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# Off-Grid Portable EV Charging Network Management with Dynamic Energy Pricing

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**Abstract**—With emerging of electric vehicles (EV), portable charging stations (PCSs) will play a key role to manage EV charging operations off-grid. Otherwise, charging hundreds of EVs at random locations and time instants would create major burden on a power supply network. This work addresses an outstanding issue in PCS networks: development of dynamic pricing strategies between buyer EVs and energy sellers to optimize deployment of PCSs. Constraints are formulated, and a realistic and yet simplistic energy incentive model is developed. A method for optimum PCS deployment to maximize profit for PCS service providers is developed for single-buyer-single-seller and multiple-buyers-single-seller cases, conditioned on a given pricing strategy.

## I. INTRODUCTION

Transportation alone utilizes more than two thirds of the oil consumed in the United States and is responsible for about 6,000 million metric tons of carbon dioxide emissions in 2008 [1]. With the long term increase in energy cost and the challenge of global warming, electric vehicle is fast becoming recognized not only as an economically viable alternative to traditional internal combustion engine based transportation but also offered as an effective green transportation.

According to recent projection, electric vehicles (EV) will begin its roll out in large volume in 2010, and will reach one million around 2015 in the USA [2]. At the same time, this roll out will need to be well supported by an appropriate infrastructure such as outdoor battery charging stations and by supportive government policy and regulations. In North America, electric vehicles will likely be mainly charged at home. However, there are many big cities where majority of the households do not have their own garages, less to say a vehicle charger. Hence, charging may take place at work or in and around the city. Infrastructure and appropriate methodology for charging electric vehicle being adopted to ensure minimum outage and maximum energy efficiency are vital to the success and long-term viability of the electric vehicle industry.

There are three categories of electric chargers that EVs can be charged from, namely, fixed, nomadic and mobile energy storage. A fixed energy storage is the grid infrastructure while nomadic energy storages can vary from a simple high capacity EVs to very large scale premium power storages shown in Fig. 1.

Nomadic energy storage systems are ones which can be easily relocated but not mobile. On the other hand the third category of energy storage system is mobile systems, which in

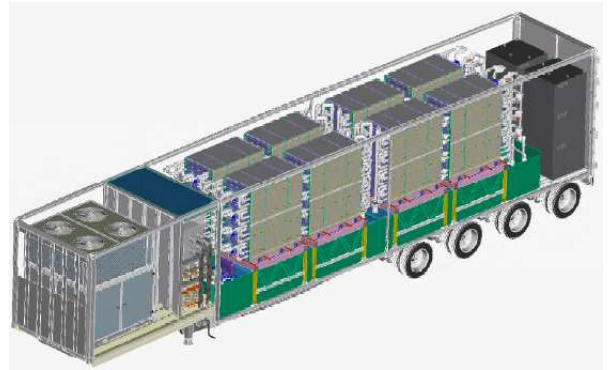


Fig. 1. Illustration of a portable energy storage [Adopted from [3]].

this paper is also referred to as portable systems. A portable storage system implies transferring the drawing of energy not from the classical concept of grid, but from moving electric-powered devices either for its own use or to sell its stored energy later on to a buyer.

The most commonly known portable energy storage is the EV which provides plenty of opportunity for utility companies to more efficiently manage their generating capacity by exploiting under-utilized energy capacity during the nights when many of the power generators have to continue to run and wind energy is usually at its peak. By allowing EVs and other forms of energy storage stations to charge up during the nights, they would help to "soak up" the energy which would otherwise be wasted. These charged vehicles and energy storage stations could then deliver the energy back to consumers, including those vehicles which have no means to charge during the nights.

Among the most important enablers of EV and portable energy storage station are advances in particularly Li-ion battery technologies, which have made longer range electric vehicles possible [4],[5].

This paper presents the use of optimum distribution of portable energy storage stations to reduce outage and maximize energy efficiency of an EV network. Here, in this paper, we classify the electric vehicle into two categories, namely, the buyers and the sellers. We will discuss the physical and virtual distance tolerances of the buyers and sellers and develop energy efficiency and incentive models for uncoordinated electric vehicles and the optimum locations for which the

buyer and sellers should meet. To date, there is almost no known literature discussing the relationship of portable energy storage stations and the potential electric vehicle network. This paper is among the first to help establish and understand the dynamics and the energy requirements of EV networks.

## II. MODELING CONSTRAINTS

### A. Physical Distance Tolerance

Assume that EV  $i$  starts its trip at a constant velocity  $v_i$  with initial energy level  $e_i$  between two points A and B, which are separated by  $r_{AB}$  and also that energy consumption per unit distance traversed is  $k_i$ . If  $e_i < k_i r_{AB}$ , recharging during its trip would be mandatory for EV  $i$  to prevent energy outage. We model physical distance tolerance (PDT) as a function of time and define it as the distance that can be traversed by an EV without recharging its battery. We express PDT as

$$T_p^{(i)}(t) \equiv \max\left(0, \frac{e_i}{k_i} - v_i(t)t\right). \quad (1)$$

In Fig.2, a typical PDT is illustrated. The tolerance decreases as an EV moves along its route to a destination, and becomes zero upon draining its battery. It is essential for an EV to recharge its battery within its PDT, if its energy reserve is not sufficient to reach its destination. Otherwise, for the EV energy outage occurs.

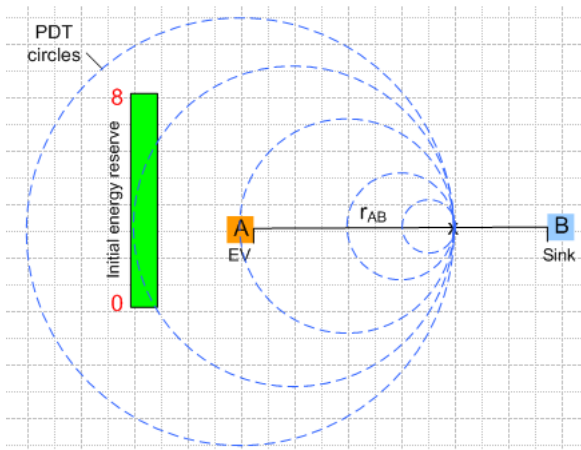


Fig. 2. Illustration of PDT circles. The initial energy reserve of the EV is shown to be 8 units. It is assumed that 1 unit energy is consumed per grid unit distance. The PDT becomes zero before reaching destination B. PDT is trip distance independent.

### B. Virtual Distance Tolerance

We assume that EV  $i$  would be willing to deviate for a certain distance from its original route to perform a critical ad-hoc task such as recharging its battery. Maximum possible deviation at a given time is called the virtual distance tolerance (VDT), and denoted by  $T_v^{(i)}(t)$ . Larger deviations are assumed to be more likely at an earlier stage of a longer trip. During

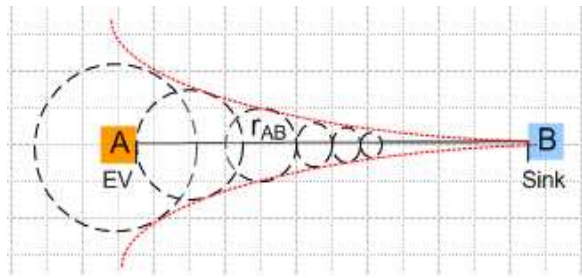


Fig. 3. Illustration of VDT circles. The VDT circles become infinitesimally small closer to destination. VDT is trip distance dependent.

shorter trips, deviations would be less than those during longer ones. We model  $T_v^{(i)}(t)$  as

$$T_v^{(i)}(t) \equiv \max\left(0, e^{-\lambda t}(r_{AB} - v_i(t)t)\right), \quad (2)$$

where  $\lambda$  is called a deviation decay constant, and determined the envelope of the VDT circles.

In Fig.3, typical VDT circles are illustrated. The tolerance decreases as the EV gets closer to its destination B.

A hard constraint on VDT tends to increase energy outage. Such EVs may drain their batteries before finding a PCS within their VDT circle. VDT can't be larger than PDT for an EV.

### C. Energy Inefficiency

Energy inefficiency (EI),  $\rho_i$ , of an EV  $i$  is defined as the energy consumption overhead due to a deviation from an original route. It is upper bounded by  $2T_v^{(i)}(T_0)/r_{AB}$ , where  $T_0$  indicates the time instant the deviation takes place. The factor of 2 is because of round-trip deviation.

$$\begin{aligned} \rho_i &= \frac{\Delta r}{r_{AB}} \\ &\leq \frac{2T_v^{(i)}(T_0)}{r_{AB}}. \end{aligned} \quad (3)$$

In the case that an EV can't move to a PCS due to any PDT or VDT constraint, a PCS should move towards the EV to charge it. Early charging is better to lower energy outage, because the feasible region of an EV determined by the VDT and PDT shrinks in time. This introduces another factor called *seller virtual distance tolerance* (SVDT),  $T_v^{(s)}(t)$ . The SVDT indicates how far a PCS is willing to relocate from its current position to assist an EV. At this point, it becomes essential to introduce an incentive mechanism for a PCS to encourage its movement towards an EV and for the EV to enlarge its VDT.

## III. ENERGY INCENTIVE MODEL

Assume that the distance between an EV that wants to buy  $E_B(x=0)$  amount of energy and a seller PCS is  $d$ , which is much longer than the VDTs of both the EV and the PCS, as illustrated in Fig.4. The shaded region indicates that they are not located within each others VDT.

Therefore, if the EV is not recharged, energy outage occurs. To prevent outage, incentives should be given to both the seller and buyer. In our model, we consider that the price of unit

energy is  $C_e$  and that it is increased by  $\alpha_s C_e$  for a segment of the normalized distance the PCS traverses towards the buyer EV. Similarly, the price of unit energy is reduced by  $\alpha_b C_e$  corresponding to the normalized distance the EV traverses towards the PCS.

The resulting revenue for the PCS is given by

$$G(x^*) = (E_B(0) + x^* k_i) C_e \left( 1 + \frac{-\alpha_b x^* + \alpha_s (d - x^*)}{d} \right), \quad (4)$$

where  $x^* \in [0, d]$  denote the location at which the EV and PCS agree to meet for energy transaction. The seller consumes energy during relocating itself. The cost of this energy is given by

$$S_c(x^*) = (d - x^*) k_s C_s, \quad (5)$$

where  $C_s$  the price of unit energy the seller pays for, and  $k_s$  is the energy consumption per unit distance for the seller. At a given meeting point  $x^*$ , the difference of movement costs for the seller and buyer is called the relative cost, and it is given by

$$\Delta S(x^*) = G(x^*) - E_B(0) C_e - S_c(x^*) \quad (6)$$

A fair meeting point that results in equal cost burden on the seller and buyer is called a *zero-relative-cost point*.

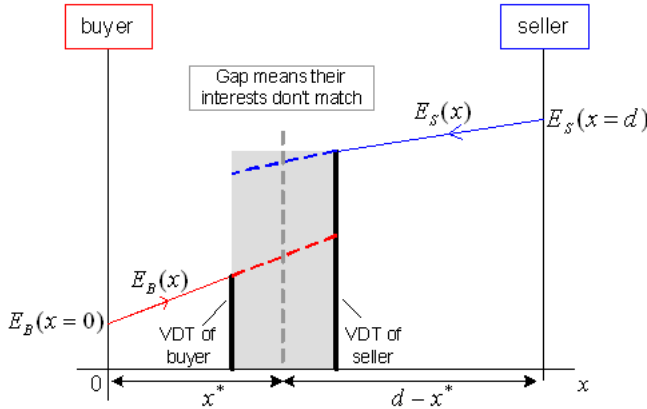


Fig. 4. Illustration of an incentive model in which neither the PCS (seller) nor the buyer (EV) is greedy. They equally compromise and move towards each other.  $E_B(x)$ : energy demand of buyer as a function of distance  $x$  from its original location,  $E_S(x)$ : available energy at the seller as a function of  $x$ .

It is reasonable to assume that there is a price upper bound from a buyer's point of view, and lower bound from the seller's point of view. To factor in these bounds, we need to define a feasible area for both  $\alpha_b$  and  $\alpha_s$ .

**Proposition:** For the price of energy (inclusive of incentives)  $C_{eff}$  to vary within  $[\frac{C_e}{2}, 2C_e]$ ,  $\alpha_s$  and  $\alpha_b$  should satisfy  $\alpha_s \leq 1$  and  $\alpha_b \leq \frac{1}{2}$ .

**Proof:** From (4), the effective energy price  $C_{eff}$  is given by

$$C_{eff} = C_e \left( 1 + \frac{-\alpha_b x^* + \alpha_s (d - x^*)}{d} \right). \quad (7)$$

Then, the following condition should be valid.

$$\frac{1}{2} \leq 1 + \frac{-\alpha_b x^* + \alpha_s (d - x^*)}{d} \leq 2. \quad (8)$$

For  $y = \frac{x^*}{d}$ , (8) is equivalent to  $-\frac{1}{2} \leq f(y) \leq 1$ , where  $f(y) = -(\alpha_b + \alpha_s)y + \alpha_s$  and  $y \in [0, 1]$ . Note that the conditions  $f(0) = \alpha_s > 0$  and  $f(1) = -\alpha_b < 0$  hold for  $\alpha_s \leq 1$  and  $\alpha_b \leq \frac{1}{2}$ .

The buyer rejects any arrangement that would require unit energy price to be higher than  $2C_e$ , and the seller would not sell energy any cheaper than  $C_e/2$ . Request for a price outside these bounds prevents reaching a consensus, and results in energy outage for the buyer. The corresponding feasible region for  $\alpha_b$  and  $\alpha_s$  forms a rectangle as shown in Fig.5.

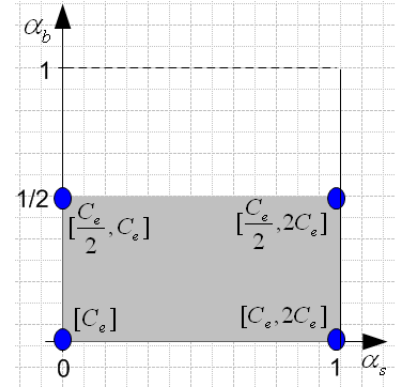


Fig. 5. Feasible region for  $\alpha_b$  and  $\alpha_s$ . Inside the feasible region the price of unit energy after incentives varies between  $C_e/2$  and  $2C_e$ .

On the one hand, the meeting location that would maximize the seller's revenue,  $x_{mr}^*$ , can be simply computed by solving  $\frac{\partial G}{\partial x^*} = 0$ , and is given by

$$x_{mr}^* = -\frac{E_B(0)}{2k_i} + \frac{d(1 + \alpha_s)}{2(\alpha_s + \alpha_b)}, \quad (9)$$

On the other hand, the zero-relative-cost point can be found by solving  $\Delta S(x^*) = 0$  for  $x^*$ , which results in a quadratic equation. A solution that  $x^* < 0$  is discarded if there is a single seller and a single buyer, because  $x^* \in [0, d]$ .

#### IV. SINGLE BUYER SINGLE SELLER (SBSS) CASE INCENTIVE ANALYSIS

##### A. Equal Incentive Case

In the equal incentive case,  $\alpha_b = \alpha_s = \alpha$ . When the seller and buyer traverse an equal distance towards each other, the impact of incentives on the energy price is canceled. Note that while relocating both the seller and buyer consume energy. Then, a zero-relative-cost point when  $C_e = C_s$  and  $k_i = k_s$  is found as

$$x_{ei}^* = \frac{d + T_v^{(b)} - T_v^{(s)}}{2} = d/2, \quad (10)$$

Note that  $\Delta S(x_{ei}^*) = 0$ . Factors such as trip delay, service time and etc would certainly induce additional costs on PCS

and EV, and impact  $x_{ei}^*$ . We ignore such external cost factors in this paper due to space limitation.

Furthermore,  $x_{ei}^*$  may not necessarily be the same as  $x_{mr}^*$ . In Fig.6, the relative cost of movement is given for various  $\alpha$  values. Note that for  $\alpha = 0.5$ , the solutions that satisfy  $\Delta S(x_{ei}^*) = 0$  are  $x_{ei}^* = 5$  and  $x_{ei}^* = 10$ .

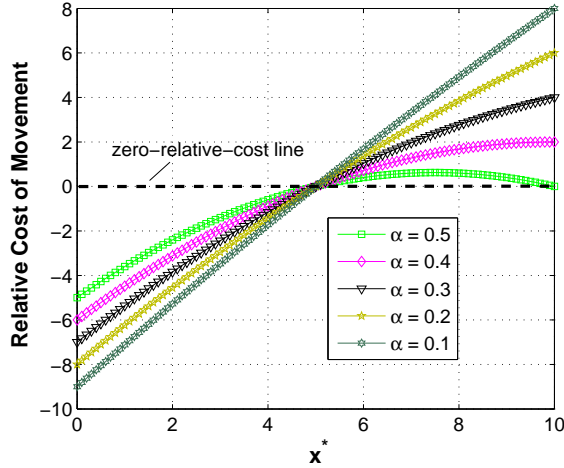


Fig. 6. Numerical analysis of a meeting point at which the relative cost of movement is zero. In other words, movement costs equally to both the seller and the buyer. The settings are as follows:  $k_i = k_s = 1$ ,  $C_e = C_s = 1$ ,  $E_B(0) = 10$  and  $d = 10$ .

### B. B-favored Case

In the b-favored case, illustrated in Fig.7, the buyer EV is willing to move towards the seller as long as the seller PCS reduces the unit energy price by  $\alpha_b C_e$  for portion of the distance  $d$  traveled by the buyer. Then, the EV enlarges its VDT to reach a meeting point. In this case, incentive for the buyer movement dominates the incentive for the seller movement. Therefore,  $\alpha_b$  is higher than  $\alpha_s$ , and bounds for a *zero-relative-cost point*  $x_{bf}^*$  conditioned on  $k_i = k_s$  and  $C_e = C_s$  is given by

$$x_{ei}^* \leq x_{bf}^* \leq T_p^{(b)}. \quad (11)$$

The *zero-relative-cost point*  $x_{bf}^*$  is upper bounded by the buyer PDT,  $T_p^{(b)}$ . In Fig.8, the relative cost of movement is shown with  $\alpha_s = 0.2$  for  $\alpha_b = \{0.3, 0.4, 0.5\}$ . Note that there is always an  $x_{bf}^*$  that is greater than  $x_{ei}^*$ .

### C. S-favored Case

In the s-favored case, the seller is given more incentive to approach a buyer. In other words, the buyer is willing to pay more for unit energy and encourages the seller relocation towards itself as shown in Fig.9. Thus,  $\alpha_b$  is smaller than  $\alpha_s$ , and the bounds for the *zero-relative-cost point*  $x_{sf}^*$  conditioned on  $k_i = k_s$  and  $C_e = C_s$  is given by

$$T_v^{(b)} \leq x_{sf}^* \leq x_{ei}^*. \quad (12)$$

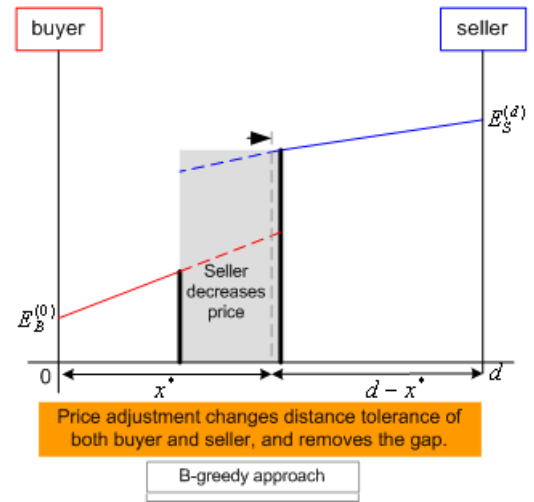


Fig. 7. Illustration of an incentive model in which the buyer EV is greedy, and it forces the PCS to lower its unit energy price. In return, the EV moves towards to PCS to buy energy.

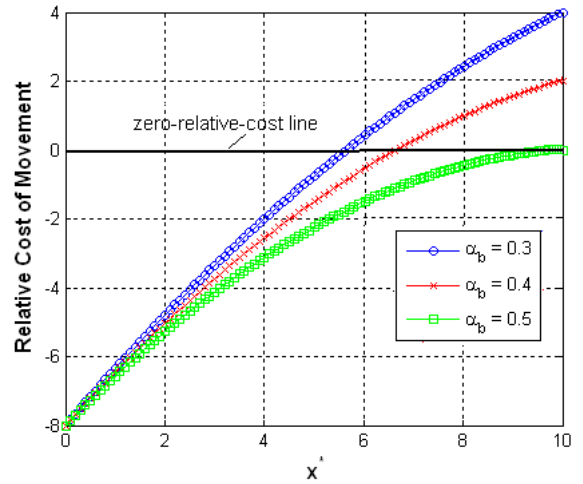


Fig. 8. Illustration of the relative cost of movement versus meeting points, when the buyer incentive  $\alpha_b$  is higher than the seller incentive  $\alpha_s = 0.2$ . Other settings are as follows:  $k_i = k_s = 1$ ,  $C_e = C_s = 1$ ,  $E_B(0) = 10$  and  $d = 10$ .

It is to the benefit of the EV to traverse a distance towards the PCS by  $T_v^{(b)}$  to reduce  $G(x^*)$ . Therefore,  $x^*$  is lower bounded by  $T_v^{(b)}$ .

In Fig.10, the relative cost of movement is shown with  $\alpha_b = 0.2$  for  $\alpha_s = \{0.3, 0.4, 0.5, 0.6\}$ . Note that there is always an  $x_{sf}^*$  that is shorter than  $x_{ei}^*$ .

## V. MULTIPLE BUYER SINGLE SELLER (MBSS) CASE INCENTIVE ANALYSIS

In the multiple buyer case, a seller shall jointly consider buyer behaviors and the incentives that they can settle for a meeting point. Let  $\Omega_V$  with cardinality  $V$  denote the entire set of buyer EVs that require energy from a portable charging station  $v$ . The location coordinates of EV  $i$  is assumed to be

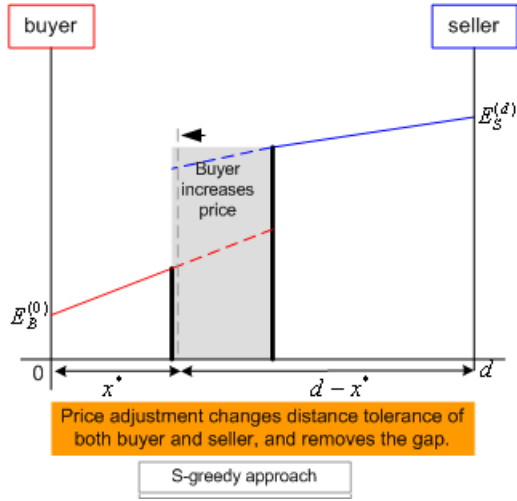


Fig. 9. Illustration of an incentive model in which the seller (PCS) is greedy, and it forces the EV to increase its unit energy purchase price. In return, the PCS moves towards to EV to sell energy.

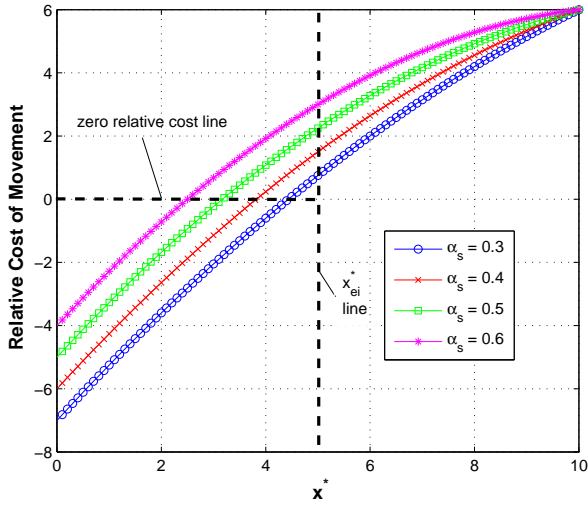


Fig. 10. Illustration of the relative cost of movement versus meeting points, when the seller incentive  $\alpha_s$  is higher than the buyer incentive  $\alpha_b = 0.2$ . Other settings are as follows:  $k_i = k_s = 1$ ,  $C_e = C_s = 1$ ,  $E_B(0) = 10$  and  $d = 10$

known and given by  $\mathbf{r}_i = [x_i, y_i]$  for  $i = 1, 2, \dots, V$  and that of PCS  $s$  by  $\mathbf{r}_s = [x_s, y_s]$ .

Assume that there is a zero-relative-cost meeting coordinate denoted by  $\mathbf{r}_{mp} \in \mathbb{R}^2$ . Analysis of the SBSS case in the previous section has revealed that any meeting point equidistant from both the seller and a buyer is a zero-relative-cost point. In a two dimensional Euclidian space  $\mathbb{R}^2$ , the set of the zero-relative-cost points form a line that is orthogonal to the shortest distance line between the seller and the buyer as shown in Fig.11. The line segments that fall outside the PDT circle of the buyer are infeasible, because the buyer simply doesn't have enough energy to travel to those locations.

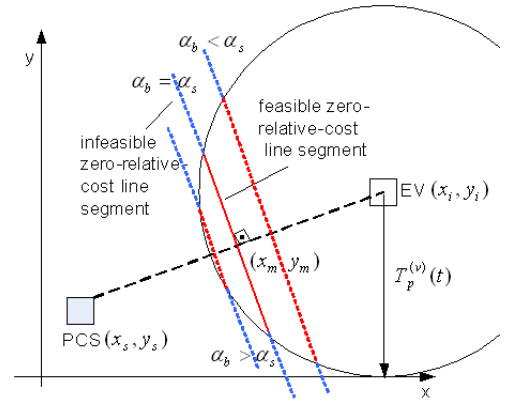


Fig. 11. Illustration of a feasible and infeasible zero-relative-cost line segments in a 2 dimensional Euclidian space.

In the MBSS case, to determine the globally zero-relative cost meeting point, a PCS first needs to find the feasible zero-relative cost line segment with each buyer, and then search for the intersection of all those segments. Let  $\Omega_F$  with cardinality  $F$  denote a set of meeting point coordinates  $\mathbf{r}_{mp}(f)$  for  $f = 1, 2, \dots, F$  such that at least two feasible segments cross through each point. Also, let  $\Omega_I^{(f)}$  and  $P_I(f)$  denote a set of buyers to be served at point  $\mathbf{r}_{mp}(f)$ , and the corresponding profit to be generated for the seller, respectively. One approach to select a meeting point is formulated in (13), which is to find the one that maximizes seller's profit.

$$\mathbf{r}_{mp}(f^*) = \arg \max_{f \in \Omega_F} \left( \underbrace{G(\mathbf{r}_{mp}(f)) - S_c(\mathbf{r}_{mp}(f))}_{P_I(f)} \right), \quad (13)$$

where  $G(\mathbf{r}_{mp}(f)) = \sum_{i \in \Omega_I^{(f)}} (E_B^{(i)} + \|\mathbf{r}_{mp}(f) - \mathbf{r}_i(f)\| k_i) C_e \left( 1 + \frac{-\alpha_b \|\mathbf{r}_{mp}(f) - \mathbf{r}_i(f)\| + \alpha_s \|\mathbf{r}_{mp}(f) - \mathbf{r}_s(f)\|}{\|\mathbf{r}_{mp}(f) - \mathbf{r}_i(f)\| + \|\mathbf{r}_{mp}(f) - \mathbf{r}_s(f)\|} \right)$  and  $S_c(\mathbf{r}_{mp}(f)) = \|\mathbf{r}_{mp}(f) - \mathbf{r}_s(f)\| k_s C_s$ . When it is not possible for all feasible segments to intersect, then to increase the number of intersecting segments, one available option is to vary  $\alpha_b$  and  $\alpha_s$ , which would jitter the feasible segment to the left or right of the segment defined for  $\alpha_s = \alpha_b$  (see Fig.11). In selecting a meeting point, other criteria, which we don't discuss in this paper, may include

- Minimizing the number of buyer EVs that would end up with energy outage.
- When sequentially serving meeting points that encapsulate all the buyer EVs, minimizing overall trip duration or distance to be traversed by the seller.
- Minimizing energy cost for the buyer EVs.

In Fig.12, results from simulation of an MBSS system with 4 buyers and 1 seller is shown together with buyers' PDT circles overlaying feasible zero-relative-cost line segments for each buyer EV and the revenue contour map along the feasible segments. Energy demands of EVs are randomly generated from a Gaussian distribution with mean of 30 and standard

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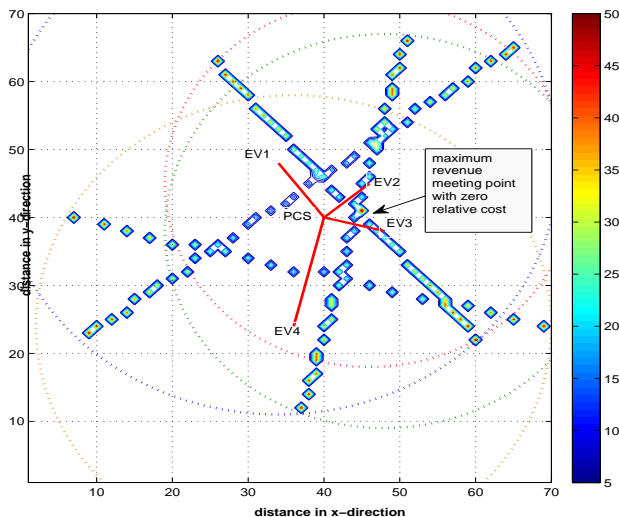


Fig. 12. Simulation of the MBSS case with 4 buyers and 1 seller, and illustration of the resulting globally zero-relative-cost meeting point with the highest revenue for the seller. Note that  $\alpha_b = \alpha_s = 0.4$ ,  $C_e = C_s = 1$  and  $k_i = k_s = 1$

deviation of 10 in this particular snapshot. The maximum revenue meeting point with the global zero-relative cost happens to be at the intersection of the feasible segments of second and third buyers.

## VI. SUMMARY AND CONCLUSIONS

The aim of this paper is to introduce a novel portable energy distribution management concept and promote research regarding this exiting field within the energy management community. First, we have formulated constraints for off-grid portable EV charging network management with dynamic pricing, including virtual and physical distance tolerances and energy inefficiency for energy buyers and sellers. Second, we have developed an energy incentive model suitable for a single buyer and single seller (SBSS) case. We have then extended the model to a multiple buyers single seller (MBSS) case and provided a formula to determine a meeting point that would result in maximum profit for the seller by using zero-relative cost criteria. This research can be easily extended in numerous directions with various other constraints and criteria that we haven't evaluated.

The insight provided in this paper suggests development of game theory based negotiation strategies among buyers and sellers particularly in a multiple and buyer multiple seller (MBMS) case. Cooperation among buyers and its impact on seller's pricing strategies is also worth looking into.

It also seems to be essential to incorporate stochastic service times and waiting delays into the analysis of dense MBMS networks for practically more valuable learning. Adoption of queuing theories and development of mobility models for buyer EVs and sellers will certainly enrich research in this field.