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TR2012-005 March 2012

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Journal of Electronic Science and Technology

A Hybrid Decoupled Power Flow Method for Balanced Power Distribution Systems

Hongbo Sun, Daniel Nikovski, Tetsufumi Ohno, Tomihiro Takano, and Yasuhiro Kojima

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Index Terms—**Direct method, distributed generations, distribution systems, indirect method, power flow, zero-impedance branches.**

1. Introduction

Power flow calculation is one of the most common computational procedures used in distribution system analysis. Planning, operation, and control of distribution systems require such calculations in order to analyze the steady-state performance of the systems under various operating conditions and equipment configurations. With the increasing penetration of various distribution generations and implementation of advanced control techniques, the analysis of distribution systems plays even more critical role than before, and the complexity of analysis has significantly increased as well.

As a special case of distribution systems, the loads and impedances of a balanced distribution system are three-phase balanced, and therefore their steady state performance is usually analyzed by using single-phase power flow analysis with positive sequence parameters.

Various methods for solving the power flow problem are known. Those methods differ in either the form of the equations describing the system, or the numerical techniques used. Methods based on bus admittance matrices are widely used. Typical methods include the Gauss-Seidel method[1], the Newton-Raphson method[2]–[6], and the fast decoupled method[7]–[12]. Those methods formulate power flow problems as linear systems, and solve them by either direct[1]–[12] or iterative techniques[13]–[16]. The method proposed in this paper also belongs to this category.

This paper proposes a new efficient and robust power flow method for balanced power distribution systems with distributed generation sources. It formulates the decoupled active power and reactive power equations in polar form. Resistance impacts are modeled in both active and reactive power equations, and the necessary trigonometric operations have been avoided by using an appropriate polynomial approximation. It includes second terms of phase angle in active power correction equations to reduce the number of required iterations for phase-angle updating. Direct and iterative solution techniques are used to handle active and reactive power corrections, respectively. This hybrid solution technique takes full advantage of different characteristics of active and reactive power updating to speed up the power-flow solution. The method accurately models zero-impedance branches by merging them with adjacent impedance branches to avoid convergence problems resulting from modeling those as small impedance branches. It seamlessly integrates various types of distribution generation sources and distribution load models with the solution process.

2. Proposed Method

2.1 Modeling of Distributed Generations and Loads

The proposed method uses polar coordinates to formulate the power flow equations for balanced distribution systems. Based on the types of connected

Manuscript received December 14, 2011; revised February 6, 2012.

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Digital Object Identifier: 10.3969/j.issn.1674-862X.2012.01.003

generation sources or loads, a bus in the system can be modeled as a constant active and reactive power (PQ) bus, a constant active power and voltage magnitude (PV) bus, or a swing bus. By converting to PV or PQ buses or shunt compensations, the distribution source and load models are seamlessly integrated into the solution process.

The generation source for the power distribution system is usually a power transmission system, and corresponding equivalent source models are expressed as a swing bus, or a PV bus in the power flow analysis.

In addition to the equivalent sources, the distribution system can also have distributed generators. Depending on the types of energy sources and energy converters, the distribution power generators are specified by a constant power factor model, a constant voltage model, or a variable reactive power model [17].

The buses connected to the constant power factor generators or the variable reactive power generators are treated as PQ buses. For a constant power factor generator, the specified values are the active power output and power factor. The reactive power output of such a generator is determined from the active power and the power factor. For a variable reactive power generator, the active power output is specified, and the reactive power output is determined by applying a predetermined polynomial function to the active power output.

The buses connected to constant-voltage generators are treated as PV buses, and the specified values are the outputs of the active powers and the magnitudes of bus voltages. These buses are also selected as master buses when the equivalent system model is constructed.

The distribution load models include a constant impedance load, a constant power load, and a constant current load.

The constant impedance load is directly treated as connected bus shunt impedance, which is embedded into the bus admittance matrix. If a bus is connected with a constant impedance load, its equivalent shunt admittance is

$$Y_i^{\mathrm{sh}} = \frac{\left(P_{Z_i}^R - jQ_{Z_i}^R\right)}{\left(V_i^R\right)^2} \tag{1}$$

where Y_i^{sh} is the equivalent shunt admittance at bus i, $P_{Z_i}^R$ and $Q_{Z_i}^R$ are the active and reactive powers of constant impedance load at bus i, and V_i^R is the rated voltage of load at bus i.

The constant power load is modeled as bus injected power. If a bus is connected with a constant power load, its equivalent bus power injection is

$$S_i = -\left(P_{S_i}^R + jQ_{S_i}^R\right) \tag{2}$$

where S_i is the equivalent power injection at bus i, $P_{S_i}^R$ and $Q_{S_i}^R$ are the active and reactive powers of constant power load at bus i.

The constant current load is converted to equivalent bus injected powers to be modeled. The equivalent injected powers are based on estimated bus voltages. The powers are recalculated when the current bus voltages become available during the iterations of the solution. The equivalent power injection for a bus connected with constant current load is

$$S_i = -\frac{\left(P_{I_i}^R + jQ_{I_i}^R\right)}{V_i^R} V_i^{(k)} \tag{3}$$

where $P_{I_i}^R$ and $Q_{I_i}^R$ are the active and reactive powers of constant current load at bus i, $V_i^{(k)}$ is the estimated voltage magnitude at bus i, and k is the current iteration number.

2.2 Modeling of Zero-Impedance Branches

Many branches in a power distribution system have very low impedance, such as voltage regulators, switches, ideal transformers, ideal phase shifters, elbows, and jumpers.

In practice, these low impedances are ignored and set to zero in conventional models. The consequence is that some entries in the resultant bus admittance matrix are infinite, and thus the admittance matrix based approaches are inapplicable. In order to use approaches based on bus admittance matrices, conventional methods have arbitrarily assigned small non-zero impedances to those branches. However, assigning such small impedances makes the analysis ill-conditioned, and power flow calculations are often difficult to converge[18]. This paper uses a different approach to handle the zero-impedance branches in power flow analysis.

Fig. 1 shows a generalized model for representing zero-impedance branches in a distribution system. A branch has a master bus m and a slave bus s. The buses are connected by an ideal transformer. The transformer has a ratio $1:a_{m,s}$, where $a_{m,s}$ is a complex number.

The complex transformer ratio becomes 1 when the branch is a switch or a small conductor, a real number when it is a voltage regulator or an ideal transformer, and a complex number with magnitude 1.0 when it is an ideal phase shifter.

The current flowing into the slave bus through the branch is equal to the current flowing from the master bus divided by the negative conjugate of the complex ratio. The voltage at the slave bus is equal to the voltage at the master bus multiplied by the complex ratio.

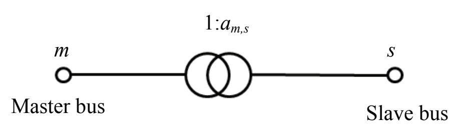

Fig. 1. Generalized zero-impedance branch model.

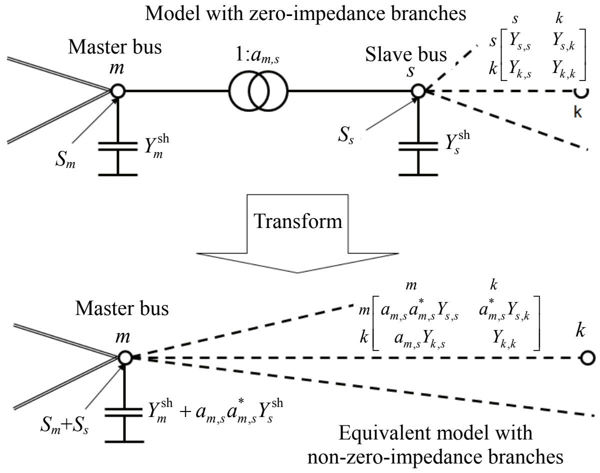

Fig. 2. Equivalent model for the distribution system with zero impedance branches.

When constructing the bus admittance matrix, only non-slave buses are considered. Zero-impedance branches are not used. The impacts of zero impedance branches are represented through the associated master buses and the branches adjacent to the slave buses, as shown in Fig. 2.

Fig. 2 shows an example construction of an equivalent distribution system model with non-zero impedances. The construction transforms a model of distribution system with zero impedance branches to the equivalent distribution system model with non-zero impedances.

In Fig. 2, a zero-impedance branch is connected to three branches (broken lines) by the slave bus and to two branches (double lines) by the master bus. Taking one adjacent branch between slave bus s and bus k as an example, the branch admittance matrix is

$$\begin{array}{cc} & \begin{array}{cc} s & k \end{array} \\ \begin{array}{c} s \\ k \end{array} & \begin{bmatrix} Y_{s,s} & Y_{s,k} \\ Y_{k,s} & Y_{k,k} \end{bmatrix} \end{array}$$

where $Y_{s,s}$ and $Y_{k,k}$ are the self admittances of the branch at the slave bus s and the bus k, $Y_{s,k}$ and $Y_{k,s}$ are the mutual admittances of the branch between the bus s and bus k, and bus k and bus s, respectively. The master bus m provides an injected complex power S_m, and a shunt compensator with admittance Y_m^{sh}. The slave bus s provides an injected complex power S_s and a shunt compensator with admittance Y_s^{sh}.

In the equivalent model, the zero-impedance branch and the slave bus s are removed. There are no changes for the branches connected to the master bus m. The branches connected to the slave bus s are reconnected to bus m, and the branch admittance matrices are modified accordingly.

The branch between buses s and k in the system is replaced with a new branch between bus m and bus k in the equivalent system and the branch admittance matrix is

$$\begin{array}{cc} & \begin{array}{cc} m & k \end{array} \\ \begin{array}{c} m \\ k \end{array} & \begin{bmatrix} a_{m,s}a_{m,s}^{*}Y_{s,s} & a_{m,s}^{*}Y_{s,k} \\ a_{m,s}Y_{k,s} & Y_{k,k} \end{bmatrix} \end{array}$$

where $a_{m,s}^{*}$ is the conjugate of zero-impedance branch ratio. The self admittance at bus m is determined from the product of self-admittance at bus s in the model and the square of the zero-impedance branch ratio. The mutual admittance for bus m to bus k is the product of the conjugate of the zero-impedance branch ratio and mutual admittance for bus s to k in the original system. The mutual admittance for bus k to bus m is the product of the zero-impedance branch ratio and mutual admittance for bus k to bus s in the original model.

The powers at bus s are directly added to bus m, and the resulting equivalent complex power at bus m is

$$S_m + \sum_{s \in M} S_s$$

where M is the set of buses that have connected with bus m through zero-impedance branches.

The shunt compensation admittance at bus s is multiplied by the square of the zero-impedance branch ratio to add to bus m, and the equivalent shunt compensation admittance at bus m is

$$Y_m^{\text{sh}} + \sum_{s \in M} a_{m,s}a_{m,s}^{*}Y_s^{\text{sh}} .$$

2.3 Decoupled Power Flow Equations with Full Impedances and Second-Order Terms

The power flow equations for any bus i are

$$P_i = V_i \sum_j V_j \left[G_{i,j} \cos\left(\theta_i - \theta_j\right) + B_{i,j} \sin\left(\theta_i - \theta_j\right) \right] \quad (4)$$

$$Q_i = V_i \sum_j V_j \left[G_{i,j} \sin\left(\theta_i - \theta_j\right) - B_{i,j} \cos\left(\theta_i - \theta_j\right) \right] \quad (5)$$

where P_i and Q_i are the active power and reactive power injections at bus i, V_i and θ_i are the voltage magnitude and phase angle at bus i, and $G_{i,j}$ and $B_{i,j}$ are the real and imaginary parts of the bus admittance matrix element associated with bus i and bus j.

Similarly to the fast decoupled method, active power is expressed as a function of bus phase angles, and reactive power is a function of bus voltage magnitudes. By applying the Taylor expansion to (4) and (5), and retaining up to second-order terms, we define

$$\frac{\Delta \mathbf{P}}{\mathbf{V}} = \mathbf{J}_{\theta}\Delta\boldsymbol{\theta} + \frac{1}{2}\Delta\boldsymbol{\theta}^{T}\mathbf{H}_{\theta}\Delta\boldsymbol{\theta} \quad (6)$$

$$\frac{\Delta \mathbf{Q}}{\mathbf{V}} = \mathbf{J}_{V}\Delta\mathbf{V} \quad (7)$$

where $\mathbf{V}$ is the bus voltage magnitude vector, $\Delta\mathbf{P}$ and $\Delta\mathbf{Q}$ are the vectors of bus active and reactive power changes, $\Delta\boldsymbol{\theta}$ and $\Delta\mathbf{V}$ are the vectors of bus phase angle and voltage magnitude changes, $\mathbf{J}_\theta$ and $\mathbf{H}_\theta$ are the Jacobian and Hessian matrices of bus active powers with respect to bus phase angles, and $\mathbf{J}_V$ is the Jacobian matrix of bus reactive powers with respect to bus voltage magnitudes. The trigonometric functions are replaced with the Taylor series up to second-order term respectively to simplify the formulation and speed up the calculation during the formulation of the Jacobian and Hessian matrices:

$$\sin(\theta) \approx \theta \tag{8}$$

$$\cos(\theta) \approx 1-\theta^2/2 . \tag{9}$$

The element of the Jacobian and Hessian matrices can be calculated using the following equations:

$$J_{\theta_{i,j}} = \frac{1}{V_i}\frac{\partial P_i}{\partial \theta_j} \approx V_j\left[G_{i,j}\left(\theta_i-\theta_j\right)-B_{i,j}-\left(\theta_i-\theta_j\right)^2/2\right] \tag{10}$$

$$J_{\theta_{i,i}} = \frac{1}{V_i}\frac{\partial P_i}{\partial \theta_i} = -\sum_{j\neq i} J_{\theta_{i,j}} \tag{11}$$

$$J_{V_{i,j}} = \frac{1}{V_i}\frac{\partial Q_i}{\partial V_j} = V_j J_{\theta_{i,j}} \tag{12}$$

$$J_{V_{i,i}} = \frac{1}{V_i}\frac{\partial Q_i}{\partial V_i} = -2B_{i,i} + \frac{1}{V_i}\sum_{j\neq i} V_j J_{V_{i,j}} \tag{13}$$

$$H_{\theta_{i,i,j}} = \frac{1}{V_i}\frac{\partial^2 P_i}{\partial\theta_i\partial\theta_j} \approx V_j\Big[G_{i,j}\left(\theta_i-\theta_j\right)+ G_{i,j}\left(\theta_i-\theta_j\right)^2/2+B_{i,j}\left(\theta_i-\theta_j\right)\Big] \tag{14}$$

$$H_{\theta_{i,i,i}} = \frac{1}{V_i}\frac{\partial^2 P_i}{\partial\theta_i^2} = -\sum_{j\neq i} H_{\theta_{i,i,j}} \tag{15}$$

$$H_{\theta_{i,j,i}} = \frac{1}{V_i}\frac{\partial^2 P_i}{\partial\theta_j\partial\theta_i} = H_{\theta_{i,i,j}} \tag{16}$$

$$H_{\theta_{i,j,j}} = \frac{1}{V_i}\frac{\partial^2 P_i}{\partial\theta_j^2} = -H_{\theta_{i,i,j}} . \tag{17}$$

2.4 Hybrid Direct and Indirect Procedures

The power flow equations are usually solved either by means of a direct solution technique[19] or an iterative solution technique[20]. Considering the characteristic difference between active power and reactive power problems, the proposed method uses a hybrid procedure to solve the power flow equations described in (6) and (7), in which the direct solution technique is used to solve the active power mismatch equations, and the iterative procedure is used to solve the reactive power mismatch equations.

For the active power mismatch problem, the following equation is used:

$$\mathbf{J}_\theta\left(\mathbf{V}^{(0)},\boldsymbol{\theta}^{(0)}\right)\Delta\boldsymbol{\theta} = \frac{\Delta\mathbf{P}^{(k)}}{\mathbf{V}^{(k)}} - \frac{1}{2}\Delta\boldsymbol{\theta}^{(k)T}\mathbf{H}_\theta\left(\mathbf{V}^{(0)},\boldsymbol{\theta}^{(0)}\right)\Delta\boldsymbol{\theta}^{(k)} . \tag{18}$$

The Jacobian matrix $\mathbf{J}_\theta(\mathbf{V}^{(0)},\boldsymbol{\theta}^{(0)})$ and the Hessian matrix $\mathbf{H}_\theta(\mathbf{V}^{(0)},\boldsymbol{\theta}^{(0)})$ are determined by using the initial bus voltage magnitude $\mathbf{V}^{(0)}$ and phase angle $\boldsymbol{\theta}^{(0)}$, which remain constant during the iterations.

The first item in the right-hand side is the bus active power mismatch divided by the corresponding bus voltage magnitude that is determined by means of the bus voltage magnitude and phase angle obtained during the previous iteration *k*. The second item is the additional mismatch added by the second-order term of phase angle changes, and it is also determined by the phase angle obtained at the previous iteration. This linear equation is solved by means of a sparse lower triangular and upper triangular (LU) decomposition with partial pivoting. The bus phase angle vector $\boldsymbol{\theta}$ is updated when the phase angle correction vector $\Delta\boldsymbol{\theta}$ is determined.

For the reactive power mismatch problem, the following equation is used

$$\mathbf{J}_V\left(\mathbf{V}^{(0)},\boldsymbol{\theta}^{(0)}\right)\Delta\mathbf{V} = \frac{\Delta\mathbf{Q}^{(k)}}{\mathbf{V}^{(k)}} . \tag{19}$$

The Jacobian matrix $\mathbf{J}_V(\mathbf{V}^{(0)},\boldsymbol{\theta}^{(0)})$ is determined from initial bus voltage magnitudes and phase angles, which remain constant during iterations. The right-hand side is the bus reactive power mismatch divided by the corresponding bus voltage magnitude that is determined from the bus voltage magnitude and phase angle obtained during the previous iteration *k*. This linear equation is solved by means of the restarted generalized minimal residual method with incomplete LU pre-conditioner. The diagonal elements of the Jacobian matrix are taken to be the preconditioned matrix. The bus voltage magnitude vector $\mathbf{V}$ is updated when the voltage magnitude correction vector $\Delta\mathbf{V}$ is determined.

The initial values for bus voltage magnitudes and phase angles are set as follows:

$$V_i^{(0)} = V_{\rm sw}\prod_{t\in N}\alpha_t \tag{20}$$

$$\theta_i^{(0)} = \theta_{\rm sw} + \sum_{p\in N}\beta_p \tag{21}$$

where $V_{\rm sw}$ and $\theta_{\rm sw}$ are the voltage magnitude and phase angle of the swing bus, N is the set of devices located on the shortest path between the swing bus and bus i, α_t is the voltage increasing ratio resulting from transformer t, and β_p is the phase angle increase resulting from the phase shifter p.

The initial voltage magnitude of a bus is set as the result of multiplying the swing bus voltage magnitude by all voltage increasing ratios resulting from the transformers along the shortest path from the swing bus to the study bus.

The bus initial phase angle is set as the swing-bus phase angle plus all phase-angle changes resulting from the phase shifters along the shortest path from the swing bus to the bus.

3. Numerical Examples

The proposed method has been tested on several sample systems, and satisfactory results have been obtained. The testing results on a sample 6.6 kV distribution system and computation performance are compared with other existing methods provided below. Five different algorithms have been implemented to calculate the load flow of the sample system, including the method proposed in this paper, the Gauss-Seidel method, the Newton-Raphson method, and the BX and XB versions of the fast decoupled method.

As shown in the Fig. 3, the test system has 7 feeders and 142 nodes. Two of the feeders, FDR1 and FDR7, are used for power generation only, and each node along the feeders has installed a photovoltaic unit. The other 5 feeders are used for both generating power and serving power demand from customers, and each node along the feeder also has a photovoltaic unit and a service load transformer installed. The load demand at each node contains 40% constant power load, 30% constant current load, and 30% impedance load. The system has 14 zero-impedance branches, including 7 closed switches, and 7 voltage regulators.

Six different test cases are simulated as shown in Table 1. Those cases represent typical operation scenarios of this system. The first three cases, Case I, Case II, and Case III, simulate normal power-supply scenarios. In those cases, the main grid satisfies the major portion of the total system load demand, and the remaining portion is satisfied by local photovoltaic units. The last three cases, Case IV, Case V, and Case VI, simulate back-feeding scenarios. Besides satisfying the load demands from local customers, the distribution system still has power surplus that can be fed back to the main grid. In test Case I and Case IV, the power factors of the photovoltaic units are set to 1.0, i.e., the units generate only active power. The power factors are set to 0.9 for Case II and Case IV, and to 0.8 for Case III and Case VI. The photovoltaic units for those cases generate both active and reactive power.

The computational performance of all implemented methods is shown in Table 2 and Table 3. The allowed maximum active and reactive power mismatch was set to 10^{-5} per unit, and the allowed maximal number of iterations was set to 5000.

Taking Case I as an example, it took 16 ms and 12 iterations for the proposed algorithm to find the final solution with the required precision. In comparison, it took 7394 ms and 1641 iterations for the Gauss-Seidel algorithm, and 640 ms and 23 iterations for the Newton-Raphson algorithm to find the solution with the same precision. The two fast decoupled algorithms, either the BX version or the XB version, failed to converge to a solution within the given maximum number of iterations. Similar results can be found for the other five cases.

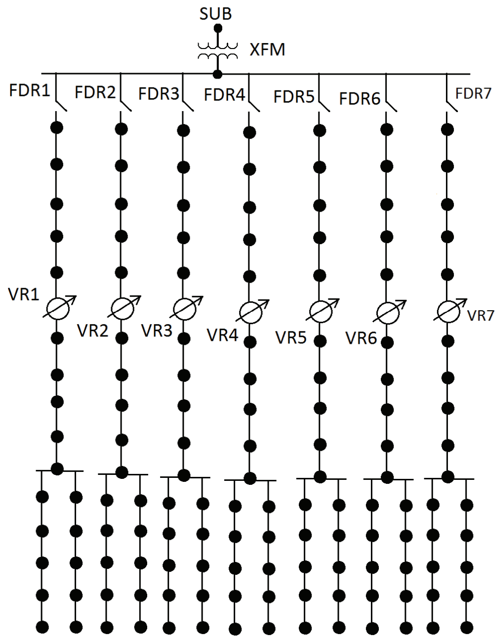

Fig. 3. Sample of the 6.6 kV distribution system.

Table 1: Test Scenarios

Scenario	Case	Load demands		PV generations	
		MVA	Power factor	MVA	Power factor
Normal power supply	I	12.0	0.8	3.5	1.0
	II	12.0	0.8	3.5	0.9
	III	12.0	0.8	3.5	0.8
Backfeed to main grid	IV	12.0	0.8	35	1.0
	V	12.0	0.8	35	0.9
	VI	12.0	0.8	35	0.8

Table 2: Computation time in seconds for test cases

Algorithm	Case					
	I	II	III	IV	V	VI
Proposed method	0.016	0.016	0.016	0.016	0.031	0.031
Gauss-Seidel method	7.394	7.457	7.332	12.62	9.313	9.032
Newton-Raphson method	0.640	0.374	0.328	0.172	0.140	0.140
Fast decoupled method, XB version	1.934	2.215	1.762	1.810	1.778	2.200
Fast decoupled method, BX version	1.794	2.028	2.090	1.872	1.809	1.825

Table 3: Total number of iterations for test cases

Algorithm	Case					
	I	II	III	IV	V	VI
Proposed method	12	12	11	13	38	61
Gauss-Seidel method	1641	1636	1641	2712	2096	1996
Newton-Raphson method	23	13	12	6	5	5
Fast decoupled method, XB version	5000	5000	5000	5000	5000	5000
Fast decoupled method, BX version	5000	5000	5000	5000	5000	5000

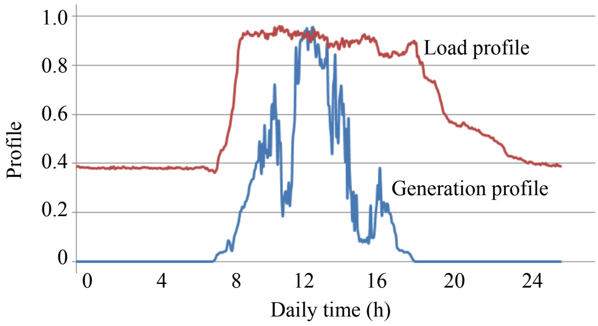

Fig. 4. Sample bus daily generation and load profiles.

Table 4: Computation performance under daily generation and load variations

Algorithm	Average computation time (s)	Average number of iterations
Proposed method	0.012	13.459
Gauss-Seidel method	6.847	1557.23
Newton-Raphson method	0.127	4.397

The proposed method was also tested against daily generation and load variations. A total of 480 test cases were created for a typical weekday by taking one sample of system loads and generations every 4 min. Both the generation output of the photovoltaic units and the load demands of the buses varied over time. The daily variation patterns of load demand and power generation for different buses may be different as well. As an example, Fig. 4 gives the daily generation and load profiles for a sample bus in the system. In the figure, the lower and upper curves represent the daily generation profile and daily load profile, respectively. The overall computational performances of three different methods are given in Table 4, including the proposed method, the Gauss-Seidel method and Newton-Raphson method. The average computation time, and the average iteration number for one single case are used to compare the computation performances among different methods.

From these test results, we can see that the proposed method is much more efficient than the Gauss-Seidel and Newton-Raphson algorithms, and has much better convergence than the fast decoupled ones.

4. Conclusions

The paper has proposed a hybrid decoupled power flow method for balanced power distribution systems with distributed generation sources.

The method formulates the power flow equations in active power and reactive power decoupled form with constant Jacobian and Hessian matrices. It uses a hybrid procedure to solve the power flow equations, in which the direct method is used to solve the active power equations, and the indirect method is used to solve the reactive power equations.

It models zero-impedance branches accurately and avoids solution divergence that is usually caused by zero or small impedance branches in conventional methods.

The test results have proven experimentally that the proposed method is much faster than both the Gauss-Seidel and Newton-Raphson algorithms, and has better convergence than the fast decoupled algorithms.

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