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Abstract: This paper considers the control design for a vibration reduction system using semi-active actuators to improve the ride quality. The main challenges come from the nonlinear dynamics, limited control authority, and lack of performance-oriented nonlinear control design results. Two nonlinear controllers are proposed and compared to a conventional semi-active control. Simulation shows the proposed controls provide a good balance of 2- and ∞-norm metrics.

Key Words: Semi-active, vibration attenuation, Hamiltonian-Jacobi-Bellman, nonlinear system, control lyapunov function

1 Introduction

Vibration reduction of transportation systems, for instance automotive, is required to achieve certain level of ride comfort. Existing architectures for the vibration reduction fall into three categories: passive, fully active, and semi-active, where passive components, active actuators, or semi-active actuators are used in the respective architecture. An active actuator may remove and inject energy to the system, whereas a passive component or a semi-active actuator only takes the energy out of the system. Due to its static design, the performance of a passive system is limited. The active architecture, including control mechanism and fully active actuators, has been successfully applied to automotive suspensions.

Active systems have superior performance at the expense of a high first cost, relatively large electric power requirements and potentially reduced reliability [12]. The semi-active architecture was originally proposed in [13] to trade off the performance of vibration reduction and the system cost. The semi-active architecture enjoys a similar form of the active counterpart except that the fully active actuators are substituted by semi-active actuators. A wide range of study on semi-active systems, mainly on automotive suspensions, demonstrates that a semi-active system can achieve comparable performance of its active counterpart at a reduced first cost and potentially simplify power supply requirements [12].

The application of semi-active actuators such as Magnetorheological (MR) or Electrorheological (ER) dampers renders a challenging problem–control of the semi-active system subject to performance criteria. The dissipative constraint on semi-active dampers not only introduces nonlinearity, but also leads to a constrained control problem. Work, e.g. [13, 15, 7], first performs control design for a fully active system, then derives semi-active control laws by ‘clipping’ active control laws to ensure that semi-active actuators generate forces as required by controller. The aforementioned two-step design approach is straightforward since the fully active system is linear time invariant. Commonly used active control strategies include Sky-Hook [13], Ground-Hook [20], LQR/LQG [10], and $\mathcal{H}_\infty$ [6]. This approach however does not address the nonlinear dynamics during the controller synthesis. Work [11, 19] represents numerous efforts to establish the control of a semi-active automotive suspension by treating it as a bilinear system. Optimal control is designed to improve the ride comfort and handling quality. The optimal control requires the solution of switching differential Riccati equations and is not in the form of state feedback. Nonlinear design such as the Lyapunov-based control [16], decentralized bang-bang control [18], establishes the semi-active control laws by maximizing the dissipative rate of distinctive energy functions. One of the disadvantages of these approaches is that the performance of the closed-loop system is not guaranteed for the lack of connection between performance costs and energy functions. Instead, this paper considers the nonlinearity during the determination of the active control. The resulting active control is performance oriented since it is established from approximate value functions of Hamiltonian-Jacobi-Bellman equations.

This paper is organized as follows. In Section 2 we introduce the semi-active system dynamics, formulate the problem, and expose the fundamental limitation of the conventional passive system. The control design of the semi-active system is carried out in Section 3. A number of controls are simulated and compared in Section 4. Section 5 concludes this note.

Notation: $\|x(t)\|_p$ is the p-norm of $x(t)$, for $1 \leq p < \infty$. A positive definite (p.d.f) matrix P is abbreviated by $P > 0$. I_k is a $k \times k$ identity matrix.

2 Preliminary

2.1 The Semi-Active System

To simplify the problem investigated, we consider a quarter car model which can be simplified into a two degree of freedom (2DOF) system as shown in Figure 1. The 2DOF system consists of a car (m_1), a frame (m_2), a controller (C), sensors (S), dampers (b_1, b_2) and springs (k_1, k_2). The semi-active actuator (b_2) is placed between the frame and the wall. The system dynamics is

$$\dot{x} = Ax + B_1 w + B_2 u, \tag{1}$$

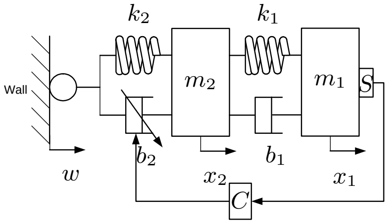


Fig. 1: A 2DOF quarter car model

where $x = (x_1, x_2, \dot{x}_1, \dot{x}_2)^T = (x_1, x_2, x_3, x_4)^T$, $u = b_2(x_4 - \dot{w})$, w is the displacement disturbance from the wall, and

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1}{m_1} & \frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{b_1}{m_1} \\ \frac{k_1}{m_2} & -\frac{k_1+k_2}{m_2} & \frac{b_1}{m_2} & -\frac{b_1}{m_2} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_2}{m_2} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{1}{m_2} \end{bmatrix}$$

A semi-active actuator can implement the control law $u = b_2(x_4 - \dot{w})$, and b_2 is the damping coefficient to adjust. The nonlinearity of the semi-active system comes from the term $b_2(x_4 - \dot{w})$. The system setup is non-unique. For instance, a semi-active actuator can be placed between the car and the frame, which appears in automotive suspension design. Placing a semi-active actuator between ground and m_2 allows the effect of the disturbance derivative.

2.2 Problem Statement

The vibration level is generally measured by norms of the car acceleration and its time derivative (jerk) [24]. The commonly used metric of ride comfort in automotive suspension designs is the 2-norm of acceleration $\|\ddot{x}_1(t)\|_2$. This note considers applications which emphasize the $\infty-$norm of acceleration $\|\ddot{x}_1(t)\|_\infty$. The design of control minimizing $\|\ddot{x}_1(t)\|_\infty$ is rather difficult. Given the abundance of 2-norm based control design techniques, we use the metric $\|\ddot{x}_1(t)\|_2$ to derive controllers but the ∞-norm to evaluate the performance of the resulting vibration reduction system.
In practice, a cost function usually reflects physical constraints such as the relative displacement between moving masses (frame and car), the dissipative rate of power [9], and the bound of control. To simplify the problem to be solved, we only consider the bounded control constraint. Denoting b_{min}, b_{max} the minimal and maximal damping coefficients of the semi-active actuator, the semi-active vibration reduction problem is formulated as follows.

> Given system (1) subject to disturbance w, find a state feedback control $u = b_2(x_4 - \dot{w})$, with $b_{min} \leq b_2 \leq b_{max}$, to minimize certain cost function J.

2.3 Fundamental Limitation with Passive Architecture

A default passive system can be depicted by Figure 1 by replacing the semi-active damper with a conventional damper and eliminating the controller and sensors. Denoting the car acceleration z, we have

$$G_p(s) = \frac{Z(s)}{W(s)} = \frac{s^2(b_1 s + k_1)(b_2 s + k_2)}{\Delta},$$

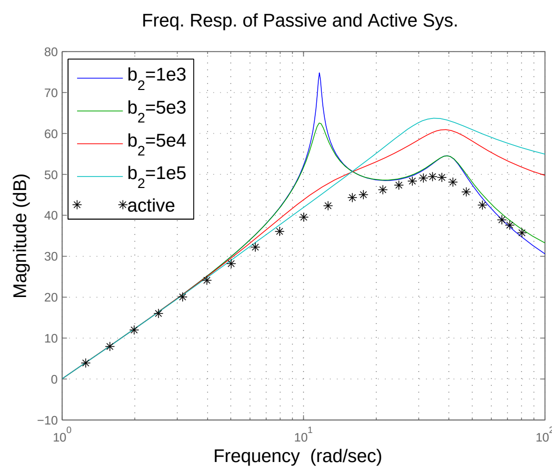
where $\Delta = (m_1 s^2 + b_1 s + k_1)(m_2 s^2 + (b_1 + b_2)s + (k_1 + k_2)) - (b_1 s + k_1)^2$. A fully active system can be similarly represented by Figure 1 but with an active actuator instead. Assuming the active actuator implements the conventional Ground-Hook strategy [21]

$$u = b_{max} x_4, \qquad (2)$$

we have the transfer function

$$G_a(s) = \frac{Z(s)}{W(s)} = \frac{s^2(b_1 s + k_1)k_2}{\Delta},$$

where b_2 is replaced by b_{max}. Bode plots of $G_p(s)$ with different b_2, and $G_a(s)$ are given in Figure 2 to show the fundamental limitation of the passive architecture. The introduction of semi-active actuators is aimed to relax this limitation by adjusting the damping according to the system state.



(a) Transfer function: $G_p(s), G_a(s)$

Fig. 2: Frequency responses of passive (solid) and active systems (*). For the passive case, good disturbance attenuation over high frequencies with a small b_2 at the expense of higher peak values of resonance. This phenomenon does not happen in the active case, which can provide a better balance between the isolation of vibration and suppression of resonance.

3 Control Design

Control design for semi-active systems has attracted a lot of attention since 1970s. The well-known Sky-Hook, Ground-Hook, Clipped optimal controls etc. first determine the active control u from the linear time invariant system (1), then clip the control to ensure the dissipative constraint. Similarly, this paper first derives nonlinear active control, then establish the semi-active control by enforcing the control constraint. The main difference is that the proposed control is based on a nonlinear augmented system, and the corresponding active control is nonlinear.

3.1 Input-to-State Stability

We will establish that the closed-loop semi-active system (1) with any semi-active control law, e.g.

$$u = \alpha(x_4 - \dot{w}), \quad b_{min} \leq \alpha \leq b_{max}, \tag{3}$$

is Input-to-State Stable (ISS), where the disturbance is treated as input.

Proposition 3.1 *Provided that* $w(t), \dot{w}(t)$ *are bounded, system* (1) *with the control* (3) *is ISS.*

Proof: The closed-loop semi-active system can be rewritten as

$$\dot{x} = A(\alpha)x + \psi(w, \dot{w}, \alpha), \tag{4}$$

where ψ is a smooth function of $w, \dot{w}$. We first study the stability of the homogenous part of (4)

$$\Sigma_1 : \quad \dot{x} = A(\alpha)x, \tag{5}$$

Taking the Lyapunov function candidate as the physical energy of the unforced system (5), we have $\dot{V} < 0, \forall x \neq 0$ because of the dampers in the physical system. Denoting $V = x^T P x, P > 0$, we have $\dot{V} \leq -x^T Q x, Q > 0$.
To show that (4) is ISS, we use the same V. Its time derivative is

$$\begin{aligned}\dot{V} &\leq x^T Q x + 2\|Px\| \|\psi\|_\infty \\ &\leq x^T Q x + \epsilon x^T P^2 x + \frac{1}{\epsilon}\|\psi\|_\infty^2,\end{aligned}$$

where $\epsilon > 0$, and ψ is bounded. One can always take a sufficiently small ϵ s.t. $x^T(Q + \epsilon P^2)x \leq -\mu\|x\|_2^2$. Hence, we have $\dot{V} \leq -\mu\|x\|_2^2 + 1/\epsilon\|\psi\|_\infty^2$, and

$$\dot{V} \leq -(1-\theta)\mu\|x\|_2^2, \quad \forall \|x\| \geq \frac{\|\psi\|_\infty}{\sqrt{\mu\theta\epsilon}},$$

where $0 < \theta < 1$. Applying [14, Thm. 4.19], we conclude that system (4) is ISS w.r.t. $w, \dot{w}$. □

3.2 Nonlinear State Feedback $\mathcal{H}_\infty$ Control Design

The control of vibration by semi-active actuators is essentially a nonlinear disturbance attenuation problem. Given the 2-norm metric, we know it can be treated under the nonlinear $\mathcal{H}_\infty$ control framework. The original system (1) is however not in the standard form due to the coupling term $\dot{w}u$. We augment the original system by including the disturbance model, which is approximated by a second order LTI system. Denoting $\xi_1 = w, \xi_2 = \dot{w}$, the disturbance dynamics is

$$\begin{aligned}\dot{\xi}_1 &= \xi_2, \\ \dot{\xi}_2 &= -a_1\xi_1 - a_2\xi_2 + v,\end{aligned} \tag{6}$$

where a_1, a_2 are positive, and v is the standard white noise. The augmented system dynamics is written as

$$\dot{x} = Ax + g_1 v + g_2(x)u, \tag{7}$$

where $x = (\xi_1, \xi_2, x_1, x_2, x_3, x_4)^T$, and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -a_1 & -a_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{k_1}{m_1} & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{k_2}{m_2} & 0 & \frac{k_1}{m_2} & \frac{b_1}{m_2} & -\frac{k_1+k_2}{m_2} & -\frac{b_1}{m_2} \end{bmatrix},$$

$$\begin{aligned} g_1 &= (0,1,0,0,0,0)^T, \\ g_2 &= (0,0,0,0,0,-\frac{x_4 - \xi_2}{m_2})^T. \end{aligned}$$

Given the disturbed system dynamics (7), the nonlinear state feedback $\mathcal{H}_\infty$ control problem is formulated as follows.

Problem 3.2 *[22, Def. 15] Find, if existing, the smallest value* $\gamma^* > 0$ *s.t.* $\forall \gamma > \gamma^*$ *there exists a state feedback* $u(x)$ *s.t.* $\mathcal{L}_2$ *gain from* v *to* $(z, u)^T$ *is less than or equal to* γ.

The $\mathcal{H}_\infty$ control is generally difficult to obtain, and the suboptimal solution is sought instead. That is to find a control s.t. given the disturbance v, the closed-loop system has $\mathcal{L}_2$ gain less than or equal to γ. The existence of such a state feedback is reduced into the solvability of the following HJB [23].

$$\begin{aligned} V_x f + \frac{1}{4}V_x\left[\frac{1}{\gamma^2}g_1 g_1^T - g_2 g_2^T\right]V_x^T \\ + h^T h = 0, \quad V(0) = 0, \end{aligned} \tag{8}$$

where $V_x = \partial V/\partial x, f = Ax$, and

$$h(x) = \left[0, 0, -\frac{k_1}{m_1}, -\frac{b_1}{m_1}, \frac{-k_1}{m_1}, \frac{b_1}{m_1}\right]x.$$

Given the solution of HJB (8), the control is given by

$$u(x) = -\frac{1}{2}V_x g_2. \tag{9}$$

Note that the control solved from (8),(9) only address the nonlinearity of semi-active actuators. Ignoring the dissipative constraint, the resulting control (9) is essentially active, and may violate the bound conditions $u \in [b_{min}, b_{max}]$. Control (9) is not implementable by the semi-active actuator and requires a clipping.

Remark 3.3 *It is difficult to obtain an exact solution of* (10)*. Numerous results are available to obtain its approximate solution, e.g. viscosity solution, basis function approach. Readers are referred to [2, 1] and references therein. For simplicity, we consider a quadratic function* $x^T P x, P > 0$ *to approximate the solution of* (10)*. Work [22] shows that locally, the solvability of the HJB* (8) *is equivalent to that of the Algebraic Riccati Equation (ARE)*

$$A^T P + PA + P\left[\frac{1}{\gamma^2}g_1 g_1^T - GG^T\right]P + H^T H = 0, \tag{10}$$

where G, H *are the linearization of* $g(x), h(x)$ *around* x_0 *respectively. When* $G = 0$*, i.e.,* $\xi_2 - x_4 = 0$*, the linear* $\mathcal{H}_\infty$ *control problem is not well posed in the neighborhood of the origin. Hence, the nonlinear state feedback* $\mathcal{H}_\infty$ *control is locally well-defined only if* $G \neq 0$. □

Remark 3.4 *To be consistent with the $\infty-$norm metric, it is more appropriate to formulate control design of the semi-active vibration reduction system as a $\mathcal{L}_\infty$ control problem instead of a $\mathcal{H}_\infty$ control problem. The objective of $\mathcal{L}_\infty$ control is to minimize the peak of the plant output w.r.t. disturbance. For an LTI system, $\mathcal{L}_\infty$ control can be solved as a $\mathcal{L}_1$ problem [25, 4, 5, 3]. The $\mathcal{L}_\infty$ control problem of nonlinear systems is however still open.* □

Considering the small magnitude of disturbance and that the semi-active system is ISS w.r.t. disturbance, the trajectory of (7) will stay inside a neighborhood of the origin. It is therefore reasonable to approximate the nonlinear $\mathcal{H}_\infty$ control by a linear $\mathcal{H}_\infty$ control, which justify the use of quadratic cost function as an approximate solution of (8). To solve a quadratic value function from (10), we need to assume a constant G. Given certain γ and a constant G, we solve the ARE (10) for P, and take the control

$$u = -\frac{1}{2} g_2^T P x. \tag{11}$$

Note that the control (11) is nonlinear. This is the main difference from existing work where a linear $\mathcal{H}_\infty$ control approach is applied and a linear controller is resulted. In our approach, the approximation is made to compute the value function instead of control. Alternatively, G can be updated along the system trajectory. We could design a set of linear $\mathcal{H}_\infty$ controls based on the different values of G (or $\xi_2 - x_4$) off-line, and schedule the control laws based on the value of $\xi_2 - x_4$.

3.3 Control Lyapunov Function Approach

A Control Lyapunov Function (CLF) can be used to construct a stabilizing state feedback control of a nonlinear system. It is also well-known that a CLF is a value function associated with certain cost [8]. That is a CLF is optimal in some sense. Consider the nonlinear dynamics

$$\dot{x} = f(x) + g(x)u, \quad f(0) = 0, \tag{12}$$

with $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ control and $f(x) : \mathbb{R}^n \to \mathbb{R}^n$ and $g(x) : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are smooth.

Definition 3.5 *[17] A control Lyapunov function is a continuously differentiable, proper, positive definite function $V : \mathbb{R}^n \to \mathbb{R}^+$ s.t.*

$$\inf_u \{V_x f(x) + V_x g(x) u\} < 0, \forall x \neq 0,$$

where $V_x = \partial V(x)/\partial x$.

Given a CLF of system (12), the resultant $u(x)$ is generally defined in $\mathbb{R}$ and violates the bounded constraint of control. A bounded state feedback $u(x)$ can be constructed from a CLF defined for a bounded control set $\mathcal{B} = \{u | \, \|u\|^2 \leq 1\}$ [17]. That is, if there exists u s.t.

$$\inf_{u\in\mathcal{B}} \{V_x f + V_x g u\} < 0, \quad \forall x \neq 0,$$

a control lying in $\mathcal{B}$ can be constructed as

$$u(x) = \begin{cases} -\frac{V_x f + \sqrt{(V_x f)^2 + (V_x g)^4}}{V_x g (1 + \sqrt{1 + (V_x g)^2})}, & V_x g \neq 0, \\ 0, & V_x g = 0. \end{cases} \tag{13}$$

Provided that the white noise v is ignored, the aforementioned result can be applied to construct a bounded control for system (7). We only need to find a V s.t.

$$\inf_{u\in\mathcal{B}} \{V_x f + V_x g_2 u\} < 0. \tag{14}$$

It is straightforward to construct a CLF satisfying (14). The bounded control is constructed according to (13). Simulation demonstrates that the control is bounded but very small. This is because the construction of CLF and the corresponding control (13) is a worst-case design. The resultant closed-loop semi-active vibration reduction system exhibits a similar performance as a passive system does.

Noticing the necessity of a saturated control to outperform passive damping systems, we focus on the search of a 'good' CLF. Since the value function is related to a meaningful cost, we can treat it as a CLF. A value function of system (7) w.r.t. a quadratic cost requires to solve a nonlinear HJB similar to (8).

Due to the difficulty of solving the HJB analytically, we take the approximate value function $V(x) = x^T P x$. This gives a HJB

$$A^T P + PA - P g_2 g_2^T P + C^T C = 0. \tag{15}$$

We make assumption further to simplify the above equation. That is to take $\bar{g}_2$ as a constant by evaluating it at the certain point of the trajectory. With $\bar{g}_2$ a constant vector, the HJB (15) is reduced to an ARE. Treating the approximate value function as a CLF, we construct the control law

$$u(x) = \begin{cases} -\frac{V_x f + \sqrt{(V_x f)^2 + (V_x g_2)^4}}{V_x g_2}, & V_x g_2 \neq 0, \\ 0, & \text{otherwise}. \end{cases} . \tag{16}$$

Remark 3.6 *Although the CLF-based design approach is based on the 2-norm metric, it can be generalized to a more general cost functional case. A realistic approximation of the $\infty-$norm cost is*

$$J = \int_0^T (\ddot{x}_1(t)^{2p} + u^2) \mathrm{d}t,$$

where p is sufficiently large. The corresponding HJB takes the form of

$$V_x f - \frac{1}{4}(V_x g_2)^2 + \ddot{x}_1(t)^{2p} = 0. \tag{17}$$

Unlike the 2-norm metric case, the approximate solution of the HJB (17) *can not be quadratic.* □

4 Simulation

Simulation is performed to compare the performance of the approximate $\mathcal{H}_\infty$ control (11), the CLF-based control (16), the default passive system, the active control (2), and the conventional semi-active control law

$$u = \begin{cases} \max\{\min\{b_{max}, \frac{b_{max} x_4}{x_4 - \dot{w}}\}, b_{min}\}, & \\ \qquad \text{if } x_4(x_4 - \dot{w}) > 0, \\ b_{min}, \quad \text{otherwise}. \end{cases} \tag{18}$$

We take $\bar{g}_2 = G = [0, 0, 0, 0, 0, 1e-1]$, $Q = 8e2 I_6$, and $\gamma = 1$ to solve P in (15), (10). Simulation results are shown in Figures 3, 4 and Table 1. In Figures 3, 4, semi-active 1, 2, 3 represent the controls (18), (11), (16) with $\bar{g}_2$ constant, respectively, and semi-active 4 is the control (16) solved from (15) with $\bar{g}_2$ updated along the system trajectory. Table 1 shows that conventional control (18) achieves lower level of the RMS of car acceleration than the proposed controls. The proposed semi-active control 4 obtain a good balance of the metrics: 2- and $\infty-$norm of the acceleration. It is worth mentioning the proposed control requires full state and disturbance information.

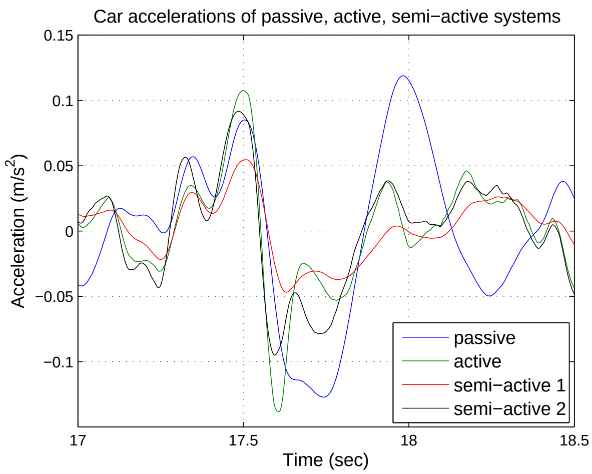


Fig. 3: Acceleration of passive, active, semi-active systems

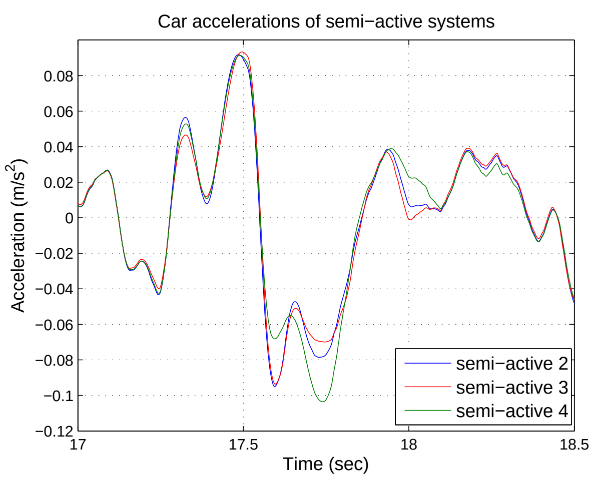


Fig. 4: Acceleration of semi-active systems

Tab. 1: Performance of passive, active, and semi-active systems

Controls	$\|\ddot{x}_1\|_2$ (m/s^2)	$\|\ddot{x}_1\|_\infty$ (m/s^2)
passive	6.6092	0.1272
active (2)	2.0770	0.0621
semi-active 1	3.4961	0.1382
semi-active 2	4.0752	0.1035
semi-active 3	4.0691	0.0950
semi-active 4	3.8550	0.0936

5 Conclusion

This note considered the nonlinear control design for a semi-active vibration reduction system to improve the ride quality. The control design was treated as a nonlinear $\mathcal{H}_\infty$ control problem. An nonlinear approximate $\mathcal{H}_\infty$ control was obtained based on a quadratic solution of the nonlinear HJB. A CLF-based control design approach was also investigated and the control were derived from solving the approximate value function of the corresponding HJB. The proposed semi-active controls are simulated and demonstrate a lower peak acceleration compared to a conventional semi-active control.

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