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Abstract

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OPTIMIZED THREE-BODY GRAVITY ASSISTS AND MANIFOLD TRANSFERS IN END-TO-END LUNAR MISSION DESIGN

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INTRODUCTION

There has been a renewed interest in the lunar mission over the past decade. The problem of designing fuel efficient trajectories from Earth to the moon has been investigated extensively, leading to mission designs with considerable savings in fuel over the traditional Hohmann transfer based missions. In this paper, we describe a modular optimization framework for GTO-to-moon mission design using the planar circular restricted three-body problem (PCR3BP) model .The three-body resonant gravity assists[1–3] and invariant manifolds in the PCR3BP[4] are used as basic building blocks of this mission design. The mission is optimized by appropriately timed delta-Vs, which are obtained by a shooting method[5] and a Gauss-Pseudospectral collocation method[6] for different phases of the mission. Depending upon the initial and final orbits, the optimized missions consume between 10-15 % less fuel compared to a Hohmann transfer, while taking around 4 to 5 months of travel time.

As an example, we computed a GTO to moon trajectory with the following conditions. The size of GTO was taken to be 36000 Km X 250 Km above the surface of the Earth. The size of LLO

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was taken to be 800 Km X 100 Km above the surface of moon. Figure 1 shows the computed trajectory in the rotating frame in the normalized coordinates. The spacecraft receives two resonant gravity assists before being inserted into the stable manifold of a L1 periodic orbit. It is then pushed off the L1 orbit along the unstable manifold, and then kicked off the manifold to be captured by moon with a final delta-V at point of insertion. The complete mission takes about 130 days, and uses 1400 m/s of delta-V, which is about 150 m/s less than a comparable Hohmann transfer, which can be completed in about two weeks. In the paper, we discuss the optimization methodology in detail, describe the results for varying mission constraints and compare the results to Ballistic lunar transfer type missions, which are solved using the bi-circular model.

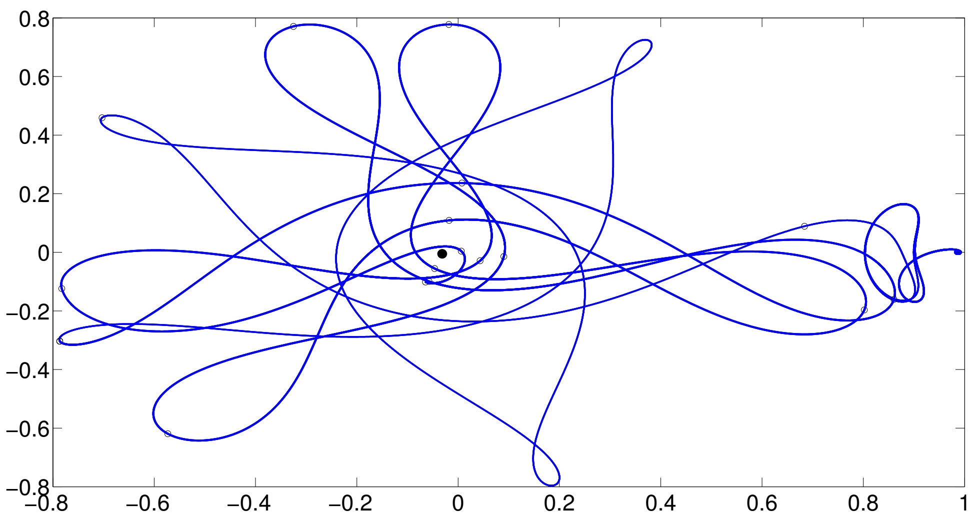


Figure 1. The optimized trajectory in the rotating frame of PCR3BP involving the Earth, moon and the space probe. The space probe receives two consecutive gravity assists, and is then pushed onto the stable manifold of L1 periodic orbit. Optimized delta-Vs then put on the low lunar orbit using the unstable dynamics near L1.

OVERVIEW OF THE MISSION DESIGN PROCESS

The Planar Circular Restricted Three-Body Problem (PCR3BP)

The motion of a *massless* object, P, in presence of two bodies, with masses m_1 and m_2, revolving around each other in a plan can be described in the rotating frame as follows,

$$\ddot{x} - 2\dot{y} = -\bar{U}_x \tag{1a}$$

$$\ddot{y} + 2\dot{x} = -\bar{U}_y \tag{1b}$$

where

$$\bar{U}(x,y) = -\frac{1}{2}(x^2+y^2) - \frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} - \frac{1}{2}\mu_1\mu_2 \tag{1c}$$

and r_1 is the distance of P from m_1 and respectively r_2 is the distance of P from m_2. Also, μ_1 and μ_2 are mass parameters. This problem, of describing the motion of the object, P, by Equation (1) is called the planar circular restricted three-body problem (PCR3BP) which will be used to describe the motion of a satellite. For a detailed description, see Reference 7.

Here, the Earth-Moon system is considered with m_1 being the earth and respectively m_2 being the moon. The resulting mass parameters are $\mu_1 = 1 - \mu$ and $\mu_2 = \mu$ where $\mu = \frac{m_2}{m_1+m_2}$.[7] We divide the task of GTO to moon mission into four phases. The first phase consists of using an optimized Hohmann transfer to an intermediate Earth orbit (IEO). The second phase consists of an optimized sequence of resonant gravity assists from the moon to increase the periapsis of the space probe orbit. The third phase is the insertion of the space probe appropriately sized L1 periodic orbit via a stable manifold of that periodic orbit, and the final phase consists of travel along the unstable manifold and insertion into a low Lunar orbit (LLO). The first two phases are optimized by a shooting method, which is a modified version of the method introduced in.[5] The third and fourth phases are optimized by a Gauss-pseudospectral collocation method.[6]

The size of IEO is determined by the amount of time available for the mission. Once IEO is fixed, a nominal trajectory that uses zero fuel can be computed from the IEO to near L1 periodic orbit, by choosing different initial conditions at IEO and doing rapid propagation via a Keplerian map approximation, as described in[1] and.[2] This map captures the effect of moon kicks at apoapse, which lead to change in the periapse of the orbit. This nominal trajectory is used to obtain initial conditions for the multiple-shooting optimization problem,[3] which is solved by converting it into a nonlinear programming problem via IPOPT. The control is modeled as instantaneous delta-Vs at periapses and apoapses. The solution of multiple-shooting problem gives us a trajectory that avoids visiting the same resonances more than once, and hence leads to shorter propagation times across various resonances using very little fuel.

The problem of optimally inserting into the stable manifold of a periodic orbit around L1 (and hence into the periodic orbit) and the optimal transfer to a LLO is solved via a Gauss-pseudospectral collocation method. This collocation method leads to a set of KKT conditions that is identical to the discretized form of the first-order optimality conditions at the collocation points. The solution thus obtained provides optimal instantaneous delta-Vs required to insert into an L1 periodic orbit of specified size. It also provides the optimal location and magnitude of delta-Vs that take the space probe off the unstable manifold of L1 periodic orbit and inserts it into the LLO. This method is capable of handling various constraints during the insertion into the LLO.

GTO TO L1

Resonant gravity assists in PCR3BP: Generation of zero-fuel paths

The first step is to determine a first control trajectory of the object from an intermediate orbit around the Earth to a neighborhood of the stable manifold of the first Lagrange point (L1). This process is accomplished by first computing several segments of zero-fuel trajectories that can be used to form the complete trajectory. These segments are computed using a kick function that approximates the effect of moon on the object when the object is within the sphere of influence of the Earth. The idea of using three-body resonant gravity assists for low energy mission design has been developed over the last several years, e.g. in the context of multi-moon orbiter for Jupiter,[1,2,5,8] and in the lunar mission design of SMART-1.[9] The kick function F is given as follows:

$$F \begin{pmatrix} \omega_{n+1} \\ K_{n+1} \end{pmatrix} = \begin{pmatrix} \omega_n - 2\pi(-2K_{n+1})^{-3/2}(mod 2\pi) \\ K_n + \mu f(\omega_n, K_n) \end{pmatrix} \tag{2}$$

The above map gives the approximate evolution of the angle of apopase , and the semi-major axis of the spaceprobe over one revolution around the Earth. The kick function f is computed off-line by integrating the perturbations of the moon over an unperturbed Keplerian orbit. A sample zero-fuel trajectory is shown in figure 2, in the a-w plane, along with the various relevant resonances.

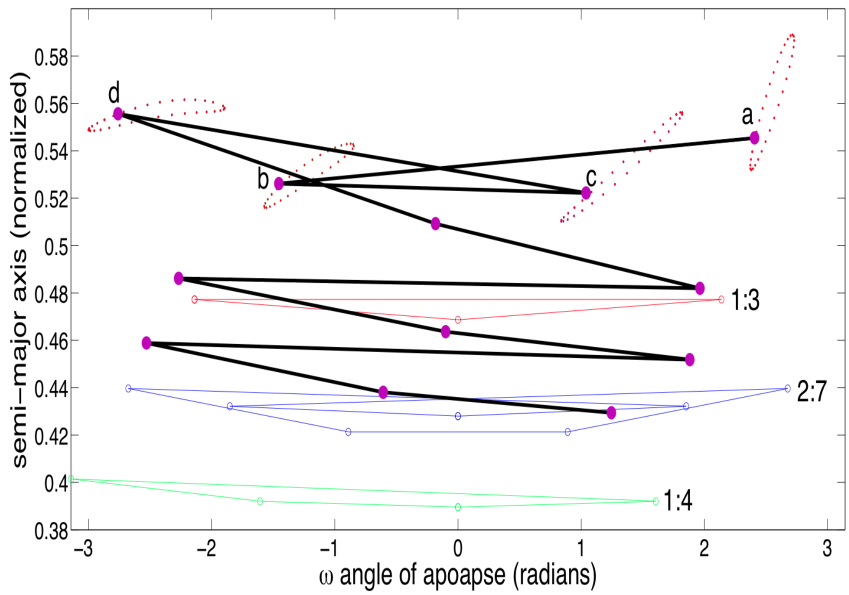


Figure 2. A zero-fuel trajectory from an intermediate Earth orbit which ends up inside the stable manifold of Lyapunov orbit around L1, shown here in the $a - w$ apoapse plane. The first apopase intersection with the 'tube manifold' is marked 'd', second is marked 'c' and so on.

Optimization of zero-fuel paths

A sequence of a initial guesses for the first control trajectory are formed by combining the various apses of these zero-fuel trajectories, according to various topologies that satisfy time constraints of the mission. It is realized that the time of flight during this phase of the mission is determined mostly by the topology of the trajectory. Forcing continuity constraints along with the guesses and including possible control inputs at the apses then form a multiple-shooting problem. At the ith apse, the following vector is to be solved for,

$$\begin{pmatrix} x & y & v_x & v_y & \delta v_x & \delta v_y & t \end{pmatrix} \tag{3}$$

We want to minimize the total fuel consumption, subject to constraint:

$$\begin{pmatrix} \phi_{t_0}(X_0 + \begin{bmatrix} 0 & 0 & \delta v_{x_0} & \delta v_{y_0} \end{bmatrix}^T) - X_1 \\ \phi_{t_1}(X_1 + \begin{bmatrix} 0 & 0 & \delta v_{x_1} & \delta v_{y_1} \end{bmatrix}^T) - X_2 \\ \vdots \\ \phi_{t_{n-1}}(X_{n-1} + \begin{bmatrix} 0 & 0 & \delta v_{x_{n-1}} & \delta v_{y_{n-1}} \end{bmatrix}^T) - X_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{4}$$

The function ϕ_t here represents the time-t map, given initial condition. The various X_i here represent the state vectors at different stations.

This multiple-shooting problem is solved by a nonlinear programming software which minimizes the total control input. It is also realized that this optimization problem is highly sensitive to initial guesses that are used during the optimization procedure, and hence it is imperative to have a systematic procedure of obtaining the initial guess. The control trajectory obtained for the zero-fuel trajectory mentioned earlier is shown in figure 3, upto the first intersection with a stable manifold of (a periodic orbit around) L1.

Next, the second control trajectory of the object from GTO to the initial condition of the first control trajectory. It is realized that since the moon is far away from this section of the trajectory, the Hohmann transfer provides a good guess for such a trajectory. The initial guess computed by using the apses from the Hohmann transfer trajectory are used along with continuity constraints and a multiple shooting problems is formed, and solved as before to minimize the control input required.

NEAR L1 TO MOON ORBIT

The Gauss Pseudo-Spectral method (GPM)[10,11] is a direct optimization method based on transcribing a continuous-time optimal control problem, Equation (5), into a nonlinear programming problem (NLP) which then is solved by dedicated software, such as IPOPT.[12] The idea is to approximate the states and controls using global polynomials which span the entire optimization horizon and perform the collocation at the roots of an orthogonal polynomial. The continuous cost functional is in turn approximated using a Gauss quadrature.

The continuous-time optimal control problem of interested is the transformed Bolza problem which is defined as to, determine the control, $u^*(t)$ and the parameters, q^*, that minimizes the cost,

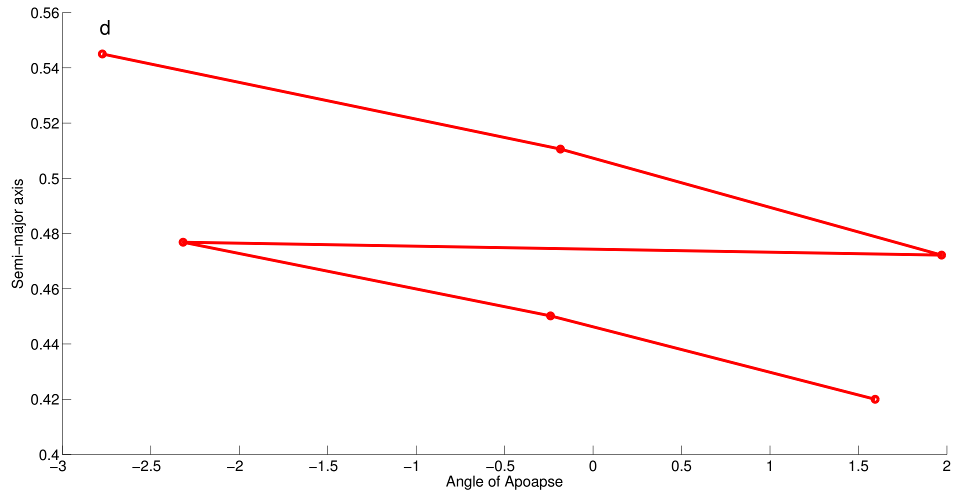


Figure 3. A control trajectory from an intermediate Earth orbit which ends up inside the stable manifold of Lyapunov orbit around L1, shown here in the $a - w$ apoapse plane. The first apopase intersection with the 'tube manifold' is marked 'd'.

$$J(u(t), q) = \Phi(x(\tau_0), t_0, x(\tau_f), t_f; q) + \frac{t_f - t_0}{2} \int_{-1}^{1} g(x(\tau), u(\tau), \tau; q) d\tau \tag{5a}$$

subject to the dynamics given by an ordinary differential equation,

$$\frac{dx}{d\tau} = \frac{t_f - t_0}{2} f(x(\tau), u(\tau), \tau; q) \tag{5b}$$

together with the boundary conditions, ϕ, and the inequality path constraints, C,

$$\phi(x(\tau_0), t_0, x(\tau_f), t_f; q) = 0 \tag{5c}$$

$$C(x(\tau), u(\tau), \tau; q) \leq 0. \tag{5d}$$

Here, x are the states, u the controls and q the parameters. The transformed Bolza problem uses the transformation $\tau = \frac{2t}{t_f - t_0} - \frac{t_f + t_0}{t_f - t_0}$ so that $\tau \in [-1, 1]$ which still allows for a free initial time and/or free final time in the optimal control problem.

In GPM, the K collocation points are chosen as the roots, τ_i, $(i = 1, \ldots, K)$, of the K^{th} degree

Legendre polynomial, $P_K(\tau)$

$$P_K(\tau) = \frac{1}{2^K K!}\frac{d^K}{d\tau^K}\left[(\tau^2-1)^K\right] \tag{6}$$

with associated Gauss quadrature weights,

$$w_i = \frac{2}{(1-\tau_i^2)\left[\dot{P}_K(\tau_i)\right]^2}, \quad (i = 1, \ldots, K). \tag{7}$$

The roots of $P_K(\tau)$ all lie in the interior of the interval [-1,1]. Figure 4 depicts the roots of $P_{10}(\tau)$.

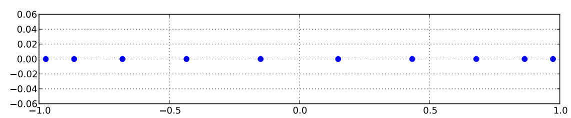


Figure 4. Roots of the 10^{th}-degree Legendre polynomial.

Given the K collocation points, the states are approximated using $K+1$ Lagrange polynomials with τ_i, $(i = 1, \ldots, K)$ as the support points together with the additional point $\tau_0 = -1$.

$$x(\tau) \approx X(\tau) = \sum_{i=0}^{K} L_i^{K+1}(\tau)x(\tau_i) \tag{8}$$

The addition of the extra point, τ_0 in the approximation is to also include the initial values. It should be noted that this extra point is only present in the approximation and is not a collocation point.

By differentiation of Equation (8), the approximation for the state derivatives at the points are given by,

$$\dot{x}(\tau_k) \approx \dot{X}(\tau_k) = \sum_{i=0}^{K} \dot{L}_i^{K+1}(\tau_k)X(\tau_i) = \sum_{i=0}^{K} D_{ki}X(\tau_k), \quad (k = 1, \ldots, K) \tag{9}$$

where D_{ki} is the *differentiation matrix* which can be calculated prior to the optimization given K. Correspondingly, the controls are approximated using K Lagrange polynomials with τ_i, $(i =$

$1, \ldots, K)$ as the support points.

$$u(\tau) \approx U(\tau) = \sum_{i=1}^{K} L_i^K(\tau) U(\tau_i). \tag{10}$$

Discretization using the Gauss Pseudo-Spectral method

Given the continuous-time optimal control problem in Equation (5) and the approximations for the states, Equation (8), the state derivatives, Equation (9), and the controls, Equation (10), the discretized GPM is as follows. Determine the control, U_k^*, $(k = 1, \ldots, K)$, the state, X_i^*, $(i = 0, \ldots, K+1)$, and the parameters, q^* that minimizes the cost:

$$J(X, U, q) = \Phi(X_0, t_0, X_f, t_f; q) + \frac{t_f - t_0}{2} \sum_{k=1}^{K} w_k g(X_k, U_k, \tau_k; q) \tag{11a}$$

subject to the discretized system dynamics,

$$R_k \equiv \sum_{i=0}^{K} D_{ki} X_i - \frac{t_f - t_0}{2} f(X_k, U_k, \tau_k; q) = 0, \quad (k = 1, \ldots, K) \tag{11b}$$

$$R_f \equiv X_f - X_0 - \frac{t_f - t_0}{2} \sum_{k=1}^{K} w_k f(X_k, U_k, \tau_k; q) = 0 \tag{11c}$$

boundary conditions, ϕ, and the inequality path constraints, C,

$$\phi(X_0, t_0, X_f, t_f; q) = 0 \tag{11d}$$

$$C(X_k, U_k, \tau_k; q) \leq 0, \quad (k = 1, \ldots, K). \tag{11e}$$

As many optimal control problems include constraints on the final states, an extra variable has been introduced, X_f, Equation (11c), which is constrained using a Gauss quadrature.

Multiphase Problems

An optimal control problem may have a solution with discontinuities, either in the states or in the controls. In such cases, a global polynomial approximation may not be advantageous. A solution, if the discontinuities are known in either time or in number, is to divide the optimal control problem into two or more phases with semi-global approximations. The idea is then to discretize for each phase according to Equation (11) with the additional linkage constraint,

$$L_{(s)}^{(s+1)}(X_f^{(s)}, t_f^{(s)}; q^{(s)}, X_0^{(s+1)}, t_0^{(s+1)}; q^{(s+1)}) = 0, \quad (s = 1, \ldots, S-1). \tag{12}$$

where S is the number of phases. In Figure 5, a sketch of a discontinuous problem is shown.

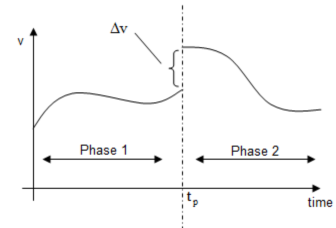


Figure 5. Linkage between phases.

IMPLEMENTATION

The Gauss Pseudo-Spectral method has been implemented into the framework of the JModelica.org[13] platform which is an "open source Modelica platform for modelling, simulation and optimization". The platform is based on the Modelica[14] language and its extension Optimica[15] for modelling of system dynamics and optimization problems.

The implementation of the discretization using the GPM follows the implementation outlined in Reference 16. For discretizing the problem formulation, CasADi[17] has been used which is able to load the model description generated by JModelica.org from the problem defined in Optimica. By using CasADi for the discretization, access to necessary derivatives, such as the gradient of the cost and the Jacobian of the constraint, is available as CasADi implements automatic differentiation. For actually solving the resulting NLP, IPOPT was chosen.

In Figure 6, the steps necessary for solving an optimization problem is shown. As a first step, the optimization problem has to be described in the Optimica language. Second, a control script in the programming language Python is needed for invoking the JModelica.org compiler and for invoking the optimization method. The compiler translates the problem description making it readable by CasADi and by using CasADi the problem is transcribed into an NLP using the GPM. Finally, the transcribed problem is solved using IPOPT.

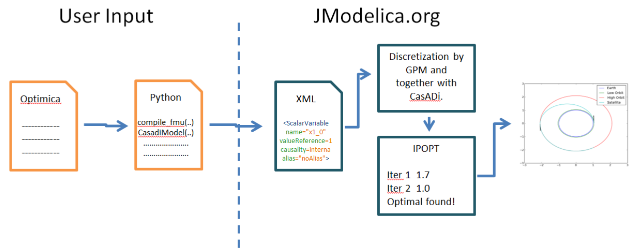


Figure 6. Intended workflow for performing an optimization using JModelica.org.

Moon Orbit Insertion Problem

The problem considered is that of, given a satellite in a halo orbit around the libration point L_1, described by the PCR3BP, find the minimum fuel consumption (minimum velocity changes) of an insertion into an elliptical moon orbit with apoapse positioned 100 *km* above the moon surface and periapse positioned 800 *km* above the moon surface. In Figure 7, the halo orbit for the starting position is shown together with the moon and the target orbit.

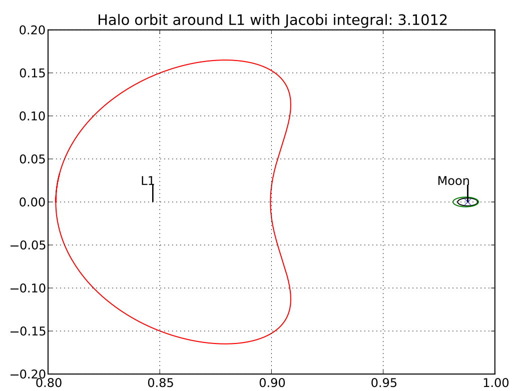
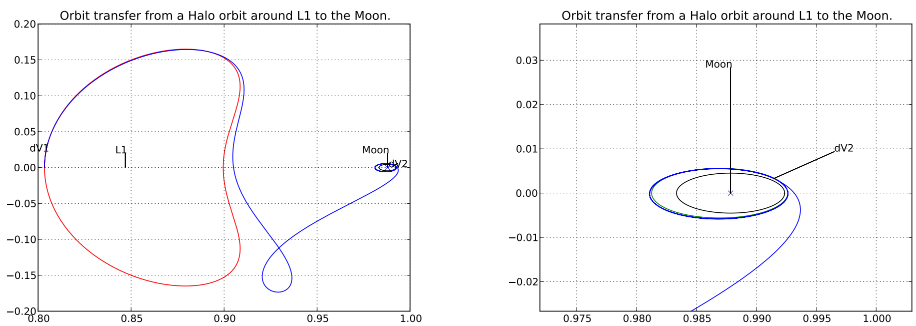


Figure 7. Halo orbit (red) around the libration point L_1 in the Eart-Moon system together with the moon (black) and the target orbit (green).

We assume that two instantaneous rocket burns, corresponding to two discontinuous changes in the velocities is required for manoeuvre a satellite from a halo orbit into a orbit around the moon. The assumption is that one rocket burn is associated with leaving the halo orbit, dV_1, and if timed correctly, one rocket burn is associated with insertion into the targeted moon orbit, dV_2.

The solution strategy is to model the system dynamics, Equation (1), and the cost, $min(dV_1 + dV_2)$, in the modelling language Optimica and discretize the problem with GPM using two phases, i.e. two semi-global polynomial approximations. The additional linkage constraint between the phases will fix the positions of the satellite and the difference in magnitude of the velocities in phase one and phase two will correspond to dV_1. dV_2 is related in the final constraints which constrain the satellites position together with its radial and tangential velocities. The final time and the time point of the intersection between phase one and phase two are set to be free, i.e, they are allowed to be changed by the NLP solver. The initial position conditions are set to a point on the halo orbit, see Figure 7, and the initial velocity constraints are set to be consistent on the halo orbit.

The problem was solved using JModelica.org and in Figure 8 the resulting satellite trajectory is shown. The resulting velocity changes were found to be $dV_1 = 1.4$ *m/s* and $dV_2 = 522.8$ *m/s* totalling $dV = 524.2$ *m/s*. The first rocket burn is ignited after $t_1 = 0.43$ *Days* and the second rocket burn, insertion into moon orbit, is ignited after $t_2 = 14.03$ *Days*.



(a) Complete trajectory. (b) Trajectory in the neighbourhood of the moon.

Figure 8. Resulting satellite trajectory given from a simulation of the optimization result. The red trajectory is the initial halo orbit. dV_1 and dV_2 are the resulting rocket burns.

Halo Orbit Insertion Problem

The problem considered is that of, given a satellite trajectory in the proximity of a given halo orbit, find the minimum fuel consumption for insertion into the halo orbit, see Figure 9.

As in the previous section, we assume that two instantaneous rocket burns is required to target the halo orbit. The assumption is that the first rocket burn takes place at the apoapse just prior to closing in on the halo orbit, as marked by the red cross in Figure 9. The second rocket burn is assumed to be somewhere in between the initial position and the targeted halo orbit.

The problem set-up and solution strategy is closely related to that of the moon insertion problem.

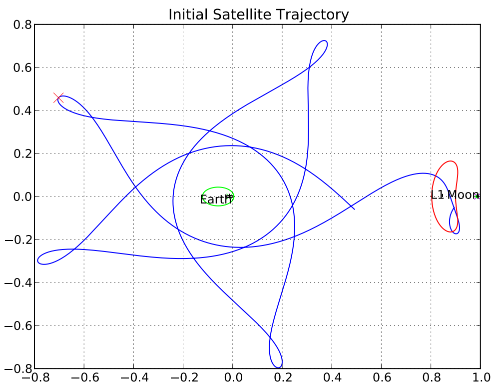


Figure 9. Initial satellite trajectory (blue) in proximity of the halo orbit (red), shown in the rotating frame. The red cross indicates the fixed initial position in the optimization.

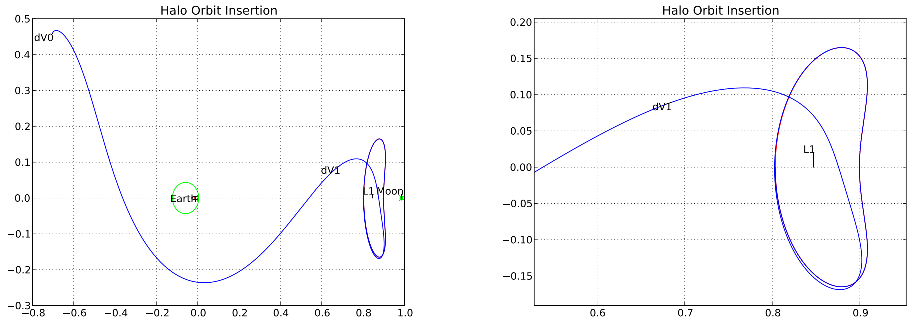
The difference is that instead of having a second rocket burn at the end of the optimization horizon, we have a rocket burn at the initial fixed position, dV_0, and another rocket burn, as before, between phase one and phase two, dV_1. The final time and the time point of the intersection between phase one and phase two are set to be free. The final constraints are set on the targeted halo orbit, both on the position and the velocity. The initial position constraints are set according to Figure 9. Finally, the objective is to minimize the cost, $min(dV_0 + dV_1)$.

As in the previous section, the problem was solved using JModelica.org and in Figure 10 the resulting satellite trajectory is shown. The resulting velocity changes were found to be 0.56 *m/s* and 1.28 *m/s* totalling 1.84 *m/s*. The first rocket burn is ignited at the fixed initial position while the second rocket burn is ignited after $t_1 = 8.19$ *Days*, taking the satellite into the halo orbit.

We show the complete control trajectory in figure 11.

CONCLUSIONS

A modular design framework for lunar mission design, capable of handling various mission constraints has been presented in this paper. This method makes systematic use of three-body gravity assists and invariant manifolds as governing dynamics. Shooting method based optimization of three-body gravity assists and collocation based optimization of manifold transfers is used to tweak initial guesses. Missions can be designed to obtain up to 150 m/s fuel savings compared to Hohmann transfers, with 4-5 months transfer time



(a) Complete trajectory.

(b) Trajectory in the neighbourhood of the halo orbit.

Figure 10. Resulting satellite trajectory given from a simulation of the optimization result. The red trajectory is the targeted halo orbit. dV_0 and dV_1 are the resulting rocket burns.

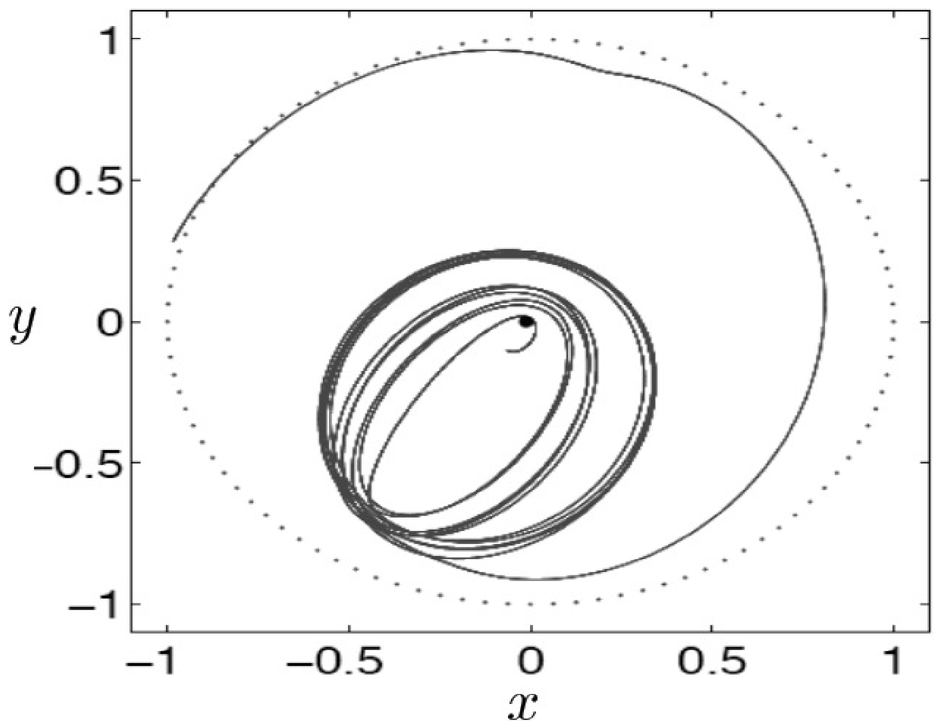


Figure 11. Full control trajectory in the inertial frame

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REFERENCES

[1] S. D. Ross and D. J. Scheeres, "Multiple gravity assists, capture, and escape in the restricted three-body problem," *SIAM Journal on Applied Dynamical Systems*, Vol. 6, No. 3, 2007, pp. 576–596.

[2] P. Grover and S. D. Ross, "Designing trajectories in a planet-moon environment using controlled Keplerian Map," *AIAA Journal of Guidance,Control and Dynamics*, Vol. 32(2), 2009, pp. 437–444.

[3] S. Campagnola and R. Russell, "Endgame Problem Part 2: Multi-Body Technique and T-P Graph," *AIAA Journal of Guidance, Control and Dynamics*, Vol. 33(2), 2010.

[4] W. S. Koon, M. W. Lo, J. E. Marsden, and S. D. Ross, "Heteroclinic connections between periodic orbits and resonance transitions in celestial mechanics," *Chaos*, Vol. 10, 2000, pp. 427–469.

[5] G. Lantoine, R. Russell, and S. Campagnola, "Optimization of Low-Energy Resonant Hopping Transfer Between Planetary Moons," *Acta Astronautica*, Vol. 68(7-8), 2010.

[6] D. A. Benson, G. T. Huntington, T. P. Thorvaldsen, and A. V. Roa, "Direct Trajectory Optimization and Costate Estimation via an Orthogonal Collocation Method," *AIAA Journal of Guidance, Control and Dynamics*, Vol. 29(6), 2006.

[7] W. S. Koon, M. W. Lo, J. E. Marsden, and S. D. Ross, "Dynamical Systems, the Three-Body Problem and Space Mission Design," April 2011.

[8] S. D. Ross, W. S. Koon, M. W. Lo, and J. E. Marsden, "Design of a Multi-Moon Orbiter," *13th AAS/AIAA Space Flight Mechanics Meeting*, Ponce, Puerto Rico, February 2003. Paper No. AAS 03-143.

[9] G. D. Racca, "SMART-1 mission description and development status," *Planetary and Space Science*, Vol. 50(14-15), 2002.

[10] D. Benson, *A Gauss pseudospectral transcription for optimal control*. PhD thesis, Massachusetts Institute of Technology, Cambridge, MA, USA, 2005.

[11] G. T. Huntington, *Advancement and analysis of Gauss pseudospectral transcription for optimal control problems*. PhD thesis, Massachusetts Institute of Technology, Cambridge, MA, USA, 2007.

[12] A. Wächter and L. T. Biegler, "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming," *Mathematical Programming*, Vol. 106, 2006, pp. 25–57.

[13] J. Åkesson, K.-E. Årzén, M. Gäfvert, T. Bergdahl, and H. Tummescheit, "Modeling and Optimization with Optimica and JModelica.org—Languages and Tools for Solving Large-Scale Dynamic Optimization Problem," *Computers and Chemical Engineering*, Vol. 34, November 2010, pp. 1737–1749.

[14] "Modelica - A Unified Object-Oriented Language for Physical Systems Modeling," language specification, Modelica Association, March 2010.

[15] J. Åkesson, "Optimica—An Extension of Modelica Supporting Dynamic Optimization," *In 6th International Modelica Conference 2008*, Modelica Association, March 2008.

[16] A. V. Rao, D. A. Benson, C. Darby, M. A. Patterson, C. Francolin, I. Sanders, and G. T. Huntington, "Algorithm 902: GPOPS, A MATLAB software for solving multiple-phase optimal control problems using the gauss pseudospectral method," *ACM Trans. Math. Softw.*, Vol. 37, April 2010, pp. 22:1–22:39.

[17] J. Andersson, B. Houska, and M. Diehl, "Towards a Computer Algebra System with Automatic Differentiation for use with Object-Oriented modelling languages," *3rd International Workshop on Equation-Based Object-Oriented Modeling Languages and Tools*, Oslo, Norway, October 2010.