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Bingnan Wang, David Ellstein, and Koon Hoo Teo
Mitsubishi Electric Research Laboratories, 201 Broadway, Cambridge, MA 02139 USA

Abstract— **We describe a wireless power transfer (WPT) system based on electromagnetic coupling of resonators in an array. The system can not only extend the effective range of a conventional resonant coupling based WPT system, but also provide power continuously to multiple moving devices, such as electric vehicles and robots. In this paper, the system is presented and analyzed with a transmission line model. The performance and characteristics of the system are studied and discussed.**

Keywords-wireless power transfer; resonant coupling; array; resonators

I. INTRODUCTION

Wireless power transfer (WPT) based on resonant coupling has promising applications for short-distance power transfer [1-4]. Much progress has been made on the study of the resonant coupling based WPT since 2007 [1]. By using resonance, the effective range of power transfer is greatly improved, compared with the more commonly used inductive method. However, the range and mobility of power-consuming devices are still limited. Efficiency drops rapidly with increasing distance between transmitter and receiver. Recently it has been shown that metamaterials can be used in a WPT system to improve the power transfer efficiency and extend the effective range [5-8]. However, current WPT technologies are mainly developed for static power transfer, where the power receiving devices do not move or have very limited mobility. They are not suitable for wireless powering of moving devices, such as electric vehicles, robots, and elevators.

In this paper, we study a flexible WPT system based on the coupling of an array of resonators. We will show that the system can not only extend the effective range of a conventional resonant coupling based WPT system, but also provide power continuously to moving devices. In this paper, the system will be presented and analyzed with a transmission line model. The performance and characteristics of the system will be studied and discussed.

II. SYSTEM CONFIGURATION

In conventional resonant-coupling based WPT system, power transfer is achieved between two coupled resonators, one as transmitter and one as receiver, tuned to have the same or similar resonant frequency. High-frequency power can be sent to the transmitter and extracted from the receiver via conduction or induction. To achieve efficient power transfer, the receiver needs to be close to the transmitter, with a distance typically less than the physical size of the transmitter and receiver. Thus the system is not suitable for transmitting power to a device that moves farther than the length of the transmitter or receiver.

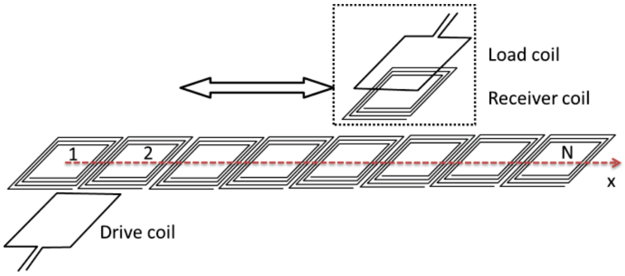

Figure 1. Schematic of a WPT system based on array of resonators.

In the proposed new system, an array of coupled resonators is used, as shown in Fig. 1. The array is formed by putting multiple resonators side by side. A non-resonant coil, referred to as the drive coil, is inductively coupled to one of the resonators to feed high-frequency energy, which is then distributed to the whole array due to resonant coupling. Another resonant coil referred to as receiver coil, identical to those in the array, together with a non-resonant coil referred to as load coil, is used to pick up power from the array. The system in Fig. 1 is an example of a linear array configuration. However, the array can take many forms based on applications and system requirements. For example, the array can be in a straight track, a bended or curved track; it can also be in a closed loop. The resonators composing the array may differ in geometry, as long as they have the same or similar resonant frequency, so that resonant coupling between them can be realized.

Power can be delivered to a device by connecting its two leads across the load coil. The position of receiver is flexible; as long as it is in close range of the array, power can be coupled to it. The effective power transfer range is largely extended through this array system than by the common two-resonant-coil system. Also, the array system can be used to provide power to multiple receivers at the same time. Lastly, the receiver in the array system can receive power while in motion relative to the array; thus, this system is useful for powering mobile devices.

III. TRANSMISSION-LINE MODEL

In order to understand the power transfer performance of the system when the receiver is at different positions, a transmission line model based on circuit analysis and analytical calculations is developed. Each resonant coil is modeled as a tank circuit; each non-resonant coil is modeled as the series combination of an inductor and a resistor. The drive coil is assumed to be powered by a linear voltage source described by its open load voltage V_s and internal resistance R_s. The load coil is assumed to be loaded by a resistor R_L. The circuit models of these building blocks of the system are shown in Fig. 2. All the coils are modeled as inductively coupled. The inductive coupling is modeled as mutual inductance between the inductors of the circuit model. The inductance, capacitance, resistance and resonant frequency for resonant coils, the inductance and resistance for non-resonant coils, as well as the mutual inductance between coils are calculated analytically based on the geometry and relative positions of the coils.

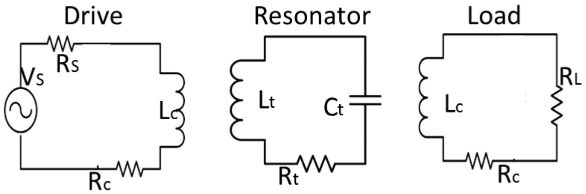

Figure 2. Circuit model for the non-resonant drive coil, resonant coil in the array and receiver, and non-resonant load coil.

For simplicity, we consider all resonant coils in the array and the resonant receiving coil as identical, thus having the same inductance, capacitance and resistance values. Similarly, the two non-resonant coils are also identical. Also, we assume the mutual inductance between coils depends only on their separation distance, not on their position in the array system. Thus all nearest neighboring coupling in the array is the same. Lastly, we assume the mutual coupling is reciprocal. With these in mind, we can reduce the circuit elements in the system to those listed in Table 1.

Table 1 Symbols and meanings in the equation system

Symbol	Physical Meaning
R_t	Resistance of a resonant coil
C_t	Capacitance of a resonant coil
L_t	Inductance of a resonant coil
V_s	Voltage Source
R_s	Voltage source resistance
R_L	Load resistance on load coil
R_c	Resistance of non-resonant coil
L_c	Inductance of non-resonant coil
$M_{tt}(m,n)$	Mutual inductance between coils *m* and *n* of the array
$M_{dt}(m)$	Mutual inductance between drive coil and resonant coil *m* of the array
M_{Lr}	Mutual inductance between the receiver coil and load coil
$M_{rt}(m,x)$	Mutual inductance between the receiver coil and resonant coil *m* of the array when the receiver is at position *x*
$M_{Lt}(m,x)$	Mutual inductance between the load coil and coil *m* of the array when the receiver is at position *x*
$M_{dr}(x)$	Mutual inductance between the receiver coil and the drive coil when the receiver coil is at position *x*
$M_{dL}(x)$	Mutual inductance between the load coil and the drive coil when the receiver coil is at position *x*

As shown in Fig. 1, we assume the drive loop is aligned with the first coil in the array and the receiving coil is aligned with the load coil. The receiver is above the array and is aligned with the width of the array. The receiver's center is horizontally separated from the center of the drive coil by a distance *x*. The total number of resonators in the array is denoted as *N*. Since the receiving coil and the load coil move as a unit along the array, the mutual inductance between these coils and the other coils are a function of the receiver position, *x*. The resonators are numbered in order starting at the *x*-axis origin and ending at the other end of the linear array. In order to develop an algorithm for generating the system of equations for the array with an arbitrary number of resonators, it is useful to use matrices and vectors to represent related mutual inductance terms. For instance, the mutual inductances between resonant coils are collected into a matrix, *Mtt(m,n)* where the row, *m*, and column, *n*, represent the numbers of the two resonators which the mutual inductance is between. For the mutual inductance symbols below, a positive value of mutual inductance between coil α and β, $M_{\alpha\beta}$, means that a positive current in β generates a flux through coil α that is in the same direction as the flux produced by a positive current in α. A positive current is a current in the direction of the assigned reference directions in the schematic.

Based on the schematic and the list of symbols, the following set of equations is derived based on Kirchhoff's circuit laws of voltage and current. Each coil can be described by an equation. In these equations, I_t is current in the drive coil, I_m is current in the *m*-th resonant coil of the array, I_d is current in the receiving resonant coil, and I_L is the current in the non-resonant load coil. The self-impedances for each resonant coil and non-resonant coil are, respectively

$$\begin{aligned} Z_t &= R_t + j\left(\omega L_t - \frac{1}{\omega C_t}\right) \\ Z_c &= R_c + j\omega L_c \end{aligned} \tag{1}$$

For the non-resonant drive coil, we have

$$\begin{aligned} V_s = I_t \cdot \left(Z_c + R_s\right) + \sum_{m=1}^{N}\left\{I_m \cdot j\omega M_{dt}(m)\right\} \\ + I_r \cdot j\omega M_{dr}(x) + I_L \cdot j\omega M_{dL}(x) \end{aligned} \tag{2}$$

For each resonator in the array, we have an equation of the following general form

$$\begin{aligned} 0 = I_d \cdot j\omega M_{dt}(t) + I_t \cdot Z_t + \sum_{m=1,m\neq t}^{N}\left\{I_m \cdot j\omega M_{tt}(t,m)\right\} \\ + I_r \cdot j\omega M_{rt}(t,x) + I_L \cdot j\omega M_{Lt}(t,x) \end{aligned} \tag{3}$$

For the receiving resonant coil, we have

$$\begin{aligned} 0 = I_d \cdot j\omega M_{dr}(x) + \sum_{m=1}^{N}\left\{I_m \cdot j\omega M_{rt}(m,x)\right\} \\ + I_r \cdot Z_t + I_L \cdot j\omega M_{Lr} \end{aligned} \tag{4}$$

Finally for the non-resonant load coil, we have

$$\begin{aligned} 0 = I_d \cdot j\omega M_{dL}(x) + \sum_{m=1}^{N}\left\{I_m \cdot j\omega M_{Lt}(m,x)\right\} \\ + I_r \cdot j\omega M_{Lr} + I_L \cdot \left(Z_c + R_L\right) \end{aligned} \tag{5}$$

The above system of equations can be generated to model the system. By solving the equations, the current in each coil and the power transfer efficiency will be obtained. In order to do that, the values for the many physical quantities in the equations must be calculated.

IV. CALCULATION OF CIRCUIT COMPONENTS

The spiral coil resonators in the system are modeled as tank circuits, as depicted in Fig. 2. The inductance, capacitance, and resistance of each tank circuit are calculated in the following manner.

The inductance of the spiral is the sum of the self-inductance of each square loop, plus the mutual inductance between every combination of square loops; the mutual inductance between each pair of loops is counted twice. The self-inductance of a square loop is calculated using an analytical solution to the flux generated by a uniform filament current along the centerline of the wire through the area enclosed by the inner boundary of the wire. The self-inductance is the ratio of the flux to the assumed current magnitude. The mutual inductance between two square loops is found in a similar manner. The mutual inductance between loop 1 and loop 2 (arbitrary names) can be found by assuming a current filament along the center line of the wire composing loop 1 and finding the flux generated through the inner boundary of the wire forming loop 2. The mutual inductance between this loop pair is the ratio of that flux to the assumed current magnitude.

The capacitance of the tank model of the spiral is found as the series combination of the capacitance between adjacent square loops. This model is not accurate and a more accurate model of the capacitance is being developed. No self-capacitance of a single loop of a spiral is modeled. The capacitance between adjacent loops is found by assuming a line charge along the centerline of adjacent loops, but the charge of one loop's centerline is the opposite of the other loops. Each loop is modeled as an equipotential. The electric field between the loops is found by assuming the line charges are much longer than the distance between the adjacent wires so that Gaus's Law can be applied. The medium surrounding the wires is assumed to be homogeneous with a relative permittivity of one. Thus the effect of the dielectric slab under the strips is ignored. The voltage difference between the wires is found enabling the capacitance between the loops to be calculated as the ratio of the total charge magnitude on each loop (take the average of each loop since their perimeter is not exactly the same) to the voltage difference between the loops.

The resistance of the tank model is found as the series combination of the resistance of each loop of the spiral. The resistance of each loop is found using the surface resistance expression for a slab of copper metal. This resistance is frequency dependent and is thus recalculated for each frequency at which the system is analyzed.

The mutual inductances between the different coils are calculated using similar method as the calculation of mutual inductance between loops of a single spiral. Specifically, a uniform current filament along the center line of each loop of one coil was assumed and used to find its contribution to the flux through each loop of another coil. The total flux divided by the current is the mutual inductance. This problem was more complicated than that for the single spiral because the coils are not all identical and are sometimes offcentered from each other. An algorithm was created to find and account for the relative positions of the coils.

Once all the circuit components in the transmission line model are calculated, the system of equations (Equations. 2-5) can then be solved for the current in each coil. When the position of receiving coils is changed, the mutual coupling terms in the equations need to be recalculated and updated in order to solve the system of equations. Then the current in each coil, the input and output power, and the power transfer efficiency of the system can be calculated using following equations, respectively.

$$P_t = \frac{1}{2}\Re\left[V_s I_t^*\right]$$

$$P_L = \frac{1}{2}\frac{|I_L|^2}{R_L} \tag{6}$$

$$\eta = P_L / P_t$$

V. RESULTS AND DISCUSSIONS

The approach described here is very general and can be applied to different configurations of arrays, such as different number of resonators, different designs of resonators. As an example, we calculated an array with 10 resonators aligned in a straight line. The resonator design is shown in Fig. 3. The coil is a 5-turn square spiral with 149 mm outer width, 2 mm copper trace width, and 1 mm separation between adjacent traces. We assume the spirals are fabricated on a 0.5 mm thick RO4350 substrate and the copper thickness is 35 µm. The non-resonant coils are identical to the resonant coils except they only have 1 turn. Adjacent resonators in the array are separated by 1 mm. The drive coil is aligned with the first coil in the array; the receiving coil is 50 mm above the array and moves together with load coil horizontally along the array.

Figure 3. The geometry of resonant coil used in the analysis.

With these parameters, the system is simplified by the transmission line model, and the circuit components are calculated respectively for a given receiver position. After solving the system of equations, the power transfer efficiency can be calculated for each receiver position using Equation 6. As shown in Fig.4, the calculated efficiency is plotted as functions of receiver position (*x*-axis) and the excitation frequency of the voltage source (*y*-axis).

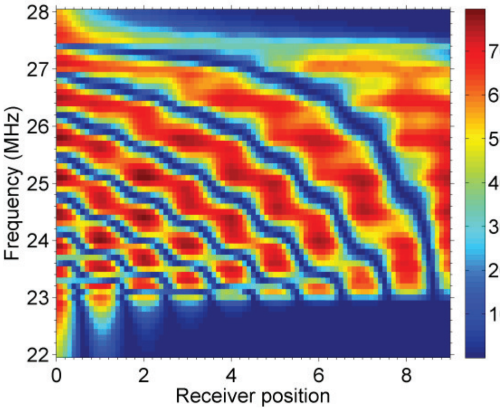

Figure 4. The power transfer efficiency calculated by the transmission-line model, as function of driving frequency and receiver position, which is in unit of lattice size of the array (150 mm).

The transmission line model can be used to quickly calculate the properties of the system. However the model itself is not rigorous; approximations are used in the calculation. In order to verify the model, a more rigorous full-wave simulation was done using COMSOL, which is commercial software based on the finite-element method. The exact system as in the previous calculations is setup, with exact physical geometries for each coil. The system is excited by a 50 Ω port on the drive coil, and power is transferred to another 50 Ω port on the load coil. The efficiency is calculated as the ratio of output and input power. The system is solved for each excitation frequency and each receiver position. The result is plotted in Fig. 5.

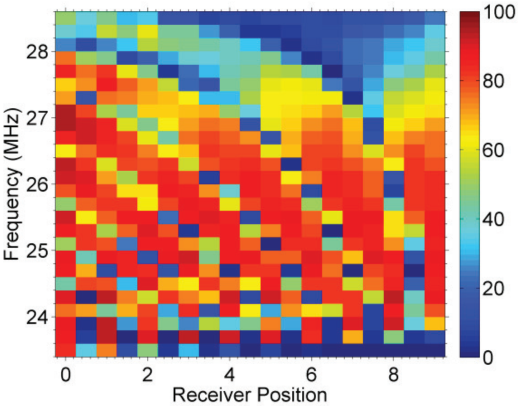

Figure 5. The power transfer efficiency calculated by numerical model with COMSOL, as function of driving frequency and receiver position, which is in unit of lattice size of the array.

From Fig. 4 and Fig. 5, it is seen that the system has high efficiency around 25 MHz, which is the resonant frequency of the coils. For each fixed frequency, nodes and anti-nodes are observed when the receiver is moving along the array, indicating a standing-wave formed on the array due to the mutual coupling in the finite-sized array. Due to the finite size and discrete nature of the coils, the permitted wave-vectors of such standing wave on the array are also discrete, which is indicated by the multiple modes at each receiver position. The results show a negative dispersion relationship, because the number of nodes decreases as the frequency increases. The negative dispersion relationship is the result of the negative mutual coupling between neighboring coils in the array.

The results show that the system can be used for wireless power transfer to moving devices and operate at a wide frequency band. System efficiency of over 80% can be achieved in the example system. However, when fixing the frequency, the efficiency fluctuates significantly when the receiver moves. This can be addressed by different approaches. For examples, multiple receivers can be used and the combined power can be higher and more stable than the single receiver case; the impedance can be adaptively matched depending on the receiver position; the operating frequency can be tuned depending on the receiver position for better performance.

VI. CONCLUSIONS

In conclusion, a system based on an array of resonators is proposed for wireless power transfer to moving devices. A transmission line model assisted by analytical calculations is developed to study the performance of the system when a power receiver is at different position. A system with 10 resonators is studied with the model, and the results are comparable with more rigorous full-wave simulations. High efficiency can be achieved with the array based system when the receiver is at different positions, although there is fluctuation in the efficiency due to the non-uniform near-field distribution in the array. Different approaches can be used to reduce the efficiency fluctuation and make the system better suited for power transfer to moving devices.

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