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Abstract

This paper proposes a minimum mean-square-error bi-directional amplify-and-forward (MMSE-BAF) relaying protocol for multi-hop wireless networks employing multi-antenna relays. MMSE-BAF is a two-phase relaying protocol which allows for two sources to exchange independent messages via a relay node equipped with multiple antennas. The latter performs a joint linear MMSE filtering of the received signal after the multiple access (MA) phase before amplifying and forwarding using a single transmit antenna, possibly through a specific antenna selection procedure, during the broadcast phase. The proposed MMSE-BAF protocol extends upon the so-called analog network coding schemes in the literature in that it inherently exploits the multiple antennas at the relay station in order to reduce the noise enhancement effects typical of an AF protocol. Owing to its joint linear MMSE filtering approach, it can also compensate for link imbalances between the relay and the sources and is agnostic to sources' modulation and coding schemes (MCS), which is especially relevant when these experience dissimilar channel conditions and wish to adapt their MCS accordingly. We derive the instantaneous signal-to-noise ratio expressions for the received signal by the source nodes in the downlink and provide extensive link-level simulation results for the MMSE-BAF protocol subject to both frequency flat and selective fading. Furthermore, we detail the modifications needed to the IEEE 802.16e orthogonal-frequency-division multiple access (OFDMA) cellular standard (mobile WiMax) to enable support of multi-antenna bi-directional communications and show that MMSE-BAF is a viable solution within that framework.

Index Terms

analog network coding, cooperative communications, multiple antennas, minimum-mean-square error (MMSE) filtering

I. INTRODUCTION

Half-duplex bi-directional relay systems in which two nodes S_1 and S_2 wish to exchange independent messages via a third node R, termed relay, give rise to some interesting challenges from a cooperative communications and information-theoretic points-of-view. This is especially true when the relay node R is equipped with multiple antennas. Such two-way relay channels have many applications in ad-hoc and cellular networks in which all mobile-to-mobile communications have to pass through a common base station. Since full-duplex operation is of little practical interest given current state-of-the-art technology, our focus is on half-duplex nodes, where each active node can either transmit or receive an information message at a given point in time. In particular, without loss of generality, we are interested in the communications part of the problem in a cellular context where two mobile stations wish to exchange data simultaneously via a common base station.

The traditional baseline approach for bidirectional communications in half-duplex mode between two sources S_1 and S_2 via a relay station R consists of a 4-phase protocol with a completion time of 4-time-slots (TSs) whereby S_1 and S_2 send N-bit packets $\mathbf{b}_1$ and $\mathbf{b}_2 \in \{0,1\}^N$ to R during TS 1 and TS 2, respectively; R decodes the received packets, and then sends $\mathbf{b}_2$ to S_1 and $\mathbf{b}_1$ to S_2 during TS 3 and TS 4, respectively. The gist of the 4-phase protocol is to avoid interference by preventing simultaneous transmissions from the sources to the relay and vice versa. However, it was shown in [1] that a three-phase protocol exploiting the network coding idea by combining packets $\mathbf{b}_1$ and $\mathbf{b}_2$ at the relay and broadcasting a single packet $\mathbf{b}_1 \oplus \mathbf{b}_2$, where $\oplus$ denotes the bit-wise exclusive-or (`XOR`) operation, is actually more attractive in terms of achievable throughput, since the desired packet at S_1 can be decoded using another `XOR` operation (and similarly at S_1). Better still, a recent concept introduced in [2] and termed analog network coding (ANC) combines the first two phases of the conventional baseline protocol into a single multiple access (MA) phase with simultaneous transmissions from the sources to the relay; the received multiple access signal at the relay is then amplified and broadcast to S_1 and S_2, thereby yielding the so-called two-phase bi-directional amplify-and-forward (BAF) protocol. A similar concept to ANC, using estimate-and-forward relaying as opposed to AF relaying, has been proposed in [3] under the terminology of physical-layer network coding (PNC). A schematic diagram illustrating the aforementioned bi-directional protocols is illustrated in Fig. 1.

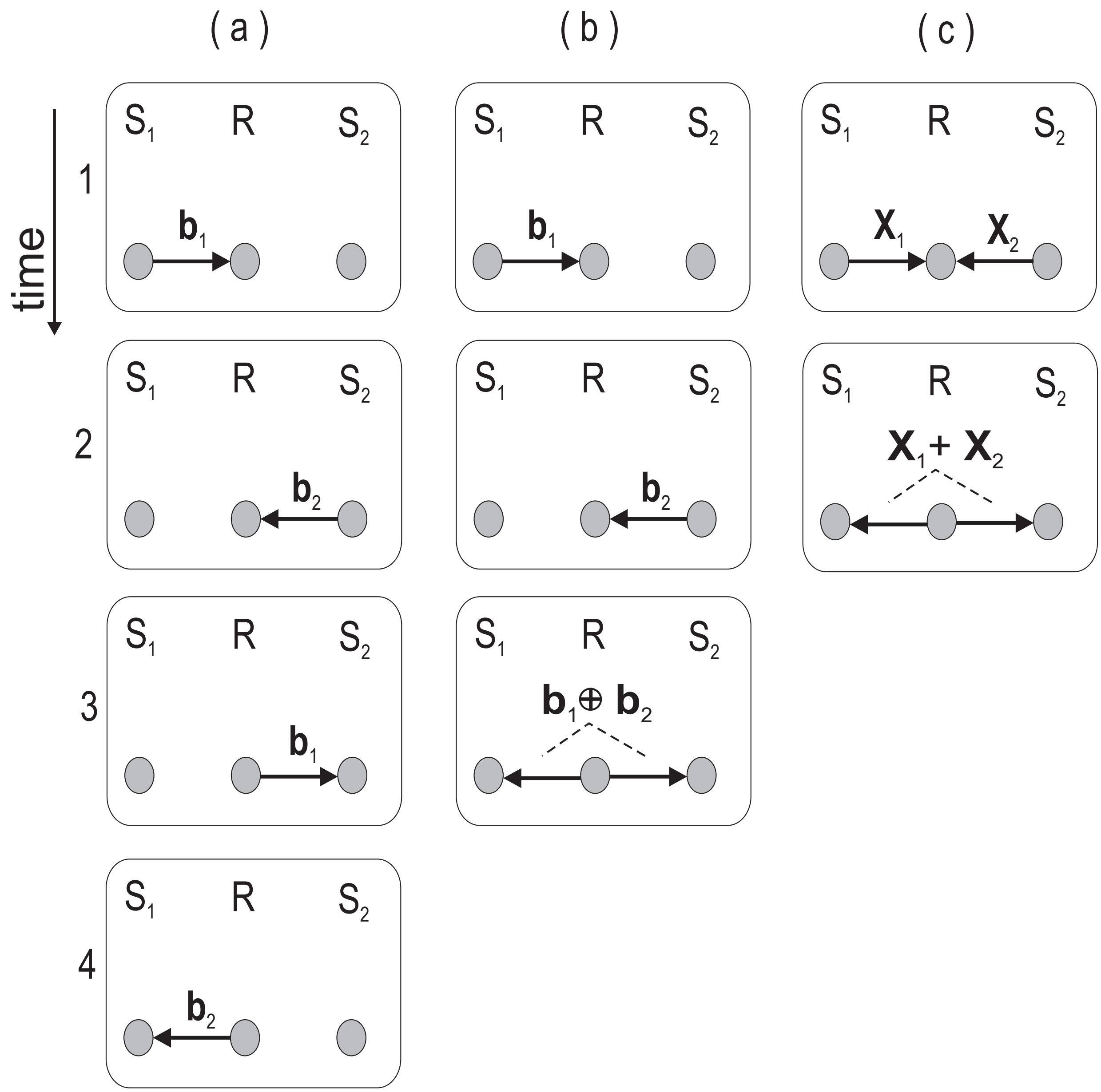


Fig. 1. Bi-directional communication protocols (a) 4-phase conventional protocol (b) 3-phase protocol with decode-and-forward network coding (c) 2-phase BAF protocol with amplify-and-forward relaying.

Information-theoretic aspects such as bounds on the achievable throughput and the capacity region of the bidirectional relay channel have been investigated in [4]–[6]. A Markov-chain-based performance analysis of several variants of the BAF protocol was carried out in [7]–[9]. Linear beamforming filter designs for bi-directional communications with multi-antenna relay stations are proposed in [10]–[12]. In this paper, we propose a simple two-phase minimum-mean-square-error (MMSE)-BAF protocol which operates by filtering the received signal at the relay station after the MA phase using a specially designed joint linear MMSE filter before amplifying and forwarding the filtered signal during the broadcast phase. Whereas optimal relay beamforming structure for bi-directional multi-antenna relay channels is sought in [12], our MMSE-BAF protocol is a simple low-complexity driven approach for bi-directional multi-antenna relay channels which exploits linear signal processing on the uplink (from S_1 and S_2 to R) and transmit antenna selection (TAS) on the downlink (from R to S_1 and S_2). Besides, MMSE-BAF is a two-phase bi-directional relaying protocol, whereas the multi-antenna relaying protocol put forward in [10] is a three-phase protocol which necessitates decoding and re-encoding of the received signals at the RS prior to the broadcast phase. Finally, MMSE-BAF differs from the so-called spatial division duplex (SDD) bi-directional relaying scheme proposed in [11] in that it allows to bias the beamforming weights in favor of one of the two source nodes as required to compensate for potential imbalance of the relay-to-source link channel gains or other parameters such as dissimilar signal constellations employed at S_1 and S_2.

The remainder of this paper is structured as follows: Section II presents the system model for MMSE-BAF relaying. In Section III, we describe the proposed MMSE-BAF protocol and derive the signal-to-noise ratio (SNR) expressions upon which our extensive simulation results provided in Section IV are based. Finally, concluding remarks are drawn in Section V.

II. System Setup and Signal Model

The following set of notations is employed throughout this paper: Boldface upper- and lower-case symbols are used to denote matrices and column-vectors, respectively. $\boldsymbol{I}_m$ denotes the identity matrix of order m. Moreover, $(.)^*$, $(.)^T$, $(.)^{\mathcal{H}}$ and $\mathrm{E}\,[.]$ stand for conjugate, transpose, transpose-conjugate and expectation operators, respectively.

Without loss of generality, we focus our attention on cellular systems, although our proposal and framework are suitable for any type of two-hop bi-directional relay setting. For that purpose, we consider an infrastructure-based wireless communications system consisting of two mobile stations (MSs), MS_1 and MS_2, and one base station (BS). A block diagram of the system under consideration is depicted in Fig. 2. Both MSs as well as the BS are equipped with multiple antennas for reception with the aim of canceling out potential other cell/user interference but may only transmit using a single transmit antenna. This assumption is dictated by the need to reduce the transmit-power requirements for user terminals and to lower the complexity and cost of a transmission chain at the base station, generally higher than that of a reception chain, especially when accounting for high-cost radio-frequency amplifiers involved in the transmission chain. This is for instance the case in current cellular standards such as IEEE 802.16e [13]. Note that our proposed scheme works equally well with both time-division duplex (TDD) as well as frequency-division duplex (FDD) modes of operation. Without loss of generality and for the sake of notational brevity, we focus on the FDD mode in the following analysis. Performance results for both TDD and FDD will be presented in Section IV.

Complex baseband transmission is assumed throughout the paper. Let M_{bs} denote the number of receive antennas at the BS, $\mathbf{h}_1[n] = [h_1^1[n], h_1^2[n], \cdots, h_1^{M_{bs}}[n]]^T$ and $\mathbf{h}_2[n] = [h_2^1[n], h_2^2[n], \cdots, h_2^{M_{bs}}[n]]^T$ denote the $M_{bs} \times 1$ uplink channels from MS_1 and MS_2 to the BS, respectively, where n is the discrete-time index. The corresponding time-varying channel vector elements $\{h_i^j[n]\}_{i=1,2; j=1,\cdots,M_{bs}}$ are realizations of a zero-mean unit-variance Gaussian wide-sense stationary process. Assuming M_{ss} antennas at each of MS_1 and MS_2 for downlink reception, we define $\mathbf{g}_1[n] = [g_1^1[n], g_1^2[n], \cdots, g_1^{M_{ss}}[n]]^T$ and $\mathbf{g}_2[n] = [g_2^1[n], g_2^2[n], \cdots, g_2^{M_{ss}}[n]]^T$ as the downlink channels from BS to MS_1 and MS_2, respectively. For the special case of $M_{bs} = M_{ss} = 1$, TDD assumption allows us to set $\mathbf{g}_1[n] = \mathbf{h}_1^T[n]$ and $\mathbf{g}_2 = \mathbf{h}_2^T[n]$. Let $x_1[n]$ be the signal transmitted from MS_1 and intended for MS_2 and $x_2[n]$ be the signal transmitted from MS_2 and intended for MS_1 at time n. Both $x_1[n]$ and $x_2[n]$ are drawn from two possibly different complex signal constellations with average energies $\sigma_1^2 = \mathrm{E}\left[|x_1[n]|^2\right]$ and $\sigma_2^2 = \mathrm{E}\left[|x_2[n]|^2\right]$, respectively. Prior to any signal processing at the BS, the $M_{bs} \times 1$ received signal at the end of the MA phase is then given by

$$\mathbf{y}[n] = \mathbf{h}_1[n]x_1[n] + \mathbf{h}_2[n]x_2[n] + \mathbf{n}[n] \quad (1)$$

where $\mathbf{n}[n]$ is $M_{bs} \times 1$ additive white Gaussian noise which is modeled as a zero-mean circularly symmetric Gaussian random vector with covariance matrix $\mathrm{E}\left[\mathbf{n}[n]\mathbf{n}[n]^{\mathcal{H}}\right] = \sigma_N^2 \boldsymbol{I}_{M_{bs}}$. For

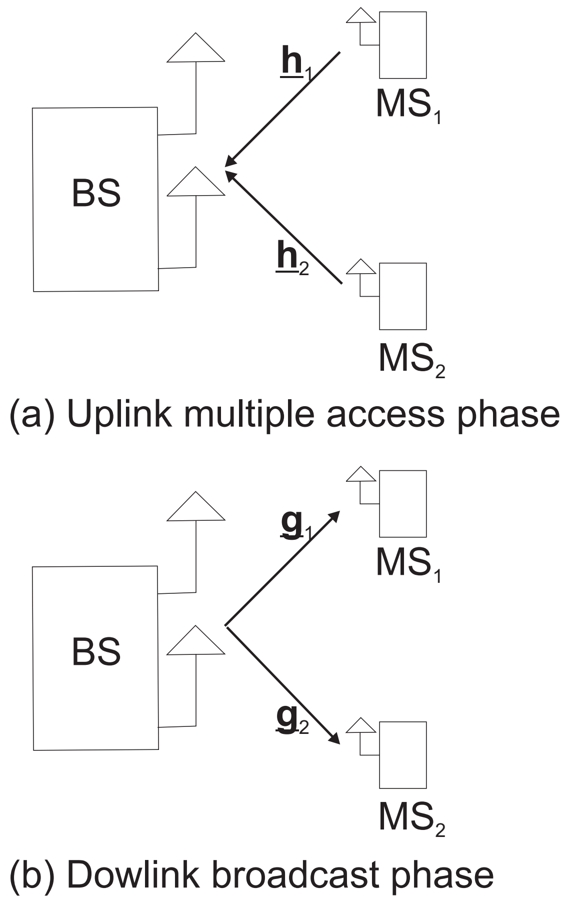


Fig. 2. Block diagram of the system setup.

notational simplicity, the discrete-time index n is omitted for the remainder of this paper.

III. MMSE-BAF: Description & Analysis

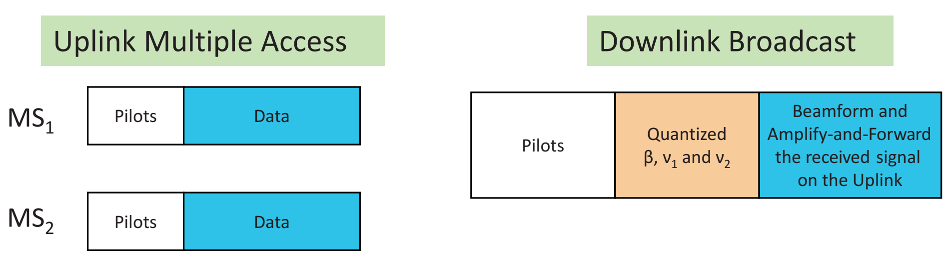


Fig. 3. Block diagram of the system setup.

A. Protocol Description

Fig. 3 illustrates the physical-layer frame structure for enabling the MMSE-BAF protocol for the uplink MA and the downlink broadcast phases. A frame consists of packets originating from the link-layer whose size depends on the chosen MCS so that each downlink or uplink frame contains a fixed number of symbols. As can be seen in the left-hand side of Fig. 3, the uplink frame structure is composed of two parts, one for pilot symbols which are chosen to be orthogonal for MS_1 and MS_2. Orthogonality of the pilot symbols can be maintained in the time, frequency or in the 2-dimensional time-frequency grid. The second part is for data symbols. The orthogonal pilot symbols are used to estimate the channels $\mathbf{h}_1$ and $\mathbf{h}_2$ corresponding to MS_1 and MS_2. A beamforming weight vector $\mathbf{w}_{\text{opt}}$ is then computed at the BS based on a joint MMSE criterion to be specified shortly. The BS then estimates an amplification factor β subject to an average power constraint. Likewise, the downlink frame structure contains pilot and data parts. Additionally, it contains a control part consisting of quantized versions of the amplification factor β (a positive scalar value) and the two complex scalars values $v_1 := \mathbf{w}_{\text{opt}}^{\mathcal{H}}\mathbf{h}_1$ and $v_2 := \mathbf{w}_{\text{opt}}^{\mathcal{H}}\mathbf{h}_2$. The downlink data symbols consist of the beamformed amplified-and-forwarded symbols received on the uplink frame in FDD mode.

The MMSE-BAF protocol at the BS with consists of the following operations:

1) Jointly minimize the MSE between the received signal at the BS $\mathbf{y}[n]$ and the transmitted signals $x_1[n]$ and $x_2[n]$, thus performing a joint linear-MMSE filtering of the received signal, using the following metric:

$$\mathbf{w}_{\text{opt}} = \underset{\mathbf{w}\in\mathbb{C}^{M_{bs}}}{\operatorname{argmin}}\{\delta_1 \mathrm{E}\left[\left|x_1 - \mathbf{w}^{\mathcal{H}}\mathbf{y}\right|^2 |\mathbf{h}_1, \mathbf{h}_2\right] + \delta_2 \mathrm{E}\left[\left|x_2 - \mathbf{w}^{\mathcal{H}}\mathbf{y}\right|^2 |\mathbf{h}_1, \mathbf{h}_2\right]\} \tag{2}$$

where $\mathbb{C}$ is the field of complex numbers and $\delta_1 \geq 0$, $\delta_2 \geq 0$, $\delta_1 + \delta_2 = 1$, are the two design constants that control the relative weight assigned to the signals of MS_1 and MS_2. The minimization problem in (2) is a modified Wiener filtering problem whose solution can be easily found using the orthogonality principal in linear mean square estimation and is given by:

$$\mathbf{w}_{\text{opt}} = \left(\sigma_1^2\mathbf{h}_1\mathbf{h}_1^{\mathcal{H}} + \sigma_2^2\mathbf{h}_2\mathbf{h}_2^{\mathcal{H}} + \sigma_N^2\boldsymbol{I}_{M_{bs}}\right)^{-1}\left(\delta_1\sigma_1^2\mathbf{h}_1 + \delta_2\sigma_2^2\mathbf{h}_2\right). \tag{3}$$

This minimization requires an estimation of both mobile stations' vector-valued channels $\mathbf{h}_1$ and $\mathbf{h}_2$.

2) Amplify the linear MMSE-filter output to maintain a constant average transmit power P_{T} which leads to computing the amplification gain factor

$$\beta = \sqrt{\frac{P_{\mathrm{T}}}{\mathrm{E}\left[\left|\mathbf{w}^{\mathcal{H}}\mathbf{y}\right|^{2}|\mathbf{h}_{1},\mathbf{h}_{2}\right]}} \tag{4}$$

$$= \sqrt{\frac{P_{\mathrm{T}}}{\sigma_{1}^{2}\left|\mathbf{w}_{\mathrm{opt}}^{\mathcal{H}}\mathbf{h}_{1}\right|^{2}+\sigma_{2}^{2}\left|\mathbf{w}_{\mathrm{opt}}^{\mathcal{H}}\mathbf{h}_{2}\right|^{2}+\sigma_{N}^{2}\left\|\mathbf{w}_{\mathrm{opt}}\right\|^{2}}}. \tag{5}$$

3) Transmit the amplified signal back to the MSs on one of the antennas using an appropriate downlink transmit antenna-selection (TAS) algorithm, based on the uplink channel. One approach inherent to the MMSE-BAF protocol is to select the antenna that has the largest beamformer weight.[1]

B. Performance Analysis

Define $z := \mathbf{w}_{\mathrm{opt}}^{\mathcal{H}}\mathbf{y}$, where $\mathbf{y}$ is the uplink received signal (1), as the output of the MMSE filtering operation at the BS. The AF transmitted signal on the downlink is

$$x_{r} = \beta z = \beta\mathbf{w}_{\mathrm{opt}}^{\mathcal{H}}\mathbf{y}. \tag{6}$$

The received signal on the downlink for MS_1 and MS_2 is therefore given by

$$\mathbf{y}_{1} = \mathbf{g}_{1}x_{r} + \mathbf{n}_{1}, \tag{7}$$

and

$$\mathbf{y}_{2} = \mathbf{g}_{2}x_{r} + \mathbf{n}_{2}, \tag{8}$$

where $\mathbf{n}_1$ and $\mathbf{n}_2$ are the zero-mean AWGN at MS_1 and MS_2 respectively, with covariance matrix $\sigma_N^2\boldsymbol{I}_{M_{ss}}$. Without loss of generality, let us focus on the signal received by MS_1 (7). A similar signal processing is required at MS_2. Incorporating (1) and (6) into (7) yields

$$\mathbf{y}_{1} = \mathbf{g}_{1}\beta\mathbf{w}_{\mathrm{opt}}^{\mathcal{H}}\left(\mathbf{h}_{1}x_{1}+\mathbf{h}_{2}x_{2}+\mathbf{n}\right)+\mathbf{n}_{1}. \tag{9}$$

Now, assuming that MS_1 i) is able to perfectly estimate its own downlink channel vector $\mathbf{g}_1$ owing to the downlink pilot symbols sent by the BS, ii) knows its own transmitted signal x_1 and iii) is able to extract the value of the amplification factor β as well as the couple (v_1, v_2),

[1]TAS using the largest beamformer weight is applicable for TDD only owing to the channel reciprocity.

all of which are sent on the downlink control channel as explained in Section III-A, then MS$_1$ can subtract the so-called self-interference component of the received signal, i.e. $\mathbf{g}_1\beta v_1 x_1$, hence winding up with a processed received signal of the form

$$\begin{aligned}
\mathbf{r}_1 &= \mathbf{y}_1 - \mathbf{g}_1\beta v_1 x_1 && (10)\\
&= \mathbf{y}_1 - \mathbf{g}_1\beta \mathbf{w}_{\text{opt}}^{\mathcal{H}}\mathbf{h}_1 x_1 && (11)\\
&= \mathbf{g}_1\beta \mathbf{w}_{\text{opt}}^{\mathcal{H}}\mathbf{h}_2 x_2 + \underbrace{\mathbf{g}_1\beta \mathbf{w}_{\text{opt}}^{\mathcal{H}}\mathbf{n} + \mathbf{n}_1}_{:=\widetilde{\mathbf{n}}_1}. && (12)
\end{aligned}$$

Note that $\widetilde{\mathbf{n}}_1$, defined in the previous equation, is a zero-mean colored noise vector with a conditional covariance matrix given by

$$\begin{aligned}
\mathbf{\Sigma}_1 &:= \mathrm{E}\left[\widetilde{\mathbf{n}}_1\widetilde{\mathbf{n}}_1^{\mathcal{H}}|\mathbf{g}_1, \beta, \mathbf{w}_{\text{opt}}\right] && (13)\\
&= \beta^2\mathbf{g}_1\mathbf{w}_{\text{opt}}^{\mathcal{H}}\mathrm{E}\left[\mathbf{n}\mathbf{n}^{\mathcal{H}}\right]\mathbf{w}_{\text{opt}}\mathbf{g}_1^{\mathcal{H}} + \mathrm{E}\left[\mathbf{n}_1\mathbf{n}_1^{\mathcal{H}}\right] && (14)\\
&= \sigma_N^2\left(\boldsymbol{I}_{M_{ss}} + \beta^2\mathbf{g}_1\mathbf{w}_{\text{opt}}^{\mathcal{H}}\mathbf{w}_{\text{opt}}\mathbf{g}_1^{\mathcal{H}}\right). && (15)
\end{aligned}$$

In the absence of knowledge of the conditional covariance matrix $\mathbf{\Sigma}_1$ at the receiver of MS$_1$, an estimate $\hat{x}_2$ of x_2 can be obtained as follows:

$$\hat{x}_2 = \frac{(\mathbf{g}_1\beta v_2)^{\mathcal{H}}}{(\mathbf{g}_1\beta v_2)^{\mathcal{H}}(\mathbf{g}_1\beta v_2)}\mathbf{r}_1, \tag{16}$$

which after simplification becomes

$$\hat{x}_2 = x_2 + \underbrace{\frac{1}{\beta v_2}\frac{\mathbf{g}_1^{\mathcal{H}}\widetilde{\mathbf{n}}_1}{\mathbf{g}_1^{\mathcal{H}}\mathbf{g}_1}}_{:=\check{n}_2}. \tag{17}$$

The conditional variance of the newly defined scalar noise term $\check{n}_2$ can be expressed as

$$\begin{aligned}
\mathbf{\Sigma}_1' = \mathrm{E}\left[|\check{n}_2|^2\right] &= \frac{1}{\beta^2|v_2|^2}\frac{\mathbf{g}_1^{\mathcal{H}}\mathrm{E}\left[\widetilde{\mathbf{n}}_1^{\mathcal{H}}\widetilde{\mathbf{n}}_1\right]\mathbf{g}_1}{(\mathbf{g}_1^{\mathcal{H}}\mathbf{g}_1)^2} && (18)\\
&= \frac{1}{\beta^2|v_2|^2}\frac{\mathbf{g}_1^{\mathcal{H}}\mathbf{\Sigma}_1\mathbf{g}_1}{(\mathbf{g}_1^{\mathcal{H}}\mathbf{g}_1)^2}. && (19)
\end{aligned}$$

Now, we are in a position to determine the SNR γ_2 at MS$_1$ as

$$\begin{aligned}
\gamma_2 &= \frac{\mathrm{E}\left[|x_2|^2\right]}{\mathrm{E}\left[|\check{n}_2|^2\right]} && (20)\\
&= \frac{\sigma_2^2\beta^2|v_2|^2(\mathbf{g}_1^{\mathcal{H}}\mathbf{g}_1)^2}{\mathbf{g}_1^{\mathcal{H}}\mathbf{\Sigma}_1\mathbf{g}_1}. && (21)
\end{aligned}$$

Note that after incorporating (13) into (21) and further simplification, (21) can be expressed as follows

$$\gamma_2 = \frac{\sigma_2^2 \beta^2 |v_2|^2}{\sigma_N^2} \times \frac{\|\mathbf{g}_1\|^2}{1 + \beta^2 \|\mathbf{w}_{\text{opt}}\|^2 \|\mathbf{g}_1\|^2}. \tag{22}$$

Similarly, one can evaluate the SNR γ_1 for the signal x_1 received at MS_2 which is found to be

$$\gamma_1 = \frac{\sigma_1^2 \beta^2 \left|v_1\right|^2 \left(\mathbf{g}_2^{\mathcal{H}} \mathbf{g}_2\right)^2}{\mathbf{g}_2^{\mathcal{H}} \mathbf{\Sigma}_2 \mathbf{g}_2}, \tag{23}$$

where $\mathbf{\Sigma}_2$ is a noise covariance matrix (analogous to $\mathbf{\Sigma}_1$) defined as

$$\mathbf{\Sigma}_2 := \sigma_N^2 \left(\boldsymbol{I}_{M_{ss}} + \beta^2 \mathbf{g}_2 \mathbf{w}_{\text{opt}}^{\mathcal{H}} \mathbf{w}_{\text{opt}} \mathbf{g}_2^{\mathcal{H}} \right). \tag{24}$$

Again, upon incorporation of (24) into (23), the latter can be simplified to

$$\gamma_1 = \frac{\sigma_1^2 \beta^2 |v_1|^2}{\sigma_N^2} \times \frac{\|\mathbf{g}_2\|^2}{1 + \beta^2 \|\mathbf{w}_{\text{opt}}\|^2 \|\mathbf{g}_2\|^2}. \tag{25}$$

It is worthwhile to mention that the above SNR expressions for γ_2 (22) and γ_1 (25) have been obtained without exploiting the colored nature of $\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_2$. Surprisingly, as shown in Appendix-A, even by whitening the colored noise, the SNR expressions for γ_1 and γ_2 remain the same, which is a good news in some sense because it means that the signal processing cost associated with the whitening operation can be completed eliminated.

IV. Simulation Results and Discussion

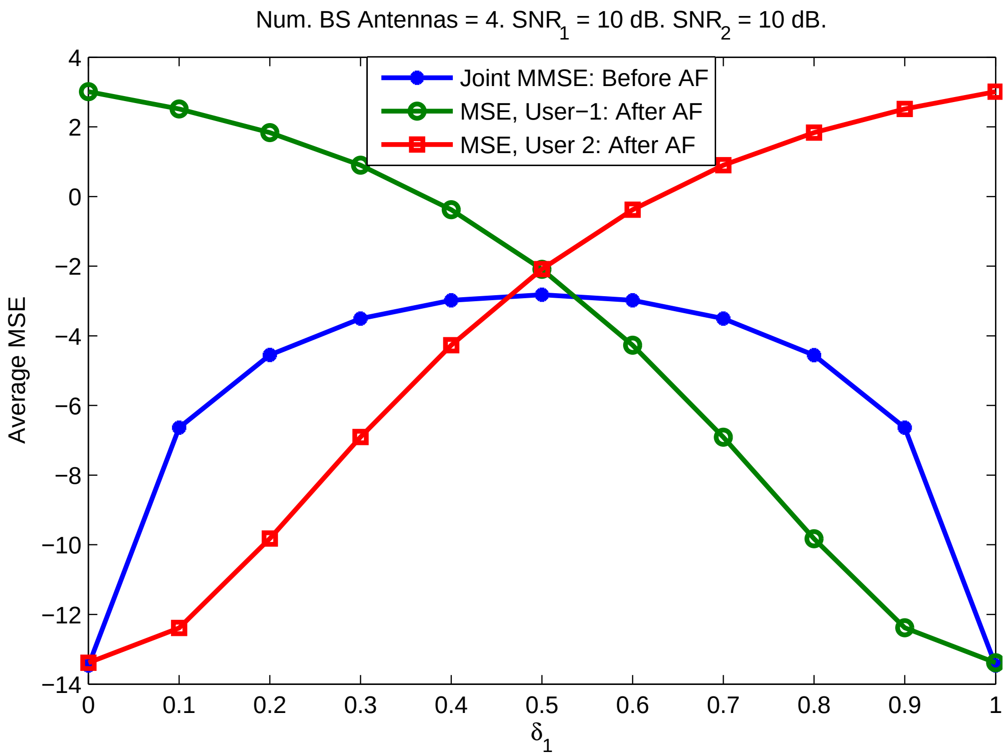
In this section, we present some simulation results on the performance of the proposed MMSE-BAF system. First, Fig. 4 shows the MSE performance of MMSE-BAF as a function of the relative loading of user-1 over user-2, δ_1, with $M_{bs} = 4$ antennas at the base-station. Two scenarios are considered: In Fig. 4(a), both the users' average received SNRs at the base-station are set to 10 dB, whereas in Fig. 4(b) the average received SNR of user-1 is set to 20 dB whereas it is 40 dB for user-2. The weighted average MSE immediately after the application of beamformer, the average MSE of user-1 after MMSE beamformer followed by an AF gain, and the average MSE of user-2 after MMSE beamformer followed by an AF gain are obtained by drawing independent channel realizations over 100000 trials. From Figs. 4(a) and 4(b), we observe that increasing δ_1 minimizes the MSE of user-1 at the expense of an increase in MSE for user-2, whereas an optimum δ_1 exists that jointly minimizes the MSE of both users. Interestingly, from an implementation point-of-view, the range of δ_1 is broad to arrive at this optimum overall

MSE. Comparing Figs. 4(a) and 4(b), we notice that, due to unequal average received SNRs, the individual MSEs as well as the overall MSE are not symmetric functions of δ_1. Thus, one should take into account the knowledge of the average uplink SNRs to arrive at an appropriate δ_1 to maintain desired MSE levels for each of the two users.

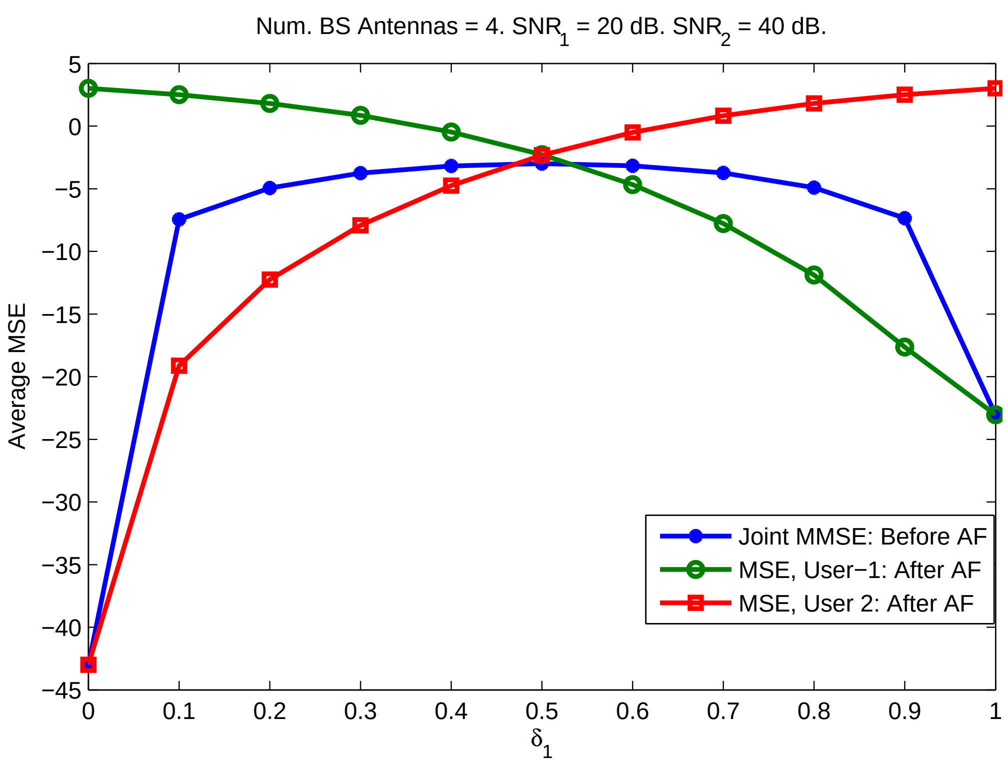
In Fig. 5, the empirical cumulative distribution function (CDF) of two-time-slots based MMSE-BAF is compared against the four-time-slot based baseline system. Here, the base-station and the mobile station receivers are each equipped with four receive antennas. The uplink average received SNRs, per antenna, of users 1 and 2 are set to 5 dB and 10 dB, respectively, and the average received SNR per antenna at each mobile station is set to 5 dB. We also assume $\delta_1 = \delta_2$. Fig. 5 shows that the two-time-slots based MMSE-BAF system outperforms the baseline system by an order of magnitude.

As argued earlier, the proposed MMSE-BAF protocol is equally attractive to both TDD and FDD systems. Figs. 6 and 7 show uncoded symbol error rate (SER) performance of MMSE-BAF on block-fading TDD channels when the two users employ dissimilar modulation formats. In both Figs. 6 and 7 (i) user-1 employs QPSK modulation whereas user-2 employs 16-QAM modulation, (ii) a data frame contains 100 modulation symbols and 20 pilot symbols for channel estimation, and (iii) the channel remains constant over the duration of at least two frames (TDD assumption). In Fig. 6 the base-station as well as the users all have single transmit/receive antenna. With equal average received SNRs at the base-station, Fig. 6(a) shows that at lower average received SNRs pilot-based channel estimation matches very closely the performance achieved in case of perfect channel knowledge for both users. Since each user has to subtract its own channel-compensated transmitted symbol to decode the other user's modulation symbol, a user transmitting using a higher order constellation has the potential to generate higher self-interference in the presence of channel estimation errors. Fig. 6(a) shows that with equal average received SNRs, the average SER of BPSK exhibits an error floor for an average SNR higher than 30 dB. When user-2 transmits at an SNR that is 20 dB higher than user-1's SNR, Fig. 6(b) shows that the error floor for the BPSK modulation occurs much earlier.

The advantages of transmit antenna selection over transmitting from an arbitrary antenna is investigated in Fig. 7 when the base-station has four antennas for reception. Exploiting the channel reciprocity of TDD systems, we first compute the element-wise magnitude of the estimated beamformer and downlink transmission is directed from the antenna that has the highest



(a) Equal average uplink received SNRs



(b) Unequal average uplink received SNRs

Fig. 4. Average MSE of MMSE-BAF system with 4 antennas at the base-station.

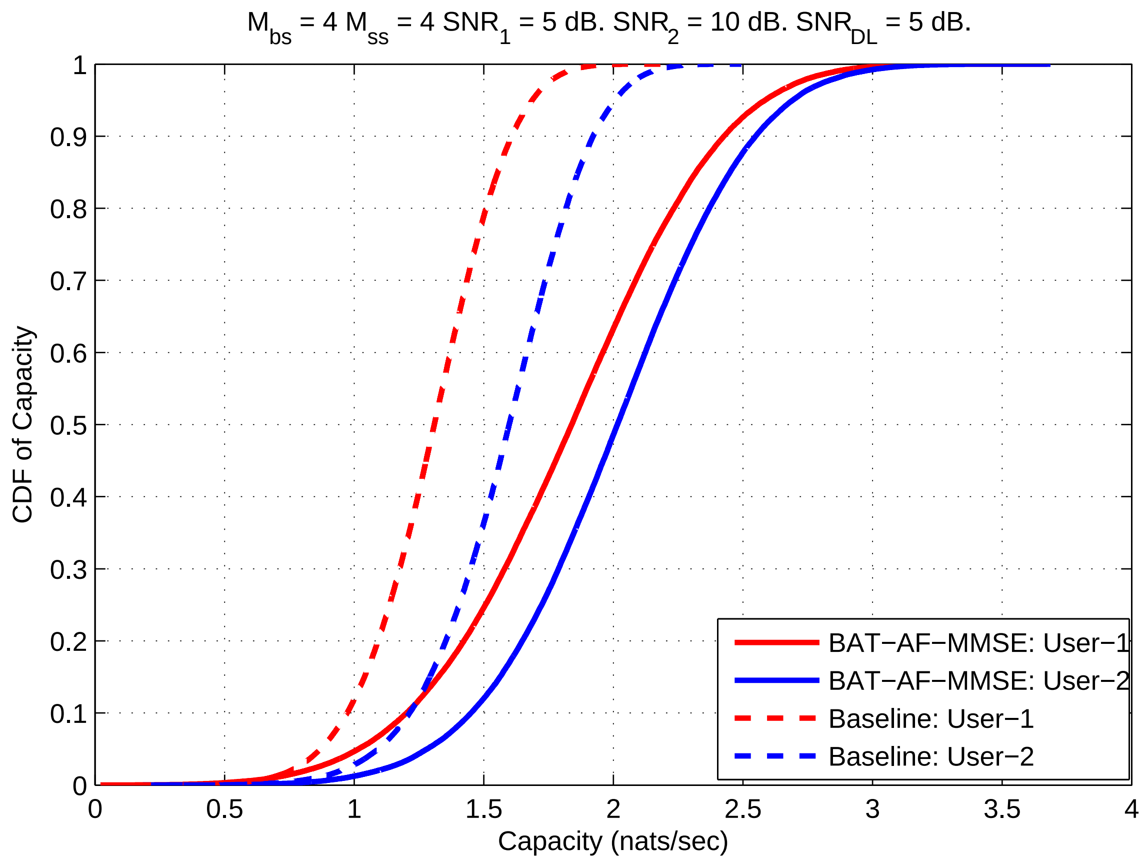
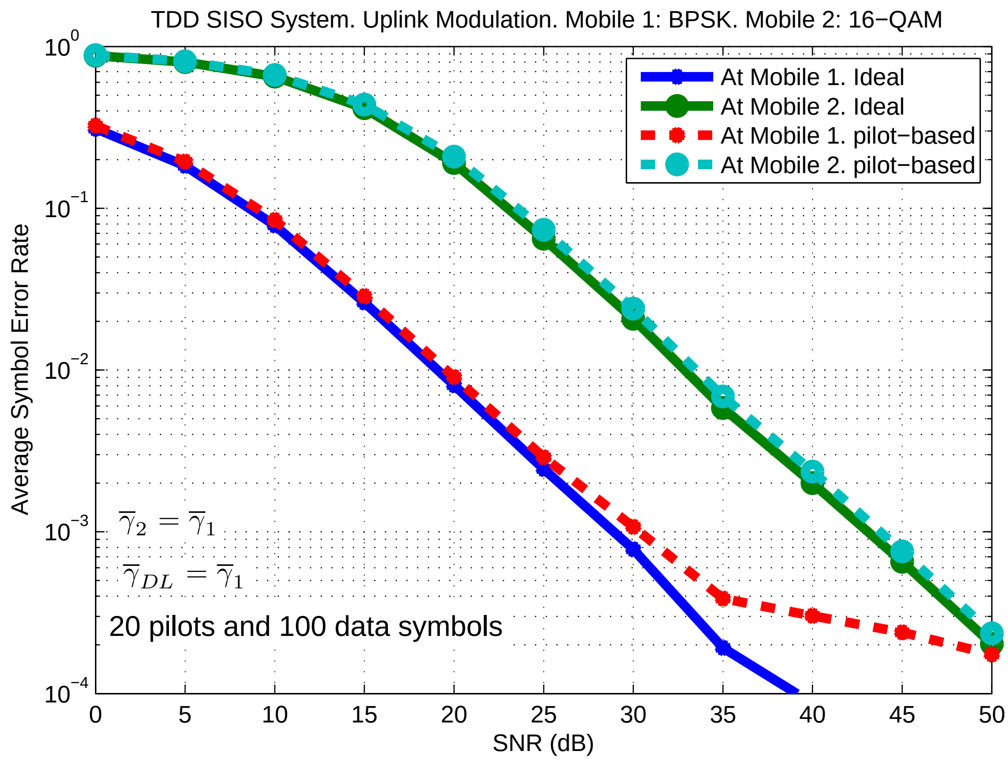


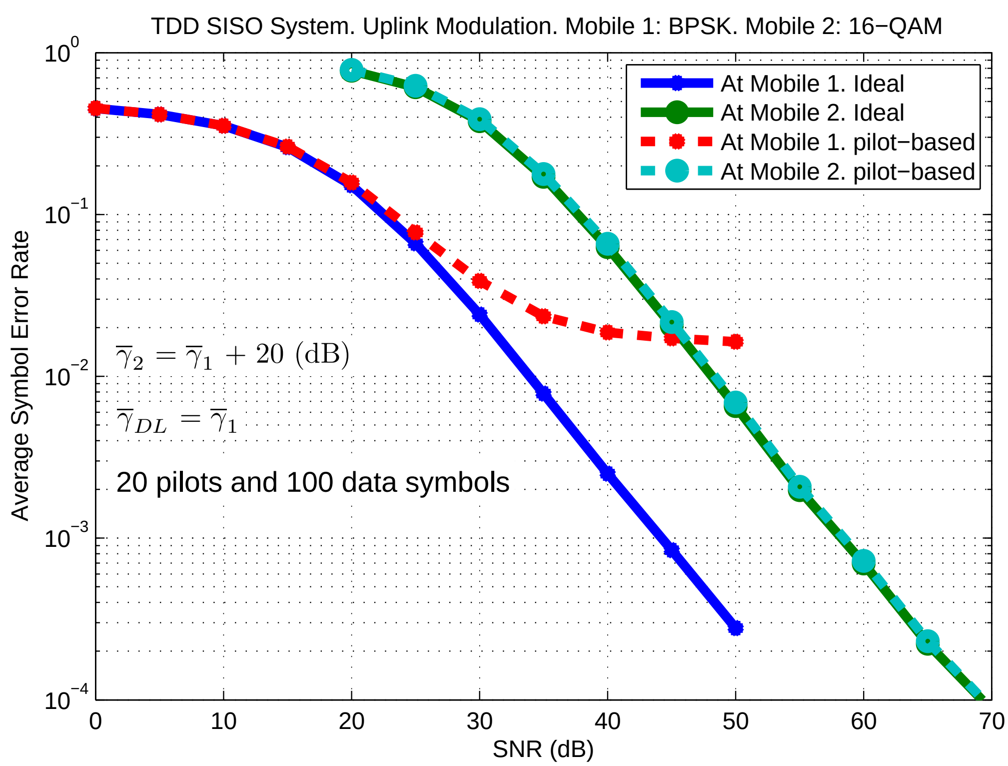
Fig. 5. Comparison of empirical capacity CDFs of the MMSE-BAF system against the baseline system with 4 antennas at the base-station as well as the mobile receivers.

magnitude. It is important to note that once the base-station computes the MMSE beamformer, no additional computation complexity for TAS is required. From Fig. 7, we observe that the pilot-based channel estimation has excellent performance in comparison with the ideal performance and our proposed simple TAS yields an impressive gain of close to 3 dB at an average SER of 10^{-4}.

We have also investigated the feasibility of bidirectional relaying for OFDM/OFDMA-based 4G cellular standards such as IEEE 802.16e [13]. The IEEE 802.16e system is based on OFDMA physical layer for both uplink and downlink. Current mobile WiMax standard supports various sub-channelization procedures, in both uplink and downlink directions, for data transmission in time (OFDM symbols) and frequency (OFDM subcarriers). One such uplink sub-channelization procedure is termed partially utilized sub-channelization (PUSC) wherein the modulation symbols



(a) Equal average uplink received SNRs



(b) Unequal average uplink received SNRs

Fig. 6. Average uncoded SER of MMSE-BAF system with QPSK modulation for user 1 and 16-QAM modulation for user 2.

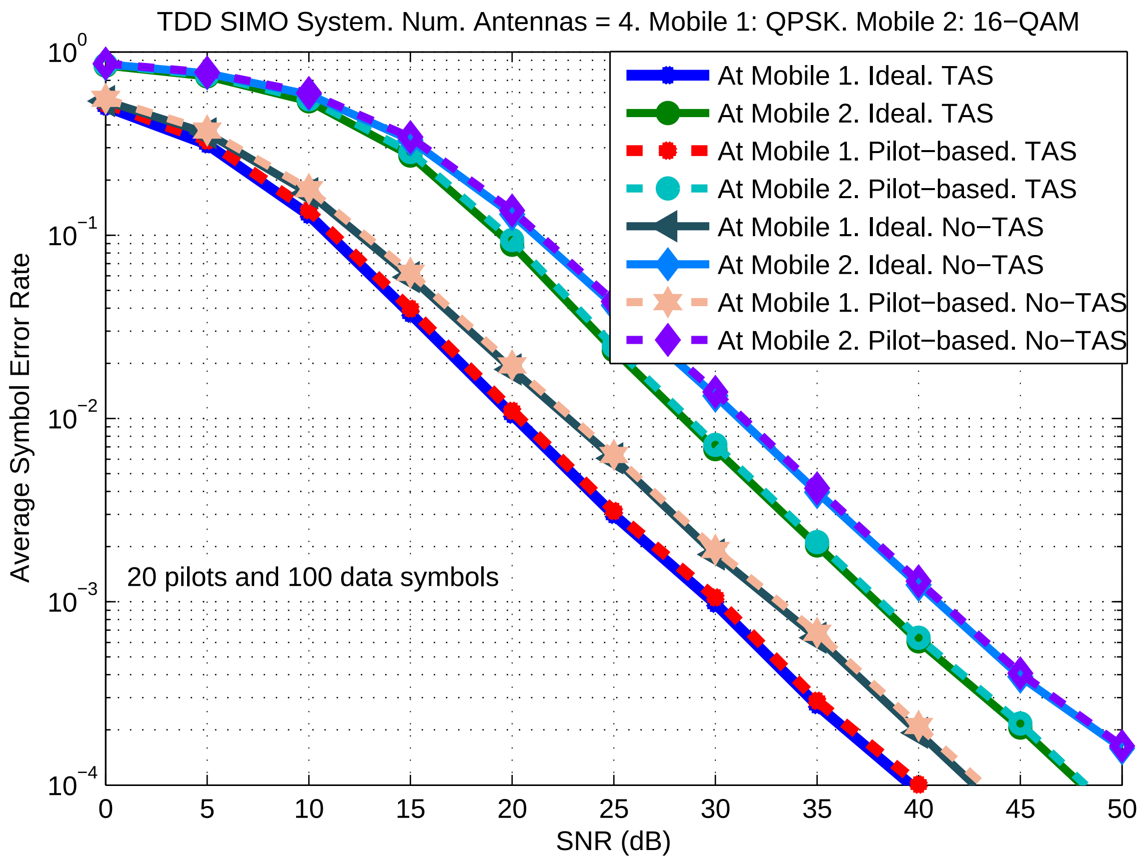


Fig. 7. Performance of transmit antenna selection for TDD-based MMSE-BAF system with 4 antennas at the base-station. User-1 employs QPSK modulation whereas user-2 employs 16-QAM modulation. An uncoded system is considered with realistic channel estimation over block fading channels with 20 pilot and 100 data symbols per fading block.

of a given user are pseudorandomly spread over the frequency band to extract frequency diversity and to average interference across neighboring cells/sectors. Briefly, one slot in UL-PUSC is defined as 48 modulation symbols spanning over three consecutive OFDM symbols (which is a PUSC slot duration). The modulation symbols together with the pilot symbols needed to estimate the uplink channel are sent over 6 tiles distributed over frequency, where a tile is defined as four consecutive subcarriers over three consecutive OFDM symbols. Each tile contains 4 pilot symbols, placed at the corners of the tile, and 8 data symbols. An FEC block in WiMax comprises of a given number of slots and the maximum FEC block size is a function of the modulation order and channel coding rate. The WiMax standard supports 8 modulation order and coding rate combinations. These are QPSK modulation with code rates $1/2$ and $3/4$, 16-QAM

TABLE I

SIMULATION PARAMETERS

Parameter	Value
Bandwidth	10MHz
Sampling Rate	11.2Msps
FFT Size	1024
Subcarrier Spacing	10.9375KHz
Useful Symbol Duration (T_u)	91.4286μsec
Cyclic Prefix (T_G)	$T_u/8$
Useful Subcarriers	840
Left Guard Subcarriers	92
Right Guard Subcarriers	91
Channel Coding	Convolutional Turbo Coding (with 8 iterations)
Carrier Frequency	2.0GHz

modulation with code rates $1/2$ and $3/4$ and 64-QAM modulation with rates $1/2$, $2/3$, $3/4$ and $5/6$. Fig. 8 shows the modified UL-PUSC structure to support bidirectional communications. Each user employs Hadamard sequences as pilot symbols to enable the base-station to estimate the individual channels without interference. For downlink transmission, the base-station can use any sub-channelization procedure. However, to render our proposal valid for FDD as well as TDD systems, the downlink sub-channel structure is set identical to the uplink one and the broadcast pilots from each tile are used for channel estimation at the mobile stations. For efficient cancellation of self-interference, each mobile station requires the knowledge of uplink channel-related parameters β (a positive scalar value) and the two complex scalars values $v_1 := \mathbf{w}_{\text{opt}}^{\mathcal{H}}\mathbf{h}_1$ and $v_2 := \mathbf{w}_{\text{opt}}^{\mathcal{H}}\mathbf{h}_2$. Additional simulation parameters are listed in Table I.

In Figs. 9 and 10, we show the performance of MMSE-BAF when applied to an IEEE 802.16e system in an FDD mode of operation. These simulations are conducted for a base-station with four receive antennas and for a two-antenna mobile station receiver. In these plots, block error rate (BLER) performances of genie-aided perfect channel knowledge are compared against pilot-based realistic channel estimation schemes. For each tile, one channel estimate is obtained by sample averaging the received pilots over that tile. Knowledge of neither the fading statistics

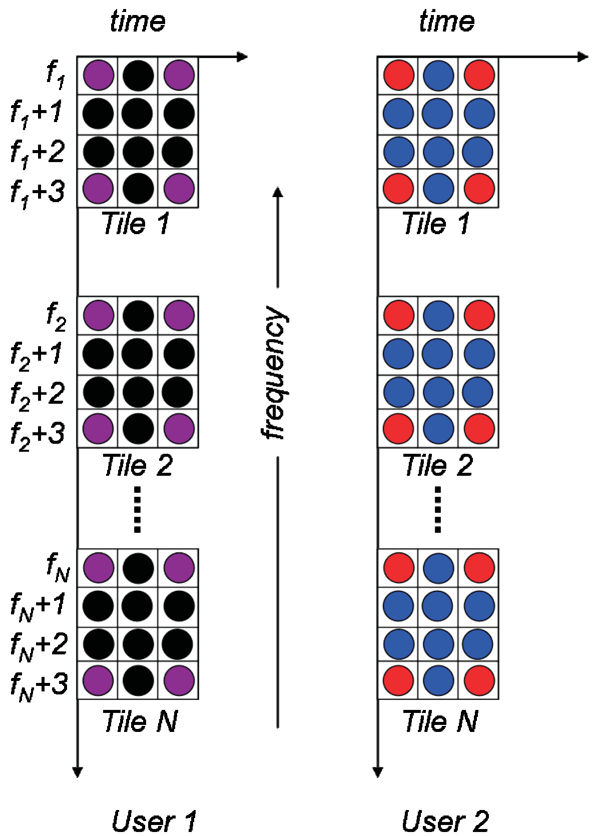


Fig. 8. Sub-channelization procedure for bidirectional communication using the IEEE 802.16e protocol. In uplink, each user is allocated K slots spanning N_f sub-channels in frequency over N_t slot-durations. Each slot-duration comprises of 3 consecutive OFDM symbols, whereas each sub-channel contains 6 tiles distributed throughput the useful frequency band. A tile contains 4 subcarries over a slot-duration with the 4 pilots at the corners of the tile and the remaining 8 tones used for data. In the above figure, there are N tiles with $N_t = 1$, $N_f = N/6$ and $K = N_t N_f = N/6$. The circles filled with violet and red colors indicate the pilot tones of user 1 and 2, respectively, whereas the circles filled with black and blue colors indicate the data tones of users 1 and 2, respectively.

nor the delay/Doppler spread is assumed at the receivers. In Fig. 9, both users are assumed to encode their data using QPSK modulation with rate-$1/2$ convolutional turbo coding (CTC). For this MCS, the FEC block size is set to the maximum allowed, which is equal to 10 slots or $10 \times 48 \times 2 \times 1/2 = 480$ information symbols. We have considered ITU Vehicular-A channel model with both low and high Doppler spread values of 32Hz and 256Hz, respectively. Fig. 9 shows that, under both high and low Doppler scenarios, pilot channel estimation incurs a loss of about 2 dB, and MMSE-BAF works extremely well in supporting data exchanges in high-mobile environments. Fig. 10 shows the performance of MMSE-BAF with 64-QAM modulation and a CTC with code rate of $1/2$. With this MCS level, supporting a per-user over-the-air spectral efficiency of 3 bits/sec/Hz, we have employed the maximum possible FEC block size of 2 slots, or $2 \times 48 \times 6 \times 1/2 = 288$ information bits. Fig. 10(a) shows the block error performance over ITU-Vehicular-A channel with 32 Hz Doppler, whereas Fig. 10(b) shows the performance on a Pedestrian-B channel with 6 Hz Doppler. Due to higher frequency-selectivity of Pedestrian-B channel, compared with the Vehicular-A channel where the channel estimation-based BLER performance (at 1 percent BLER) is about 2 dB away from the ideal performance, the simple sample-average based channel estimation has a performance degradation of about 3.5 dB compared to the performance with perfect channel knowledge. It is expected that the BLER performance can be significantly improved by incorporating a more complex two-dimensional channel estimation scheme, such as Wiener filtering, which requires knowledge of fading statistics as well as Delay/Doppler spread information.

V. Conclusion

In this paper, we have introduced a so-called MMSE-BAF protocol for bi-directional communications over two-way relay channels with multi-antenna relay nodes. The features of this protocol include the usage of analog network coding at the relay node and the evaluation of a receive weight vector for the relay node using a joint linear MMSE filtering operation on the received uplink multiple access signal. Transmit antenna selection using the largest MMSE weight branch on the downlink is also an inherent feature of the proposed protocol in the TDD mode of operation. Extensive link-level simulations have been proposed for both TDD and FDD modes of operations and required modifications to the existing IEEE 802.16e standard have been proposed to accommodate the MMSE-BAF protocol. It has been shown through simulation results that the MMSE-BAF protocol is a simple yet efficient solution to the problem of bi-directional communications in two-way relay channels with multi-antenna relays and half-duplex nodes.

APPENDIX A
RECEIVED SNRS WITH NOISE WHITENING

We first re-write the noise covariance matrices $\mathbf{\Sigma}_1$ (13) and $\mathbf{\Sigma}_2$ (24), using their eigenvalue decompositions, as

$$\begin{aligned} \mathbf{\Sigma}_1 &= \mathbf{P}_1^{\mathcal{H}} \mathbf{\Lambda}_1 \mathbf{P}_1 \quad (26) \\ \text{and} \qquad \mathbf{\Sigma}_2 &= \mathbf{P}_2^{\mathcal{H}} \mathbf{\Lambda}_2 \mathbf{P}_2, \quad (27) \end{aligned}$$

where $\mathbf{P}_1$ and $\mathbf{P}_2$ are unitary matrices and $\mathbf{\Lambda}_1$ and $\mathbf{\Lambda}_2$ are diagonal matrices containing the eigenvalues of $\mathbf{\Sigma}_1$ and $\mathbf{\Sigma}_2$, respectively.

Let us now focus on demodulating x_2 from $\mathbf{r}_1$ by whitening the noise $\widetilde{\mathbf{n}}_1$. Let

$$\begin{aligned} \mathbf{s}_1 &= \mathbf{\Lambda}_1^{-\frac{1}{2}} \mathbf{P}_1 \mathbf{r}_1 \\ &= \mathbf{\Lambda}_1^{-\frac{1}{2}} \mathbf{P}_1 \left(\mathbf{g}_1 \beta v_2 x_2 + \widetilde{\mathbf{n}}_1 \right). \quad (28) \end{aligned}$$

Since, conditioned on $\mathbf{\Lambda}_1$ and $\mathbf{P}$,

$$\mathrm{E}\left[\left(\mathbf{\Lambda}_1^{-\frac{1}{2}} \mathbf{P}_1 \widetilde{\mathbf{n}}_1 \right) \left(\mathbf{\Lambda}_1^{-\frac{1}{2}} \mathbf{P}_1 \widetilde{\mathbf{n}}_1 \right)^{\mathcal{H}} \right] = \boldsymbol{I}_{M_{ss}} \quad (29)$$

it follows from (28) that the instantaneous received SNR of x_2 by whitening $\mathbf{r}_1$ is simply

$$\begin{aligned} \gamma_2' &= \sigma_2^2 \left\| \mathbf{\Lambda}_1^{-\frac{1}{2}} \mathbf{P}_1 \mathbf{g}_1 \beta v_2 \right\|^2 \\ &= \sigma_2^2 \beta^2 |v_2|^2 \mathbf{g}_1^{\mathcal{H}} \mathbf{P}_1^{\mathcal{H}} \mathbf{\Lambda}_1^{-\frac{1}{2}} \mathbf{\Lambda}_1^{-\frac{1}{2}} \mathbf{P}_1 \mathbf{g}_1 \\ &= \sigma_2^2 \beta^2 |v_2|^2 \mathbf{g}_1^{\mathcal{H}} \mathbf{\Sigma}_1^{-1} \mathbf{g}_1. \quad (30) \end{aligned}$$

In a similar manner, upon whitening $\mathbf{r}_2$ to demodulate x_1, the instantaneous received SNR of x_1 becomes

$$\gamma_1' = \sigma_1^2 \beta^2 |v_1|^2 \mathbf{g}_2^{\mathcal{H}} \mathbf{\Sigma}_2^{-1} \mathbf{g}_2. \quad (31)$$

Using the following matrix inversion lemma (MIL) [14]

$$\left(\boldsymbol{I}_{M_{ss}} + \mathbf{x}\mathbf{x}^{\mathcal{H}} \right)^{-1} = \boldsymbol{I}_{M_{ss}} - \frac{\mathbf{x}\mathbf{x}^{\mathcal{H}}}{1 + \|\mathbf{x}\|^2}, \quad (32)$$

where $\mathbf{x}$ is a column-vector of appropriate size, it is possible to further simplify (30) as

$$\begin{aligned} \gamma_2' &= \frac{\sigma_2^2 \beta^2 |v_2|^2}{\sigma_N^2} \times \mathbf{g}_1^{\mathcal{H}} \left(\boldsymbol{I}_{M_{ss}} - \frac{\beta^2 \|\mathbf{w}_{\text{opt}}\|^2 \mathbf{g}_1 \mathbf{g}_1^{\mathcal{H}}}{1 + \beta^2 \|\mathbf{w}_{\text{opt}}\|^2 \|\mathbf{g}_1\|^2} \right) \mathbf{g}_1 \\ &= \frac{\sigma_2^2 \beta^2 |v_2|^2}{\sigma_N^2} \times \frac{\|\mathbf{g}_1\|^2}{1 + \beta^2 \|\mathbf{w}_{\text{opt}}\|^2 \|\mathbf{g}_1\|^2}. \quad (33) \end{aligned}$$

In a similar manner, application of MIL in (31) leads to

$$\gamma_1' = \frac{\sigma_1^2 \beta^2 |v_1|^2}{\sigma_N^2} \times \frac{\|\mathbf{g}_2\|^2}{1 + \beta^2 \|\mathbf{w}_{\rm opt}\|^2 \|\mathbf{g}_2\|^2}. \tag{34}$$

As we mentioned earlier, expressions for γ_2' (33) and γ_1' (34) are respectively identical to SNRs γ_2 (25) and γ_1 (22) derived in Section III-B without performing noise whitening. This somewhat counterintuitive result leads us to conclude that SNR improvement is not an option with noise whitening when employing the MMSE-BAF protocol.

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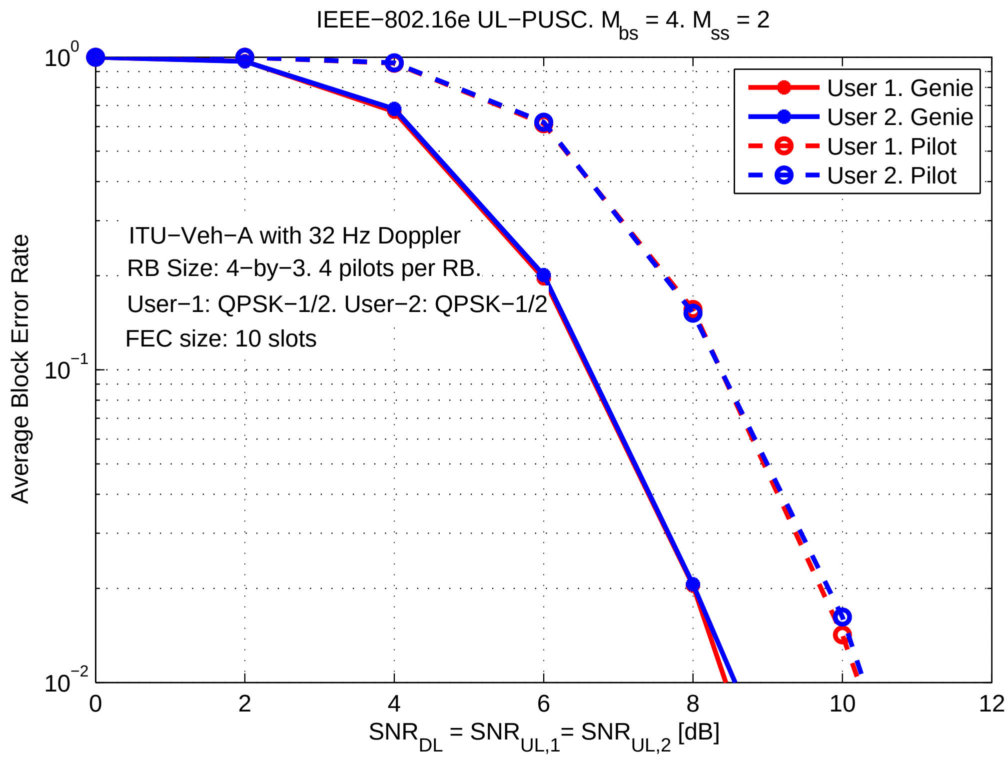
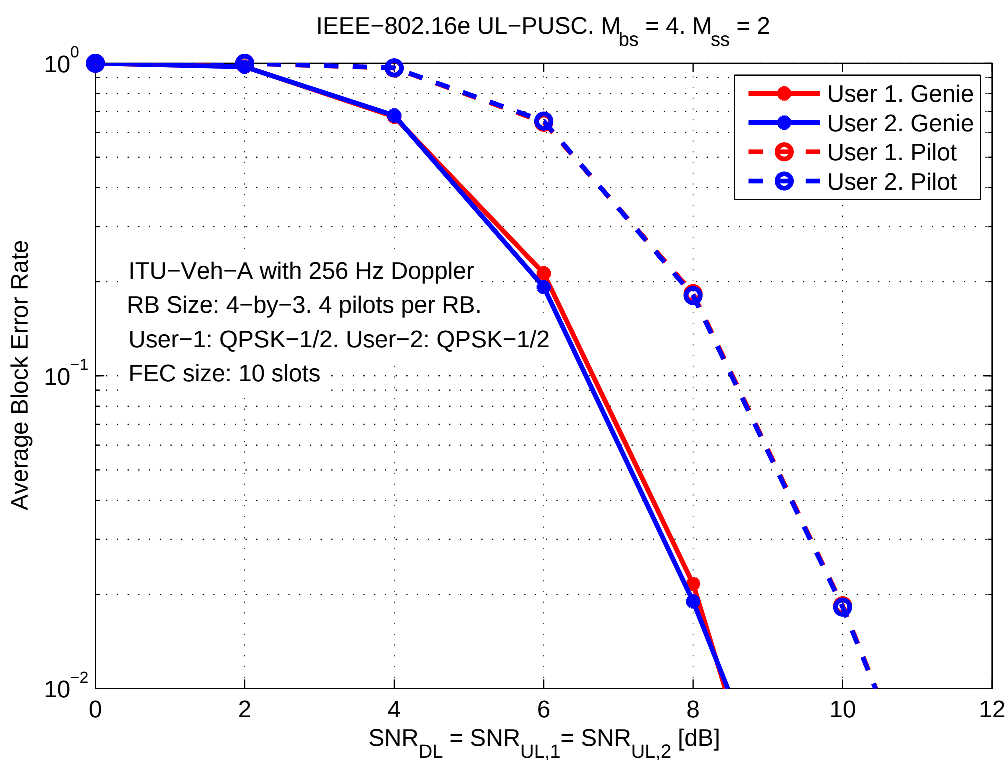
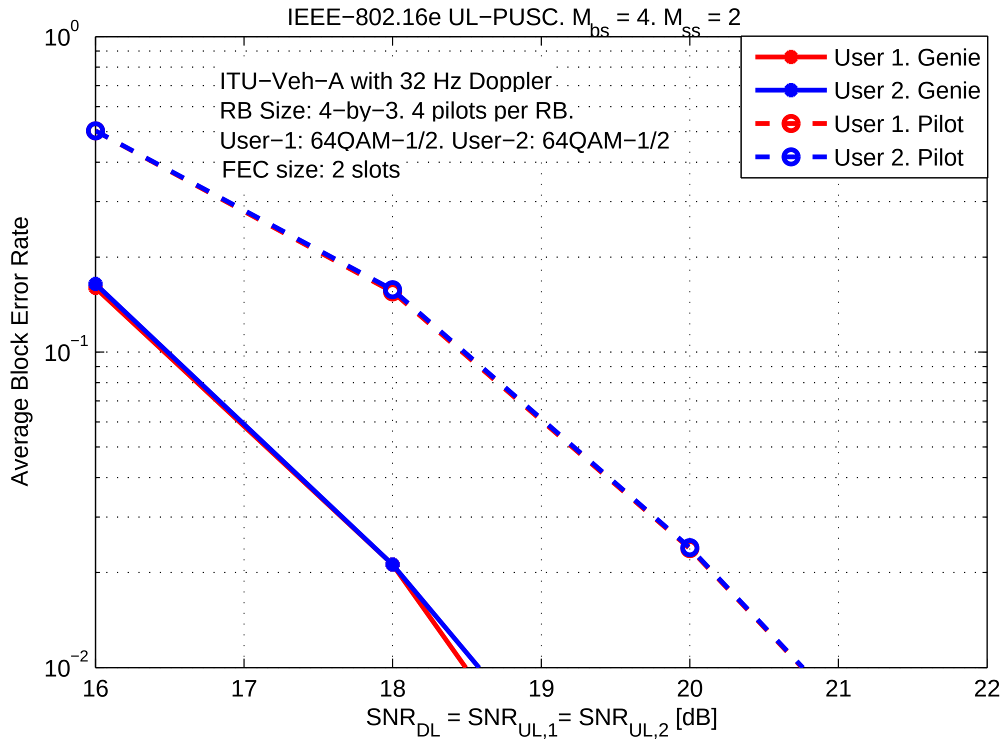


(a) With 32 Hz Doppler

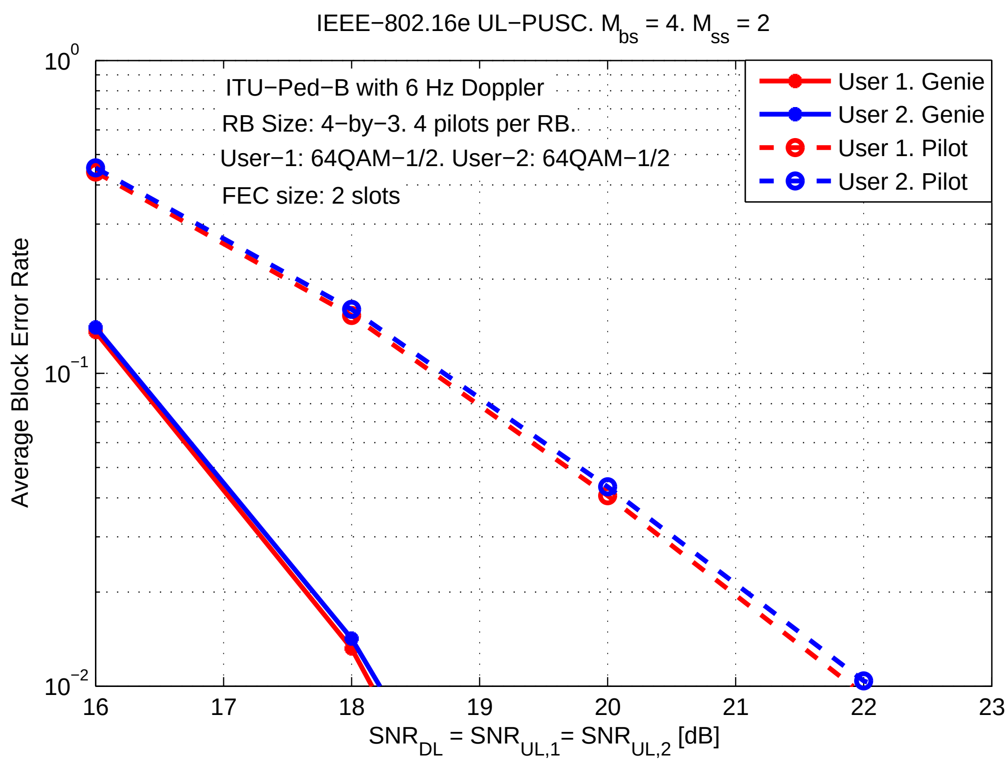


(b) With 256 Hz Doppler

Fig. 9. Performance of MMSE-BAF on MIMO-OFDMA based IEEE 802.16e system using uplink partial utilization of sub-channelization (PUSC) permutation. Each user employs rate-1/2 convolutional turbo coding (CTC) with QPSK modulation. The FEC block length is 480 information bits which corresponds to 10 slots, with 48 modulation symbols per slot, as per the terminology in [13].



(a) ITU-Veh-A channel with 32 Hz Doppler



(b) ITU-Ped-B channel with 6 Hz Doppler

Fig. 10. Performance of MMSE-BAF on MIMO-OFDMA based IEEE 802.16e system using uplink partial utilization of sub-channelization (PUSC) permutation. Each user employs rate-1/2 convolutional turbo coding (CTC) with QPSK modulation. The FEC block length is 288 information bits which corresponds to 2 slots, with 48 modulation symbols per slot, as per the terminology in [13].