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Abstract

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Nonlinear Model Predictive Control (NMPC)

An Industry Perspective on MPC in Large Volumes Applications: Potential Benefits and Open Challenges

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Abstract: Model predictive control has been originally developed for chemical process control, where plants are expensive, have slow dynamics, and a large number of inputs and outputs. Furthermore, in chemical process control each control system is usually deployed to a single plant, and hence can be specifically tuned. In recent years there has been a growing interest towards MPC in other industries, such as automotive, factory automation, and aerospace, where the plants have faster dynamics, fewer inputs and outputs, reduced costs, and each controller is deployed to a large number of plants, i.e., it is deployed in large volumes. These applications also presents several classes of nonlinearities. While there are several benefits for using MPC in these industries, the difference in the plant characteristics and in production volume targets pose several challenges to the widespread use of MPC that are still partially unsolved. In this paper we discuss the benefits of MPC in large volumes industries, by using examples from automotive, aerospace, and mechatronics that also present several specific nonlinearities that can be efficiently handled by MPC. We then discuss the unsolved challenges for these application domains, and the related ongoing research.

Keywords: Model predictive control, control applications, automotive, aerospace.

1. INTRODUCTION

Model predictive control (Garcia et al. [1989]) has been developed primarily for chemical applications, to control the transients of dynamic systems with hundreds of inputs and outputs, subject to constraints (Qin and Badgwell [2003]). Such applications are characterized by slow dynamics, presence of expert human supervision, and expensive plants, causing the cost of the control system to be practically irrelevant. Also, each controller is usually deployed into a single plant, which means that each control design is customized and specifically tuned.

In the last 15 years, due to the increase of the available computational power in microprocessors, MPC has been investigated in different industries, such as automotive, aerospace, and factory automation/robotics, where each control design is deployed in hundreds, or even thousands, of final products. Here we refer to these domains as "large volumes" application domains. The applications in large volumes domains differ from the ones in chemical process control for system size, significantly smaller, dynamics, significantly faster, control system supervision, unsupervised, and plant cost, significantly reduced, meaning that the control system has to have low cost. Another major difference is that, due to the large number of plants where it is deployed, the controller cannot be tuned for a single plant, and hence needs to be flexible and robust to accommodate the plant-to-plant differences.

While the availability of cheap powerful processors and the development of new MPC algorithms address some of the problems in large volumes applications, for some cases, other problems are yet to be solved and limit still today the widespread use of MPC in favor of traditional control strategies, like PIDs. Some of these problems are the computational complexity of the algorithm and the induced microprocessor cost, the need of robust calibration, the applicability to limited classes of dynamics, the lack for guaranteed robustness and bounded computational load. On the other hand there are multiple reasons for the large volumes industries to look with increased interest at MPC, such as the performance optimization, the enforcement of constraints on outputs and inputs, the ease of design for multivariable systems, the capability of dealing with time delays and future information, and the capability of easily handling certain nonlinearities.

In this paper we discuss the potential benefits and the unsolved challenges of MPC in large volumes applications. After reviewing the fundamentals of MPC in Section 2, in Section 3 we analyze some of the potential benefits by using examples developed in automotive, aerospace, and mechatronics. In Section 4 we discuss some of the remaining challenges, and the related ongoing research. Conclusions are summarized in Section 5.

2. FUNDAMENTALS OF MPC

The development of model predictive control (Garcia et al. [1989]) is an attempt to counteract some of the limitations of classical optimal control. With respect to indirect optimal control, MPC improves robustness by implementing feedback through receding horizon planning. With respect to LQR feedback, MPC explicitly accounts for plant and control constraints, while restricting the cost function integration to a finite interval to make the associated optimization problem computationally tractable.

Model predictive control is based on the iterative receding horizon solution of a finite horizon optimal control problem formulated basing on a model of the system dynamics, plant and control constraints, and performance objective. At any control cycle the MPC controller: (i) sets a finite horizon optimal control problem using the current state estimate as initial state; (ii) solves the optimal control problem obtaining the optimal input sequence over the future horizon; (iii) applies the computed optimal input sequence until new information on the system is available. Then, the process is repeated from (i).

The general MPC optimal control problem is

$$\min_{U(t)} F(x(N|t)) + \sum_{k=0}^{N-1} L(x(k|t), y(k|t), u(k|t)) \quad (1a)$$

$$\text{s.t. } x(k+1|t) = f(x(k|t), u(k|t)) \quad (1b)$$

$$y(k|t) = h(x(k|t), u(k|t)) \quad (1c)$$

$$x_{\min} \leq x(k|t) \leq x_{\max}, \quad k = 1, \ldots, N_c \quad (1d)$$

$$y_{\min} \leq y(k|t) \leq y_{\max}, \quad k = 0, \ldots, N_c \quad (1e)$$

$$u_{\min} \leq u(k|t) \leq u_{\max}, \quad k = 0, \ldots, N_{cu} \quad (1f)$$

$$u(k|t) = \kappa(x(k|t)), \quad k = N_u, \ldots, N-1, \quad (1g)$$

$$x(N|t) \in \mathcal{X}_N \quad (1h)$$

$$x(0|t) = x(t) \quad (1i)$$

where t is the discrete-time index, and $v(h|t)$ denotes the value of v predicted h steps ahead from t, based on information up to t. Equations (1b), (1c) are the discrete time model of the system dynamics with sampling period T_s, where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$ are the system state, input, and output, respectively. The model is initialized at the current state estimate $x(t)$ by (1i). The optimizer in (1) is the control input sequence $U(t) = (u(0|t), \ldots, u(N-1|t))$, where N is the finite problem horizon. The cost function (1a) is composed of a stage cost L, and a terminal cost F. The constraints on states and outputs, and inputs are enforced along the horizons N_c and N_{cu}, respectively. The control horizon N_u is the number of optimized steps, before terminal control law (1g) is applied. The terminal constraint (1h) forces the state at the end of the horizon to belong to the set $\mathcal{X}_N \subseteq \mathbb{R}^n$.

Problem (1) is formulated as the mathematical program

$$\min_{U(t)} \mathcal{J}(U(t), x(t)) \quad (2a)$$

$$\text{s.t. } \mathcal{G}(x(t), U(t)) \leq 0, \quad (2b)$$

which is often non-convex, and hence difficult to solve for the global optimum. However, for particular classes of systems, (2) becomes computationally tractable. For linear systems with linear constraints and quadratic cost, (2) reduces to the quadratic program

$$\min_{U(t)} U'(t)\mathcal{H}U(t) + x(t)\mathcal{F}'U(t) \quad (3a)$$

$$\text{s.t. } \mathcal{G}U(t) \leq \mathcal{W} + \mathcal{S}x(t), \quad (3b)$$

that under mild assumptions on (1) is convex, and hence can be solved efficiently. Tractable formulations can be also achieved for linear systems with linear constraints and linear cost functions (Bemporad et al. [2002a]), and linear hybrid systems (Bemporad and Morari [1999]).

It is relatively evident from (1) that MPC results in a (nonlinear) static state-feedback,

$$u(t) = g_{\text{MPC}}(x(t)), \quad (4)$$

since at every cycle the only changing element in (1) is the initial state (1i). If the feedback law (4) can be computed, the control algorithm is simplified, since (2) does not need to be solved at every control cycle. Furthermore, by constructing the closed-loop system

$$x(t+1) = f(x(t), g_{\text{MPC}}(t)), \quad (5)$$

the control-theoretic properties can be analyzed.

While the explicit feedback law (4) is impossible to compute for the general case, in Bemporad et al. [2002b] it was shown that that for linear systems with linear constraints and quadratic cost, (4) is the piecewise affine function

$$u(t) = F_{i(t)}x(t) + G_{i(t)} \quad (6a)$$

$$i(t) \in \{1, \ldots, s\} \, : \, H_{i(t)}x(t) \leq K_{i(t)}, \quad (6b)$$

which is easy to synthesize even in simple microprocessors, although it may requires a significantly large amount of memory and computations to execute, when s is large.

3. POTENTIAL BENEFITS

In this section we discuss the MPC benefits in large volumes applications, in particular, performance optimization, handling of multivariable systems, time delay, future information, and enforcement of constraints. We discuss this benefits by introducing examples where these capabilities are highlighted. Since these are real-world examples, the plants show some specific nonlinearities, such as time-varying constraints, delays, switching behavior, and we discuss how MPC efficiently handle these.

As mentioned, the MPC feedback law (4) is a static (state) feedback. The testing and verification procedures for static control laws are significantly simpler than for dynamic ones. For MPC, the output of the controller can be tested by using static test vectors (basically, the system state), as opposed to dynamic feedback laws, e.g., adaptive control, which require test vectors composed of system state histories. The simpler verifiability increases the confidence of the engineers in the correctness of the algorithm, thus simplifying the technology transfer from R&D laboratories to products.

3.1 Performance objective optimization

An appealing element of MPC is that cost function (1a) is optimized over a future horizon. Thus, in principle it is possible to obtain optimal/close-to-optimal performance by encoding the performance specification of the closed-loop system in (1). However, in order to obtain a computationally feasible optimization problem, the choices for (1a)

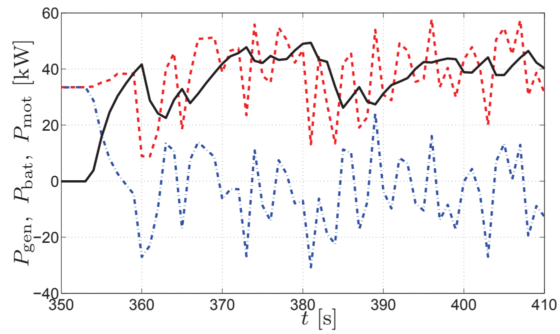


Fig. 1. Effects of MPC energy manager on an interval of the US06 (Highway) cycle. Generator power (solid), battery power (dash dot), motor power (dash).

are restricted, and it is not easy to obtain a cost function that directly encodes the desired performance. However, the optimizing approach of MPC can still be beneficial through indirect principles.

Recently, there has been a significant interest in the automotive industry in energy management of Hybrid Electric Vehicles (HEVs). Among other techniques, MPC has also been applied to this problem. In Ripaccioli et al. [2009], Borhan et al. [2012] controllers that directly optimize the explicit performance criterion, i.e., the fuel consumption, were designed. The authors obtain interesting results, but the complexity of the controllers is prohibitive for the application, since a long horizon is needed, and nonlinear, linear-time varying, and hybrid models are involved. These limitations are overcome in Di Cairano et al. [2011] where an indirect performance objective is applied. Instead of minimizing the fuel consumption, the MPC controller was minimizing the engine transient aggressiveness, hence minimizing also the efficiency losses. The controller is computationally feasible and it was implemented in a production-like environment in a prototype Series HEV. The controlled achieved improvements in overall fuel economy in the order of 5% with respect to advanced base strategies, and very close to the (theoretical) optimal ceiling.

An example of the controller behavior is reported in Figure 1, where a part of the simulation of the city drive cycle for the controller in closed-loop with an industry-grade high-fidelity complete vehicle simulation model is reported. The generator power, battery power, and motor power are reported, where for the Series HEV

$$P_{\rm wh}(t) = \eta_{\rm wh}(t) P_{\rm mot}(t), \tag{7a}$$

$$P_{\rm mot}(t) = \eta_{\rm mot}(t)(P_{\rm gen}(t) + P_{\rm bat}(t)), \tag{7b}$$

$$P_{\rm gen}(t) = \eta_{\rm gen}(t) P_{\rm eng}(t), \tag{7c}$$

$P_{\rm wh}$ is the power at the wheels, requested by the driver, $P_{\rm mot}$ is the motor power, which is the unique source of traction and is powered through a bus from battery and generator powers $P_{\rm bat}$, $P_{\rm gen}$, respectively, and where the generator converts the whole engine power $P_{\rm eng}$. The variables $\eta_{\rm wh}$, $\eta_{\rm mot}$, $\eta_{\rm gen}$ are the efficiencies of driveline, motor, and generator. In Figure 1, while the motor power changes aggressively, the engine power variations are smoothed by the MPC using the battery, while still enforcing constraints on battery charge and battery power.

3.2 Design for MIMO systems

In contrast with many classical control design techniques, MPC allows to design control systems for multi-input multi-objective [1] (MIMO) systems with the same complexity for the engineer than for single-input single-objective (SISO) systems. In MIMO systems, MPC tends to provide significant improvements in performance with respect to classical techniques, since with more degrees of freedom, the use of an optimal criterion rather than a hands-on gain calibration allows to achieve higher performance over a larger range of operating conditions.

A practical example of this was demonstrated in Di Cairano and Tseng [2010] where MPC was used to coordinate active front steering (AFS) and differential braking system (DBS) to improve vehicle stability and cornering control. The control system has two actuators (AFS, DBS) and two objectives (vehicle stability and cornering performance). The problem is particularly complex because both AFS and DBS are subject to constraints, and the vehicle dynamics are nonlinear, and approximated by the piecewise affine system

$$x(t+1) = A_{i(t)} x(k) + B_{i(t)} u(t) + f_{i(t)} \tag{8a}$$

$$i(t) \in \{1, \ldots, s\} \, : \, H_{i(t)} x(k) \leq K_{i(t)} \tag{8b}$$

where the state is composed of the tire slip angles and vehicle steering angle $x = [\alpha_f \ \alpha_r \ \delta]'$ the control inputs are the steering angle rate and yaw moment provided by the DBS, $u = [\varphi \ Y_m]'$. The steering angle is the sum of the AFS actuator and the driver steering, $\delta = \delta_{\rm drv} + \delta_{\rm AFS}$. Model (8) is obtained by approximating the tire forces as piecewise affine functions of the slip angles.

Figure 2 shows experimental data from a vehicle stabilization test where while driving in circle at an approximately constant yaw rate, drift is induced by stepping on the accelerator pedal. The drift events are shown by the positive yaw rate peaks. When drifts occur, the driver-assist system actuates AFS and brakes to countersteer (see the negative yaw rate peaks), so that the vehicle is returned to a stable condition. MPC takes advantage of the actuator differences. The AFS is actuated only to countersteer, due to its dynamics, while the DBS is actuated first to stabilize the vehicle, then, in the opposite direction, to rapidly resume yaw rate tracking.

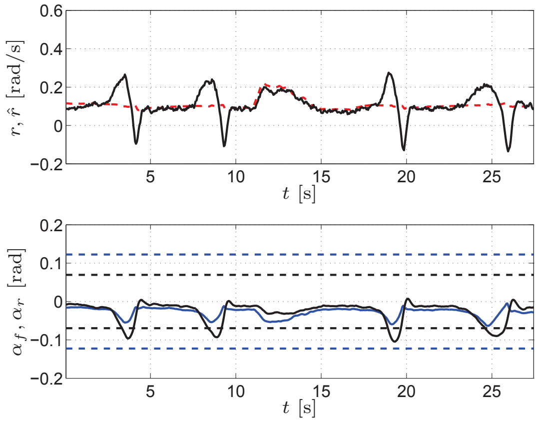
3.3 Time delays and future information

MPC can take into account any available future information along the prediction horizon. This proves to be very important for reference tracking problems and for systems with delays or non-minimum phase zeros.

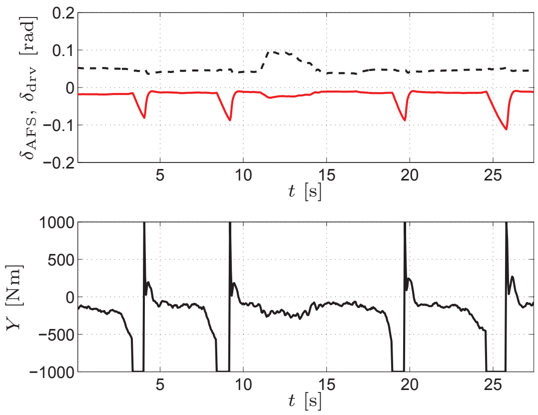
The nonlinearity due to time delay is particularly significant in the regulation of the engine speed while idling. Idle speed is controlled by modulating engine airflow and spark timing, where the airflow-to-torque generation delay is particularly long. The dynamics are described by (Hrovat and Sun [1997]),

$$Y(S) = G_{\rm air}(s) e^{-\delta_a s} U_{\rm air}(s) + G_{\rm spk}(s) e^{-\delta_s s} U_{\rm spk}(s)\,, \tag{9}$$

[1] We use here the word *Objective* rather than *Output* in accordance with the optimization nature of MPC.



(a) Upper plot: Yaw rate reference (dash) and yaw rate (solid). Lower plot: Slip angles (solid) and saturation angles (dash), front in blue, rear in black.



(b) Upper plot: AFS actuator steering angle (solid), driver steering angle (dash). Lower plot: Yaw moment from differential braking.

Fig. 2. Experimental validation of the MPC-based AFS-DBS control in a vehicle stability experiment.

where $G_{\mathrm{air}}(s)$, $G_{\mathrm{spk}}(s)$ are second order functions from the corresponding input (airflow and spark timing) to the engine speed (Y), U_{air}, U_{spk} are the airflow and spark timing control inputs, and δ_a, δ_s are the delay in the corresponding channels, with the first significantly larger than the second one (e.g., 120ms vs 15ms).

Since MPC predicts the system dynamics along a future horizon, the effects of the delay are compensated for. Thus, the controller bandwidth can be increased above the limit that causes instability in a classical (causal) controller. In Figure 3 we show the performance of a baseline controller (PID-type) controlling both channels, of a single input MPC controlling the airflow with the baseline controller on the other channel, and of a multivariable MPC. We consider a disturbance rejection test, where a large torque load from power-steering hits at approximately $t = 10.5$s. It can be seen that indeed even the single input MPC greatly improves the performance, which is due to the capability of counteracting the time-delay.

By acting more aggressively on the airflow without losing stability, the authority of the spark modulation can be safely reduced and as a consequence the engine can run in

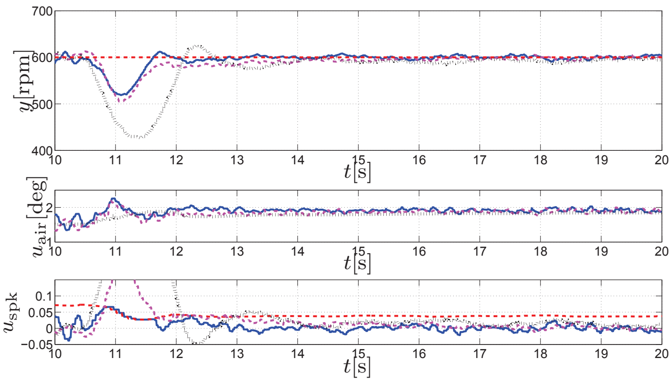


Fig. 3. Idle speed regulation in a disturbance rejection test. Base controller (dot), single-input MPC (dash), multi-input MPC (solid).

nominal conditions closer to the optimal efficiency point thus reducing fuel consumption Di Cairano et al. [2012b].

Other applications were the MPC behavior has been useful are the vehicle stability control described in Section 3.2, where the dynamics associated to the understeering vehicle (front tire force saturated) are non-minimum phase, and for engine speed deceleration control (Di Cairano et al. [2012]) where the desired deceleration rate is known, so that the tracking performance is improved by using a first order model of the reference dynamics

3.4 Enforcement of constraints

Besides the optimization capabilities, MPC is investigated for its capability of handling input and, especially, state constraints. Due to the use of a prediction horizon, MPC usually achieves better performance than other constrained control techniques such as reference governor (Gilbert and Kolmanovsky [2002]).

Indeed, MPC can handle complex constraints involving combinations of states and inputs. In (Di Cairano et al. [2007]) it was shown how MPC can be designed to enforce soft landing in an electromagnetic actuator for engine valve control. By defining the constraints

$$-\varepsilon - \beta(d - x) \leq \dot{x} \leq \varepsilon + \beta(d - x)\,, \qquad (10)$$

that restrict the admissible velocity ($\dot{x}$) to reduce as the moving armature position (x) gets close to the coil (distance d from the origin), soft landing is enforced. Parameters β and ϵ specify the constraints so that these are not restrictive when the coil is far from the armature, while they restrict the velocity range, when the coil is close.

The approach has been recently extended in (Park et al. [2011]) to control the proximity maneuvering and docking of a spacecraft to a rotating target. In this case, besides the soft landing constraints, the MPC controller must maintain the spacecraft within the Line-of-Sight cone of the target sensors.

Since the target is rotating, such cone is rotating as well, and the LoS constraints are mathematically defined by

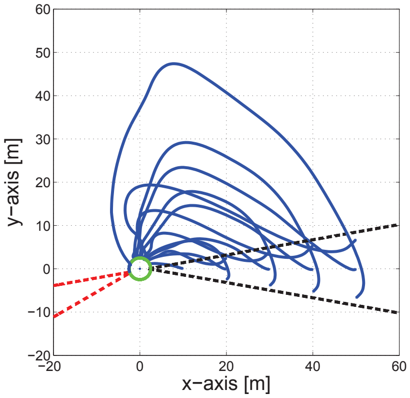


Fig. 4. Radial approach trajectories from various initial positions for a spacecraft docking to a rotating platform within the (rotating) LoS cone (initial position in black dashed lines, final position in the red dashed lines, counterclockwise rotation).

$$\begin{cases} \dfrac{\sin(\varphi(t)+\gamma)}{(r_p - r_{tol})\sin\gamma}\delta x(t) - \dfrac{\cos(\varphi(t)+\gamma)}{(r_p - r_{tol})\sin\gamma}\delta y(k) \geq 1 \\ -\dfrac{\sin(\varphi(t)-\gamma)}{(r_p - r_{tol})\sin\gamma}\delta x(t) + \dfrac{\cos(\varphi(t)-\gamma)}{(r_p - r_{tol})\sin\gamma}\delta y(t) \geq 1 \\ \dfrac{\cos\varphi(t)}{r_p \sin\gamma}\delta x(t) + \dfrac{\sin\varphi(t)}{r_p \sin\gamma}\delta y(t) \geq 1 \quad , \end{cases} \tag{11}$$

where δx, δy are the spacecraft relative coordinates with respect to the target in Hill's frame, φ is the angle of the docking port in the frame, γ is the half of the LoS cone angle, r_p and r_{tol} are physical parameters of the target. Constraints (11) are nonlinear and a linear time-varying approximation is applied.

Figure (4) shows the trajectories from different initial conditions for a spacecraft approaching a rotating platform within the (rotating) LoS cone. The results of (Park et al. [2011]) show that the capability of MPC to enforce constraints coupled with the use of future information allows to achieve improved performance. In particular it was shown in simulations that the fuel consumption of the docking maneuver can be reduced by up to 20%.

4. OPEN CHALLENGES

Despite the many benefits of MPC strategy, some challenges remain to limit its use in large volumes applications. These are tightly related to the intrinsic differences between large volumes applications and chemical process control, for which MPC was developed. While in Section 3 several specific classes of nonlinearities were successfully addressed (delays, switches, varying constraints), for general nonlinear plants these challenges are even harder.

4.1 Applicability: linear, hybrid, and nonlinear systems

While the theory of MPC is well established (Mayne et al. [2000]) even for general nonlinear systems, its applicability is still mostly restricted to linear systems subject to linear constraints and linear/quadratic cost. However, some applications have been developed for hybrid systems with (piecewise) linear continuous dynamics and for switched systems (Geyer et al. [2008], Borrelli et al. [2006], Di Cairano et al. [2007], Bemporad et al. [2010], Di Cairano and Tseng [2010], Di Cairano et al. [2012a]).

In Di Cairano et al. [2007] hybrid MPC was applied to control an electromagnetic actuator. By decoupling mechanical and electromagnetic dynamics by feedback linearization, and by approximating the (non-convex) magnetic force constraint

$$0 \leq F_{\text{mag}}(k) \leq \frac{k_a i^2}{(x(k) - d + k_b)^2}, \tag{12}$$

by a set of piecewise linear constraints

$$0 \leq F_{\text{mag}}(k) \leq r_i x(k) + q_i \quad \text{if} \quad x(k) \in [\bar{x}_i, \bar{x}_{i+1}], \tag{13}$$

the control law can be explicitly computed and meet stringent chronometric requirement (sampling period 0.5ms).

The vehicle stability control system discussed in Section 3.2 was initially designed by hybrid MPC (Di Cairano et al. [2012a]), while the final implementation was a switched MPC, where a piecewise affine system with frozen mode

$$x(h+1|k) = A_{i(k)} x(h|k) + G_{i(k)} \tag{14a}$$

$$i(k) \in \{1, \ldots, s\} \,:\, H_{i(k)} x(k) \leq K_{i(k)}, \tag{14b}$$

is used as prediction model. The switched MPC controller could be implemented and executed in vehicle tests with sampling period 50ms. In general, hybrid MPC based on mixed-integer programming is complex and slow. The last problem is somewhat reduced by the work in (Di Cairano et al. [2008]), but still hybrid MPC seems to be mostly inapplicable to (general) fast systems.

While research is ongoing on general approaches to apply MPC to nonlinear dynamics (Ohtsuka [2004], Houska et al. [2011]), alternative approaches are based on finding parametrization of the system dynamics that are linear/closer to linear. An example is the transformation applied for engine speed deceleration in Di Cairano et al. [2012]. The indicated engine torque (M_{ind}) is described by the product of the base airflow torque (M_{base}) and the torque ratio (κ), $M_{\text{ind}} = \kappa(t - \delta_{\text{spk}}) \cdot M_{\text{base}}(t - \delta_{\text{air}})$, both of which are affected by time-delays (δ_{air}, δ_{spk}) and box-constraints. In Di Cairano et al. [2012] the simple multiplicative nonlinearity $x(t+1) = x(t)u(t)$, where $u \in [\underline{u}\ \overline{u}]$, is converted for $x > 0$ into

$$x(t+1) \ = \ v(t) \tag{15a}$$

$$\underline{v}x(t) \leq x(t) \leq \overline{v}x(t) \tag{15b}$$

hence converting a nonlinear, box-constrained system, into a linear, linearly constrained system. Thus, a deceleration MPC controller that could executed at more than 60Hz was designed. A test of stationary deceleration in drive is shown in Figure 5, where MPC enforces the constraints and achieves high performance tracking.

Major researches is ongoing to apply MPC to uncertain and stochastic systems. Advances in these domains have been achieved in Langson et al. [2004], Mayne et al. [2011], Bemporad and Di Cairano [2011], Bernardini and Bemporad [2009], Bichi et al. [2010], Chatterjee et al.

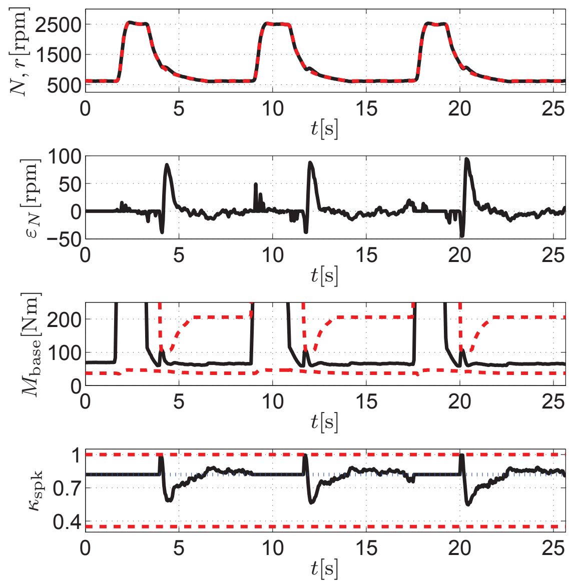


Fig. 5. Engine deceleration in drive using MPC. From the top: Engine speed and reference, tracking error, base torque and constraints, torque ratio and constraints

[2011]. Although not yet mature enough for industrial applications, research efforts are promising.

4.2 Closed-loop properties

From a control theory perspective it is still rather difficult to analyze and enforce stability and robustness of the MPC controllers. Several conditions have been obtained for designing MPC controllers using terminal set and terminal cost (Mayne et al. [2000]), but these usually require long horizons, and hence tend to excessively increase the computational requirements. Thus, they are often not viable for the class of applications discussed here.

When the explicit feedback law (4) is known, the closed loop system (5) can be analyzed a posteriori, and the controller design updated, if needed. For linear systems with linear-quadratic MPC, the closed-loop system (5) is the piecewise affine system

$$x(t+1) = (A + BF_{i(t)})x(k) + G_{i(t)} \tag{16a}$$

$$i(t) \,:\, H_{i(t)}x(t) \leq K_{i(t)}, \tag{16b}$$

whose stability can be studied (a posteriori) globally, by piecewise quadratic Lyapunov functions, or locally, by eigenvalues analysis. Indeed, when (16) is known, robustness margins can also be studied.

It is certainly more complex to guarantee stability by design, without resorting to long horizons. An approach presented in Scokaert et al. [1999] is based on including a control Lyapunov function

$$\mathcal{V}(f(x(0|t), u(0)) \leq \rho\mathcal{V}(x(0|t)), \tag{17}$$

where $0 < \rho < 1$, as an additional constraint in the optimal control problem (1). Indeed, when the problem remains feasible the system is enforcing a control Lyapunov function and hence is asymptotically stable. However, to design (17) such that (1) with (17) remains feasible, is non-trivial. Recently, in Lazar [2009] a design method based on infinity-norm flexible Lyapunov functions has been proposed. In this implementation, (17) is formulated by linear constraints, and can be relaxed to maintain feasibility. Also, if a 1-step horizon is used, for general input-affine nonlinear systems the approach only requires the solution of a linear program. This technique has achieved interesting performance in automotive applications, including driveline oscillation control (Caruntu et al. [2011]) and magnetic valve control (Hermans et al. [2009]).

The application of control Lyapunov functions to hybrid systems has been recently proposed in Di Cairano et al. [2008] resulting in the so called hybrid Lyapunov functions, which allow for the synthesis of stabilizing dynamic controllers based on MPC for hybrid systems.

MPC designs with guaranteed robustness based on min-max approaches (Kothare et al. [1996], Magni et al. [2003], Bemporad et al. [2003]) and tubes (Langson et al. [2004], Limon et al. [2010], Mayne et al. [2011]) have been proposed. Although often too complex for the applications considered here, significant progresses have been achieved.

4.3 Computational load

A major drawback of MPC with respect to more classical control schemes (i.e., PID) is the significant amount of computations required to solve (1). When used in large volumes applications, likely inexpensive and with limited microprocessor capabilities, such an amount of computations may exceed the availability. This also considering that the majority of large volume applications have significantly faster dynamics than the chemical ones.

The use of multiparametric programming to compute the explicit feedback law (6) allows to apply MPC to systems with relative high control bandwidth (up to 100Hz) and with low power processors (10–100MHz and 10–100Kb ROM memory). Major advantages of explicit MPC are that very little active memory (RAM) is required, and that the worst case maximum number of computations can be bounded, as well as the closed-loop system (5) can be computed and analyzed. In fact, most of the applications described here were synthesized by explicit MPC.

However, explicit MPC has the intrinsic limitation that the number of regions (and hence memory occupancy and worst case FLOPS) increases exponentially with the number of constraints. Thus in certain conditions the usage of customized solvers that may take advantage of the MPC structure is still to be preferred. Recently, several techniques have been proposed to exploit the receding horizon nature of MPC, such as Ferreau et al. [2008], where active set with warm-start is used, and Wang and Boyd [2010], where interior point methods are used.

When we consider applications in factory automation, precision engineering, and even certain applications in automotive components, bandwidths above 1kHz in inexpensive hardware are desired. Thus, hardware solutions based on FPGA and ASIC, such as in Bleris et al. [2006], Johansen et al. [2007], Bemporad et al. [2011], Wills et al. [2012], are investigated.

4.4 Design and tuning procedures

A reason for the success of PID controllers are the simple and intuitive calibration procedures that required little control theory expertise. Simple calibration techniques still lack for MPC, which is critical for instance in the automotive industry, where the final controller is usually not calibrated by an expert control engineer.

MPC design allows for several decisions parameters: (i) the complexity of the prediction model, (ii) the cost function, (iii) the constraints, (iv) the length of the horizon. Some of these choices are (partially) dictated by the specifications ((ii), (iii)), and some by the physics of the plant ((i), (iv)). However, there are considerable degrees of freedom in the final choices. In Di Cairano et al. [2008], Di Cairano et al. [2012b] the authors discuss a procedure to perform those decisions, that is appropriate for several automotive problems, and has been applied in the later works (Di Cairano and Tseng [2010], Di Cairano et al. [2011]). However, further investigations are needed to optimize and streamline such procedures.

The cost function tuning process needs to be defined as well, since the relation between the cost function parameters (especially the weighting matrices) and the closed-loop performance, is difficult to characterize. To this end some approaches have been proposed in Al-Ghazzawi et al. [2001], Garriga and Soroush [2008], Di Cairano and Bemporad [2010]. The approach proposed in Di Cairano and Bemporad [2010] is based on the inverse design from an existing (local) controller, that provides the desired performance around the equilibrium. By solving the BMI

$$\min_{\mathcal{K},Q,R,P} \; J(\mathcal{K},Q,R,P) \tag{18a}$$

$$\text{s.t. } Q \geq 0, \; P \geq 0, \; R \geq \sigma I \tag{18b}$$

$$(\mathcal{R}+\mathcal{S}'\mathcal{Q}\mathcal{S})\mathcal{K}+\mathcal{S}'\mathcal{Q}\mathcal{T}=0 \tag{18c}$$

$$\kappa_0 = K, \tag{18d}$$

where $\mathcal{K} = [\kappa_0' \;\ldots\; \kappa_{N-1}']'$, and $J : \mathbb{R}^{Nm\times n}\times\mathbb{R}^{n\times n}\times\mathbb{R}^{m\times m}\times\mathbb{R}^{n\times n}\to\mathbb{R}$ is an (arbitrary) objective function and $\mathcal{H}=(\mathcal{R}+\mathcal{S}'\mathcal{Q}\mathcal{S})$, $\mathcal{F}=\mathcal{T}'\mathcal{Q}\mathcal{S}$, the MPC matrices are designed such that the MPC behaves like the desired controller when the constraints are not active. However, the MPC controller provides optimal performance and semi-global asymptotic stability also when the constraints are active. In Di Cairano and Bemporad [2010] several procedures are proposed to solve (18). Inverse-design approaches are being investigated also by other researchers (Hartley and Maciejowski [2009]).

5. CONCLUSIONS

Model predictive control provides several features that are extremely desirable for industries with large volumes applications, such as automotive, factory automation, and aerospace. However, MPC still faces several challenges, since these applications are significantly different from the ones MPC was developed for, namely those arising in chemical process control. We have illustrated benefits and open challenges of MPC for large volumes applications through applications in automotive, mechatronics, and aerospace, and we have provided reference to related relevant research aimed at overcoming such challenges.

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