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Static Power Allocation in Two-Hop MIMO Amplify-and-Forward Relay Systems

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***Abstract*—In this paper, we propose a static power allocation algorithm for a two-hop *multi-input-multi-output* (MIMO) *amplify-and-forward* (AF) relay system in which the interim channel state information over the first and the second hops is unavailable. Based on the path losses over the first and the second hops, this algorithm performs static power allocation between the source and relay nodes to maximize the equivalent received SNR of the system. We further investigate the optimal location of the relay node when the conventional fixed and the proposed optimal static power allocation schemes are applied. Our comparison between direct transmission and relay-based two-hop transmission indicates that whether the latter outperforms the former depends on a tradeoff between the received SNR gain and the multiplexing loss in the relay-based two-hop transmission scheme.**

I. Introduction

Relay-based cooperative communication has become a hot topic in wireless communication. A typical single-user relay-based cooperative communication system consists of a *source node* (SN), one or more *relay nodes* (RN's), and a *destination node* (DN). As a repeater, the RN relays the signal from the SN to the DN after appropriate processing. It has been demonstrated that relay techniques can increase the communication coverage, decrease the overall transmit power, and enhance the capacity or reliability of the communication links [1]- [3].

Depending on how much signal processing is performed at the RN, the existing relay techniques can be broadly categorized as *decode-and-forward* (DF) and *amplify-and-forward* (AF) [1]. In the DF scheme, the RN detects and demodulates its received signals, decodes the encoded data, re-modulates the data, and forwards them to the DN. In contrast, the RN operating in AF mode only amplifies and forwards its received signals without any further processing and hence has much simpler implementation than that in DF mode.

Depending on the implementation complexity permitted and the channel information available, appropriate signal processing can be performed at the RN in AF mode to improve the system performance. For a two-hop *multi-input-multi-output* (MIMO) AF relay system, the optimal amplifying matrix at the RN and dynamic power allocation between the SN and the RN that maximize the instantaneous capacity of the system have been investigated in [4] and [5]. While this optimal dynamic power allocation algorithm optimizes the system performance, it is based on the instantaneous interim channels over the SN-RN and the RN-DN hops and needs to continuously vary the amplifying matrix at the RN accordingly. However, in a practical two-hop MIMO AF relay system with minimum-complexity RN's, the channels over the two interim hops are unknown and each RN has a fixed amplifying matrix. In this paper, we propose a static power allocation algorithm especially suitable for such a practical two-hop MIMO AF relay system. This algorithm performs power allocation between the SN and the RN based on the static path loss information over the SN-RN and the RN-DN hops and therefore has little complexity. Mathematically, this static power allocation algorithm is designed to maximize the equivalent received SNR of the two-hop MIMO AF relay system. Based on a simple but yet representative path loss assumption, we further analyze the optimal location of the RN when the optimal and the fixed static power allocation schemes are applied. Furthermore, our comparison between direct transmission and relay-based two-hop transmission indicates that whether the latter outperforms the former depends on the location of the RN and the available total transmit power, which reflects a tradeoff between the received SNR gain and the multiplexing loss in the relay-based two-hop transmission scheme.

The rest of this paper is organized as follows. The two-hop MIMO AF relay system is described in Section II. In Section III, we develop a new static power allocation algorithm for this system. Then we analyze the optimal location of the RN and present numerical results in Sections IV and V, respectively. Finally Section VI concludes this paper.

II. System Model

In this paper, we are concerned with a two-hop MIMO AF relay system consisting of an SN, multiple RN's, and a DN. In this system, there is no direct communication link between the SN and the DN and data are conveyed from source to destination via two orthogonal channels by time-division or frequency-division. Since the application of multiple RN's effectively enhances the reliability of the communication link between the SN and the DN, such a system can be deployed to realize long-range communication in a wireless cellular or sensor network. As will be demonstrated in this paper, there exists an optimal location for an RN with a given average transmit power. In practice, RN's usually have the same transmit power and hence have the same optimal location as well. Considering this, we assume that different RN's in this system have the same path losses from the SN and to the DN. Consequently, multiple RN's are equivalently regarded as one composite RN for notational convenience in this paper.

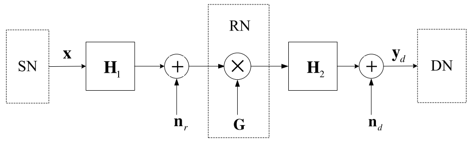


Fig. 1. Two-hop MIMO AF relay system model

Figure 1 shows a block diagram of the two-hop MIMO AF relay system. Suppose there are N_s transmit antennas at the SN, N_r receive and transmit antenna pairs at the composite RN, and N_d receive antennas at the DN. Denote $\mathbf{H}_1$, $\mathbf{G}$, and $\mathbf{H}_2$ to be the $N_r \times N_s$ channel matrix between the SN and the RN, the $N_r \times N_r$ amplifying matrix at the RN, and the $N_d \times N_r$ channel matrix between the RN and the DN, respectively, then the received signal vector at the DN is given by

$$\mathbf{y}_d = \mathbf{H}_2\mathbf{G}\mathbf{H}_1\mathbf{x} + \mathbf{H}_2\mathbf{G}\mathbf{n}_r + \mathbf{n}_d = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{x}$ denotes the transmitted signal vector of the SN, $\mathbf{n}_r$ and $\mathbf{n}_d$ denote the local white noise vectors at the RN and the DN, respectively, $\mathbf{H} = \mathbf{H}_2\mathbf{G}\mathbf{H}_1$ denotes the $N_d \times N_s$ overall channel matrix between the SN and the DN, and $\mathbf{n} = \mathbf{H}_2\mathbf{G}\mathbf{n}_r + \mathbf{n}_d$ denotes the overall noise vector at the DN.

To maintain a constant power amplifying gain at the RN, we assume $\mathbf{G} = g\mathbf{P}$ throughout this paper, where g is a scalar amplifying gain of the RN and $\mathbf{P}$ is a unitary matrix. Assume that the elements of $\mathbf{n}_r$ and $\mathbf{n}_d$ are *identically and independently distributed* (i.i.d.) complex Gaussian random variables with zero mean and variances σ_r^2 and σ_d^2, respectively, then the correlation matrix of the overall noise vector is given by

$$\mathbf{R}_n = E\left\{\mathbf{n}\mathbf{n}^H\right\} = g^2\sigma_r^2\mathbf{H}_2\mathbf{H}_2^H + \sigma_d^2\mathbf{I}_{N_d}, \tag{2}$$

where $\mathbf{I}_{N_d}$ denotes the $N_d \times N_d$ identity matrix.

III. Static Power Allocation between SN and RN

In the two-hop MIMO AF relay system, both the SN and the RN work as transmitters. Thus a natural question is how to allocate transmit power between the SN and the RN so that the system performance is optimized for a given overall transmit power. In this section, we will investigate low-complexity static power allocation between the SN and the RN based on the path losses over the SN-RN and the RN-DN hops.

A. Principle of Static Power Allocation Algorithm

Suppose that the elements of the transmitted signal vector, $\mathbf{x}$, are independent and have the same average power, P_x, i.e., the correlation matrix of $\mathbf{x}$ is $\mathbf{R}_x = P_x\mathbf{I}_{N_s}$ where $\mathbf{I}_{N_s}$ denotes the $N_s \times N_s$ identity matrix. Then the instantaneous capacity of the two-hop MIMO AF relay system is given by [6], [7]

$$\begin{aligned} C(\mathbf{H}_1, \mathbf{H}_2) &= \frac{1}{2}\log_2\left|\mathbf{I}_{N_d} + \mathbf{H}\mathbf{R}_x\mathbf{H}^H\mathbf{R}_n^{-1}\right| \\ &= \frac{1}{2}\log_2\Big|\mathbf{I}_{N_d} + g^2P_x\mathbf{H}_2\mathbf{P}\mathbf{H}_1\mathbf{H}_1^H\mathbf{P}^H\mathbf{H}_2^H \\ &\quad \cdot\left(g^2\sigma_r^2\mathbf{H}_2\mathbf{H}_2^H + \sigma_d^2\mathbf{I}_{N_d}\right)^{-1}\Big|. \end{aligned} \tag{3}$$

where the factor $\frac{1}{2}$ is due to the multiplexing loss caused by two channel uses for each data transmission.

While the dynamic power allocation algorithm proposed in [4] and [5] requires interim channel state information over the SN-RN and the RN-DN hops and maximizes the instantaneous capacity of the system, $C(\mathbf{H}_1, \mathbf{H}_2)$, our proposed static power allocation algorithm does not need this information and maximizes the average capacity of the system,

$$\overline{C}(\sigma_1^2, \sigma_2^2, P_x, g^2) = E_{\mathbf{H}_1,\mathbf{H}_2}\left\{C(\mathbf{H}_1, \mathbf{H}_2)\right\}, \tag{4}$$

where σ_i^2 is the average power of the elements of $\mathbf{H}_i$, $1 \le i \le 2$*, and $E_{\mathbf{H}_1,\mathbf{H}_2}\{\cdot\}$ denotes expectation *with respect to* (w.r.t.) $\mathbf{H}_1$ and $\mathbf{H}_2$. From (4), the optimal P_x and g^2 that maximize $\overline{C}(\sigma_1^2, \sigma_2^2, P_x, g^2)$ for a given average total transmit power can be obtained, based on which the optimal static power allocation between the SN and the RN can be performed.

Since it is rather difficult to obtain a closed-form expression of $\overline{C}(\sigma_1^2, \sigma_2^2, P_x, g^2)$, we try to find an approximate average capacity of the system and perform static power allocation between the SN and the RN to maximize it. Specifically, such an approximation is based on the assumption that the total number of antennas at multiple RN's, N_r, is much larger than the number of antennas at the DN, N_d. Since $N_r \gg N_d$, the row vectors of $\mathbf{H}_2$ are approximately orthogonal according to *the law of large numbers* [8], i.e., $\mathbf{H}_2\mathbf{H}_2^H \approx N_r\sigma_2^2\mathbf{I}_{N_d}$, and thus $C(\mathbf{H}_1, \mathbf{H}_2)$ in (3) can be approximated by

$$\widehat{C}(\mathbf{H}_{eqv}) = \frac{1}{2}\log_2\left|\mathbf{I}_{N_d} + \frac{\gamma_{eqv}}{N_s}\mathbf{H}_{eqv}\mathbf{H}_{eqv}^H\right|, \tag{5}$$

where $\mathbf{H}_{eqv} = \frac{1}{\sqrt{N_r}\sigma_1\sigma_2}\mathbf{H}_2\mathbf{P}\mathbf{H}_1$ denotes the normalized equivalent channel matrix between the SN and the DN, and

$$\gamma_{eqv} = \frac{N_sN_r\sigma_1^2\sigma_2^2g^2P_x}{\sigma_d^2 + N_rg^2\sigma_2^2\sigma_r^2} \tag{6}$$

denotes the equivalent received SNR at the DN. Accordingly, $\overline{C}(\sigma_1^2, \sigma_2^2, P_x, g^2)$ can be approximated by

$$\widehat{C}(\sigma_1^2, \sigma_2^2, P_x, g^2) = E_{\mathbf{H}_{eqv}}\left\{\widehat{C}(\mathbf{H}_{eqv})\right\} = f_{\widehat{C}}\left(\gamma_{eqv}\right), \tag{7}$$

where $E_{\mathbf{H}_{eqv}}\{\cdot\}$ denotes expectation w.r.t. $\mathbf{H}_{eqv}$ and $f_{\widehat{C}}\left(\gamma_{eqv}\right)$ denotes an increasing function of γ_{eqv}. Since the approximate average capacity of the system, $\widehat{C}(\sigma_1^2, \sigma_2^2, P_x, g^2)$, increases monotonically with the equivalent received SNR, γ_{eqv}, the optimal P_x and g^2 that maximize γ_{eqv} also maximize $\widehat{C}(\sigma_1^2, \sigma_2^2, P_x, g^2)$. Thus optimal static power allocation between the SN and the RN can be found by maximizing γ_{eqv}.

B. Static Power Allocation Between SN and RN

Denote $\mathbf{x}_r = g\mathbf{P}\left(\mathbf{H}_1\mathbf{x} + \mathbf{n}_r\right)$ as the transmitted signal vector of the RN, then its correlation matrix is given by

$$\mathbf{R}_{x_r}(\mathbf{H}_1) = E\left\{\mathbf{x}_r\mathbf{x}_r^H\right\} = g^2\mathbf{P}\left(P_x\mathbf{H}_1\mathbf{H}_1^H + \sigma_r^2\mathbf{I}_{N_r}\right)\mathbf{P}^H,$$

and the average total transmit power of the overall system is

$$\begin{aligned} P &= \mathrm{Tr}(\mathbf{R}_x) + E_{\mathbf{H}_1}\left\{\mathrm{Tr}[\mathbf{R}_{x_r}(\mathbf{H}_1)]\right\} \\ &= N_sP_x + N_rg^2\left(\sigma_r^2 + N_sP_x\sigma_1^2\right) \\ &= P_s + P_r, \end{aligned} \tag{8}$$

*Here we ignore shadowing and thus $-10\lg\left(\sigma_1^2\right)$ and $-10\lg\left(\sigma_2^2\right)$ represent the path losses over the SN-RN and the RN-DN hops, respectively.

where Tr$(\cdot)$ denotes the trace of a matrix, $E_{\mathbf{H}_1}\{\cdot\}$ denotes expectation w.r.t. $\mathbf{H}_1$, and $P_s = N_s P_x$ and $P_r = N_r g^2\left(\sigma_r^2 + N_s P_x \sigma_1^2\right)$ denote the transmit powers of the SN and the RN, respectively.

Denote $a = \frac{\sigma_1^2}{\sigma_r^2}$ and $b = \frac{\sigma_2^2}{\sigma_d^2}$ as the average power gains of the first and the second hops normalized to the noise power, respectively, and substitute P_s and P_r in (6), then the equivalent received SNR can be rewritten as

$$\gamma_{eqv} = \frac{P_s a \cdot P_r b}{1 + P_s a + P_r b}. \tag{9}$$

The optimal static power allocation between the SN and the RN can be obtained by maximizing γ_{eqv} subject to $P_s + P_r \le P$ and $P_s, P_r \ge 0$. The solutions of this optimization problem, P_s^* and P_r^*, can be obtained as

$$P_s^* = \begin{cases} \frac{\sqrt{(1+aP)(1+bP)}-(1+bP)}{a-b}, & a \ne b, \\ \frac{1}{2}P, & a = b, \end{cases} \tag{10}$$

and

$$P_r^* = \begin{cases} \frac{\sqrt{(1+bP)(1+aP)}-(1+aP)}{b-a}, & a \ne b, \\ \frac{1}{2}P, & a = b, \end{cases} \tag{11}$$

respectively, and the corresponding maximum equivalent received SNR is given by

$$\gamma_{eqv}^* = \frac{P_s^* a \cdot P_r^* b}{1 + P_s^* a + P_r^* b} = ab\left(\frac{\sqrt{1+aP} - \sqrt{1+bP}}{a-b}\right)^2. \tag{12}$$

According to (10) and (11), $\frac{P_r^*}{P_s^*} = \sqrt{\frac{1+aP}{1+bP}}$, which means if $a > b$, $P_r^* > P_s^*$; otherwise, $P_r^* \le P_s^*$, where the equality holds if and only if $a = b$. In other words, the link with a larger normalized path loss will be allocated more power, which is reasonable since the overall performance of the two-hop MIMO AF relay system is restricted by the worse link.

IV. Optimal Location of RN

Obviously the location of the RN affects the performance of the two-hop MIMO AF relay system. Therefore, the RN should be appropriately located to optimize the system performance if we have such a flexibility. In this section, we will find the optimal location of the RN that maximizes the equivalent received SNR for a given average total transmit power. According to (7), such location also maximizes the approximate average capacity of the system.

A. Path Loss Environment

Figure 2 shows a simplified path loss environment of the two-hop MIMO AF relay system. To facilitate analysis, we ignore shadowing and assume that the SN, the RN, and the DN lie on one line. The distance between the SN and the DN is assumed to be unit and d represents the normalized distance between the SN and the RN. In Figure 2, σ^2 denotes the average power gain between the SN and the DN, and σ_1^2 and σ_2^2 denote the average power gains over the first and the second hops, respectively. We assume $\sigma_1^2 = \sigma^2 f_1(d)$ and $\sigma_2^2 = \sigma^2 f_2(1-d)$, where $f_1(d) = \frac{1}{d^u}$, $f_2(1-d) = \frac{1}{(1-d)^u}$, and u (> 2) denotes the path loss exponent. Although such a simplified path loss assumption may deviate from practical radio environment, it facilitates analysis and provides meaningful insights.

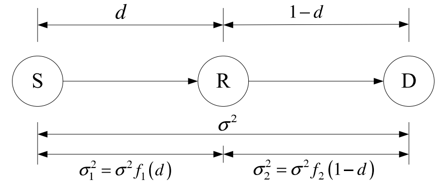


Fig. 2. Path loss environment of the two-hop MIMO AF relay system

To simplify analysis, we assume $\sigma_r^2 = \sigma_d^2 = 1$ and $P_s\sigma_1^2 + P_r\sigma_2^2 \gg 1$, which is true in high SNR region, then the equivalent received SNR, γ_{eqv}, can be approximated with

$$\gamma_{eqv} = \frac{P_s\sigma_1^2 \cdot P_r\sigma_2^2}{1 + P_s\sigma_1^2 + P_r\sigma_2^2} \simeq \frac{P_s\sigma_1^2 \cdot P_r\sigma_2^2}{P_s\sigma_1^2 + P_r\sigma_2^2}, \tag{13}$$

which, according to the path loss environment, can be further expressed as

$$\gamma_{eqv} = \frac{P_s\sigma^2 f_1(d) \cdot P_r\sigma^2 f_2(1-d)}{P_s\sigma^2 f_1(d) + P_r\sigma^2 f_2(1-d)} = \frac{P_s P_r \sigma^2}{\frac{P_s}{f_2(1-d)} + \frac{P_r}{f_1(d)}}. \tag{14}$$

B. Optimal Location for Fixed Power Allocation Scheme

In the fixed power allocation scheme, P_s and P_r are fixed regardless of the location of the RN. The optimal location of the RN in this case can be obtained by maximizing γ_{eqv} in (14) subject to $0 < d < 1$. Since $f_1(d) = \frac{1}{d^u}$ and $f_2(1-d) = \frac{1}{(1-d)^u}$, the solution of this optimization problem can be obtained as

$$d_f^* = \frac{\sqrt[u-1]{P_s}}{\sqrt[u-1]{P_s} + \sqrt[u-1]{P_r}}. \tag{15}$$

In the special case of the equal power allocation scheme, $P_s = P_r$ and hence $d_f^* = \frac{1}{2}$, i.e., the optimal location of the RN is at the midpoint between the SN and the DN.

C. Optimal Location for Optimal Power Allocation Scheme

To find the optimal location of the RN when the optimal static power allocation scheme is applied, we first get the maximum equivalent received SNR for an arbitrary location, and then find the optimal location with the largest maximum equivalent received SNR.

The optimal static power allocation between the SN and the RN for a given d can be obtained by maximizing γ_{eqv} in (14) subject to $P_s + P_r = P$ and $P_s, P_r \ge 0$. The solutions of this optimization problem are given by

$$P_s^* = \frac{\sqrt{f_2(1-d)}}{\sqrt{f_1(d)} + \sqrt{f_2(1-d)}} P, \tag{16}$$

and

$$P_r^* = \frac{\sqrt{f_1(d)}}{\sqrt{f_1(d)} + \sqrt{f_2(1-d)}} P. \tag{17}$$

The corresponding maximum equivalent received SNR is

$$\gamma_{eqv}^*(d) = \frac{P_s^* P_r^* \sigma^2}{\frac{P_s^*}{f_2(1-d)} + \frac{P_r^*}{f_1(d)}} = \frac{P\sigma^2}{\left(\frac{1}{\sqrt{f_1(d)}} + \frac{1}{\sqrt{f_2(1-d)}}\right)^2}. \tag{18}$$

Then the optimal location of the RN can be further obtained by maximizing $\gamma_{eqv}^*(d)$ subject to $0 < d < 1$. Since $u > 2$, $f_1(d) = \frac{1}{d^u}$, and $f_2(1-d) = \frac{1}{(1-d)^u}$, the solution of this optimization problem is given by $d_o^* = \frac{1}{2}$, i.e., the optimal location of the RN for the optimal static power allocation scheme is at the midpoint between the SN and the DN.

D. Comparison between Direct and Two-Hop Transmission

Since the average capacity of the two-hop MIMO AF relay system varies with the location of the RN, it is interesting to compare direct and two-hop transmission when the RN is differently located, which will be our focus in this subsection.

1) Direct Transmission: For direct transmission, the system can be described as

$$\mathbf{y}_d = \mathbf{H}_{1-hop}\mathbf{x} + \mathbf{n}_d, \tag{19}$$

where $\mathbf{H}_{1-hop}$ denotes the $N_d \times N_s$ channel matrix between the SN and the DN and $\mathbf{n}_d$ denotes the white noise vector at the DN. To facilitate analysis, we assume that the elements of $\mathbf{H}_{1-hop}$ and $\mathbf{n}_d$ are i.i.d. complex Gaussian random variables with zero mean and variances σ^2 and 1, respectively. Suppose that the total transmit power at the SN is P and the correlation matrix of the transmitted signal vector is $\mathbf{R}_x = E\left\{\mathbf{x}\mathbf{x}^H\right\} = \frac{P}{N_s}\mathbf{I}_{N_s}$, then the instantaneous capacity of the direct transmission scheme is given by [6], [7]

$$C_{1-hop}(\mathbf{H}_n) = \log_2\left|\mathbf{I}_{N_d} + \frac{\gamma_{1-hop}}{N_s}\mathbf{H}_n\mathbf{H}_n^H\right|, \tag{20}$$

where $\gamma_{1-hop} = P\sigma^2$ denotes the average received SNR at the DN in the direct transmission scheme and $\mathbf{H}_n = \frac{1}{\sigma}\mathbf{H}_{1-hop}$ denotes the normalized $N_d \times N_s$ channel matrix between the SN and the DN whose elements are i.i.d. Gaussian random variables with zero mean and unit variance.

2) Relay-based Two-Hop Transmission: In Section III, we have obtained an approximate instantaneous capacity of the two-hop MIMO AF relay system with a sufficiently large N_r in (5). Assume that $\mathbf{H}_1$ and $\mathbf{H}_2$ have i.i.d. Gaussian elements with zero mean and variances σ_1^2 and σ_2^2, respectively, then it can be shown that, when N_r is large, the elements of $\mathbf{H}_{eqv}$ are approximately i.i.d. complex Gaussian random variables with zero mean and unit variance. Thus the instantaneous capacity of the two-hop MIMO AF relay system with a sufficiently large N_r can be expressed similarly to (20) as follows

$$C_{2-hop}(\mathbf{H}_n) = \frac{1}{2}\log_2\left|\mathbf{I}_{N_d} + \frac{\gamma_{2-hop}}{N_s}\mathbf{H}_n\mathbf{H}_n^H\right|, \tag{21}$$

where $\gamma_{2-hop} = \gamma_{eqv}$ and $\mathbf{H}_n = \mathbf{H}_{eqv}$. Consider the path loss environment given in this section, then the equivalent received SNR at the DN for the proposed optimal static power allocation scheme can be obtained based on (18) as

$$\gamma_{2-hop} = \frac{P\sigma^2}{\left(\sqrt{d^u} + \sqrt{(1-d)^u}\right)^2}, \quad 0 < d < 1. \tag{22}$$

Since $u > 2$, $\gamma_{2-hop} > \gamma_{1-hop}$ for any $0 < d < 1$, i.e., the optimal static power allocation between the SN and the RN achieves a received SNR gain over direct transmission no matter where the RN is located[†]. Moreover, such a received SNR gain is maximized when the RN is located around the midpoint between the SN and the DN.

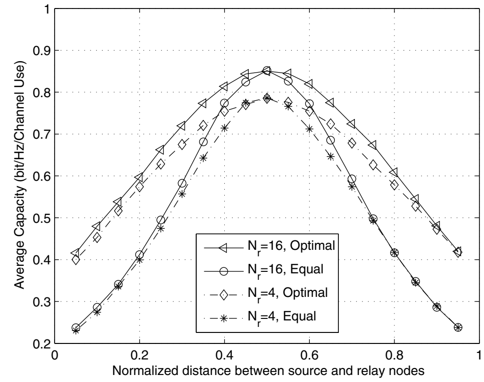


Fig. 3. Contrastive curves of the average capacity when $N_s = N_d = 2$

Comparison between (20) and (21) indicates that although two-hop transmission scheme achieves a received SNR gain, it suffers a multiplexing loss caused by two channel uses for each data transmission. Therefore, whether relay-based two-hop transmission outperforms direct transmission depends on a tradeoff between the multiplexing loss and the received SNR gain in the relay-based two-hop transmission scheme.

V. Numerical Results

In this section, we present numerical results on the proposed optimal static power allocation algorithm. When the RN is differently located, the path loss environment given in Section IV is utilized. Furthermore, we assume that the elements of $\mathbf{H}_1$ and $\mathbf{H}_2$ are i.i.d. complex Gaussian random variables with zero mean and variances σ_1^2 and σ_2^2, respectively.

Figure 3 shows the contrastive curves of the average capacity of the two-hop MIMO AF relay system, $\overline{C}(\sigma_1^2, \sigma_2^2, P_x, g^2)$, when the equal and the optimal static power allocation schemes are applied. We assume there are two antennas at the SN, the DN, and each RN, and $N_r = 4$ and 16 in Figure 3 correspond to a composite RN consisting of 2 and 8 component RN's, respectively. Furthermore, the path loss exponent, u, is set to 4 and the average total transmit power of the overall system, P, is determined by $10\log_{10}\left(\frac{P\sigma^2}{\sigma_n^2}\right) = -5$ dB, where σ_n^2 denotes the noise power at the RN and the DN. Figure 3 indicates that the performance gap between the equal and the optimal static power allocation schemes varies with the location of the RN. When the RN lies on the midpoint between the SN and the DN where the optimal static power allocation scheme reduces to the equal one, the two schemes have the same average capacity. On the other hand, as the RN deviates from the midpoint farther and farther, the optimal static power allocation scheme achieves a larger and larger capacity gain over the equal one. Furthermore, Figure 3 also indicates that, although the equivalent received SNR comes

[†]In contrast, the fixed static power allocation between the SN and the RN achieves a received SNR gain over direct transmission only when the RN is located appropriately, which is omitted in this paper due to space limit.

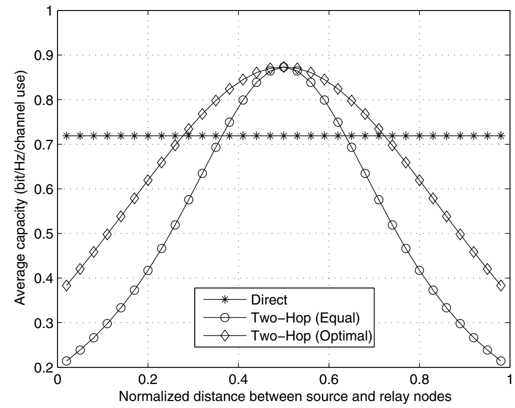
from the assumption that $N_r \gg N_s$, the optimal static power allocation scheme based on the maximization of the equivalent received SNR actually works under the general case. It can be also observed that, the average capacity of the two-hop MIMO AF relay system with a given overall average transmit power increases with the total number of antennas at the RN, N_r. This is reasonable since more relay antennas means more spatial diversity and hence means a larger average capacity.

Figure 4 shows the contrastive curves of the average capacities of the direct transmission scheme, and the relay-based two-hop transmission scheme when the optimal and the equal static power allocation between the SN and the RN are applied. Here we let $N_s = N_d = 2$ and the total transmit power, P, for both the direct and the two-hop transmission schemes is decided by $\gamma_{1-hop} = 10\log_{10}\left(\frac{P\sigma^2}{\sigma_n^2}\right) = -5$ and 5 dB in Figures 4(a) and 4(b), respectively. The average capacities of the two schemes are obtained numerically based on (20) and (21), respectively. Figure 4 indicates that for both the equal and the optimal static power allocation schemes, whether relay-based two-hop transmission outperforms direct transmission depends on the location of the RN and the available total transmit power, or, in other words, the received SNR in the direct transmission scheme, γ_{1-hop}. Specifically, there exists a threshold for γ_{1-hop} below which relay-based two-hop transmission outperforms direct transmission if only the RN is appropriately located and above which direct transmission is always preferred. Such a threshold reflects a tradeoff between the received SNR gain and the multiplexing loss in the two-hop transmission scheme. In low SNR region, the received SNR gain plays a more critical role, so the two-hop transmission has a better performance; in high SNR region, the multiplexing loss plays a more critical role, so direct transmission has a better performance. Furthermore, the specific value of this SNR threshold for a given overall transmit power increases with the path loss exponent, u, because, as u increases, the power of the transmitted signal decays with a greater and greater speed and, as a result, relay-based two-hop transmission has a greater and greater advantage over direct transmission.

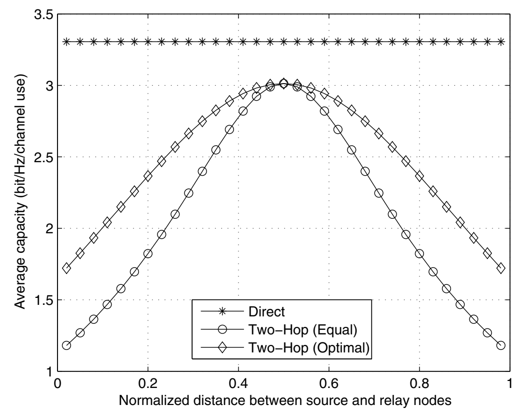
As a final remark, the above comparison is optimistic for relay-based two-hop transmission since we have assumed that there exist sufficient antennas at the component RN in the two-hop transmission scheme and, as verified, such an assumption overestimates the average capacity of a practical two-hop MIMO AF relay system. Even so, the comparison between the direct and the two-hop transmission schemes based on this assumption still provides meaningful insights.

VI. Conclusion

In this paper, we have developed an optimal static power allocation algorithm to maximize the equivalent received SNR of a two-hop MIMO AF relay system. Since this algorithm performs power allocation between the SN and the RN based on the path losses over the SN-RN and the RN-DN hops, it has little complexity. We have analyzed the optimal location of the RN that maximizes the equivalent received SNR of the system when the optimal static power allocation scheme is applied. It has also been demonstrated that whether the relay-based two-hop transmission outperforms direct transmission depends on



(a) $\gamma_{1-hop} = -5$ dB



(b) $\gamma_{1-hop} = 5$ dB)

Fig. 4. Contrastive curves of the average capacities of the direct and the relay-based two-hop transmission schemes

a tradeoff between the received SNR gain and the multiplexing loss in the relay-based two-hop transmission scheme.

References

[1] J. Laneman, D. N. C. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062-3080, Dec. 2004.

[2] J. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2415-2425, Oct. 2003.

[3] A. Wittneben and B. Rankov, "Impact of cooperative relays on the capacity of rank-deficient MIMO channels," *in IST Summit*, 2003.

[4] O. M. Medina, J. Vidal, and A. Agustin, "Linear transceiver design in nonregenerative relays with channel state information," *IEEE Trans. Signal Processing*, vol. 55, no. 6, pp. 2593-2604, June 2007.

[5] I. Hammerstrom and A. Wittneben, "Power allocation schemes for amplify-and-forward MIMO-OFDM relay links," *IEEE Trans. Wireless Commun.*, vol. 6, no. 8, pp. 2798-2802, Aug. 2007.

[6] G. J. Foschini and M. J. Gans, "On limits of wireless communications in a fading environment when using multiple antennas," *Wireless Pers. Commun.*, vol. 6, no. 3, pp. 311-335, Mar. 1998.

[7] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *AT&T Bell Lab. Internal Tech. Memo.*, June 1995.

[8] A. Papoulis, *Probability, Random Variables, and Stochastic Process*, 4th Ed., New York: McGraw-Hill, 2002.