigation, we will apply the eigensubspace untangling approach [8, 9].

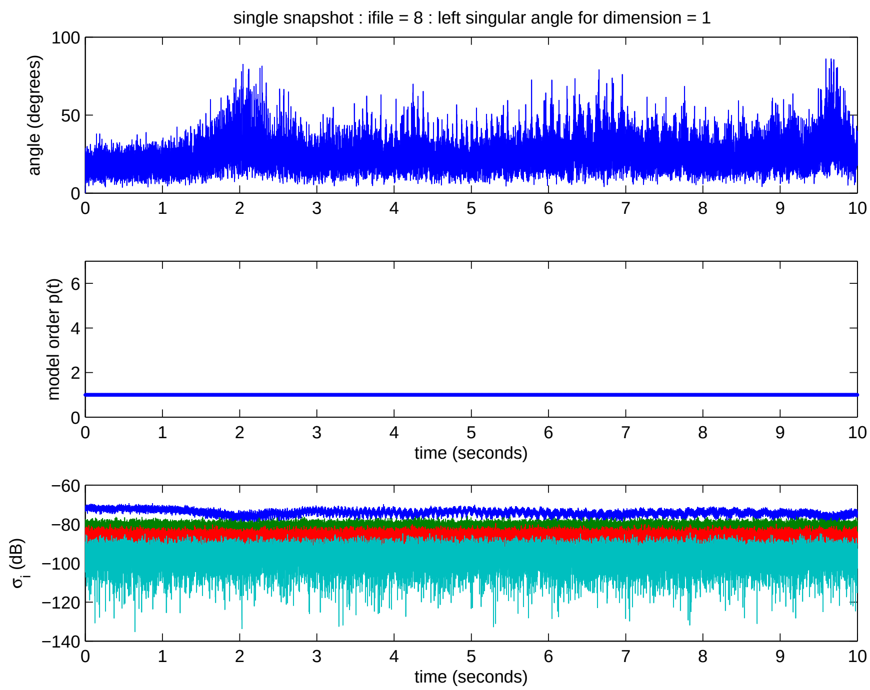

Figure 2: Highway same direction: Principle angle between left singular subspaces of dimension 1 at times t and $t + \Delta$ at 5.2 GHz.

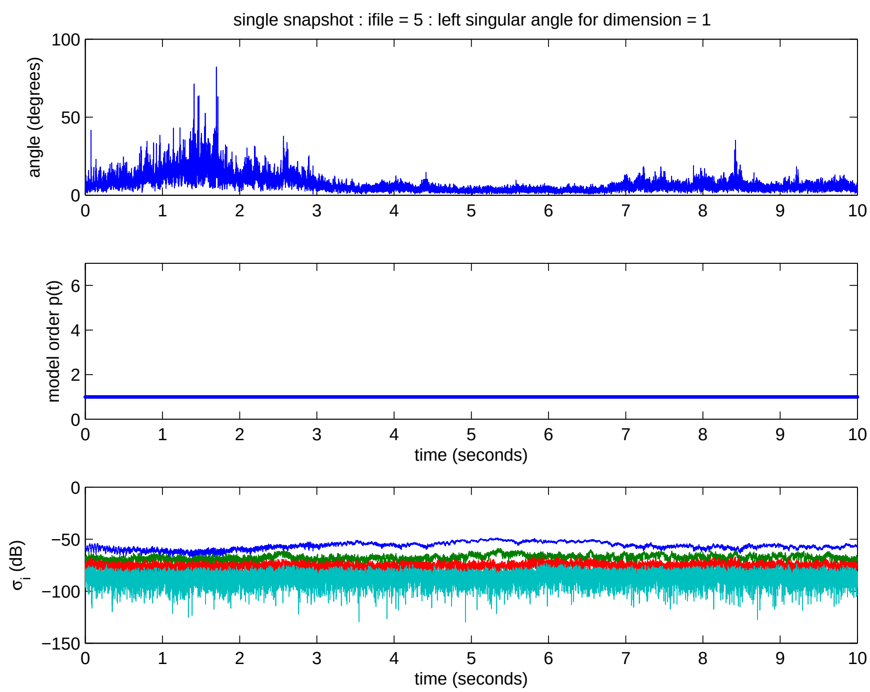

Figure 3: Urban same direction: Principle angle between left singular subspaces of dimension 1 at times t and $t + \Delta$ at 5.2 GHz.

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