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Joint Voltage and Phase Unbalance Detector for Three Phase Power Systems

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***Index Terms*—Frequency estimation, GLRT, phase unbalance, three-phase power systems, utility grid, voltage unbalance.**

I. INTRODUCTION

FOR the past several years, deployment of distributed and renewable power systems has been continuously growing. Connection of distributed generators to a power grid can lead to grid instability, if they are not properly operated. Synchronization is critical in controlling grid connected power converters by providing a reference phase signal synchronized with the grid voltage [1], [10]. The grid voltage signal often deviates from its ideal waveform due to various disturbances, resulting in unbalance. This degrades synchronization accuracy. Another important consequence of unbalance conditions is that they may generate overheating and mechanical stress on rotating machines. Therefore, unbalance needs to be detected and compensated to provision high power quality and maintain grid stability [11].

To address the grid synchronization problem, numerous techniques have been proposed, [2], [3]. The studies in [4]–[7] are all based on the separation of the positive and negative sequences through the application of symmetrical component transformation whose input signals are produced by adopting different techniques, such as all-pass filter, Kalman filter, enhanced PLL (EPLL), and adaptive notch filter (ANF). While extensive research effort has been put on designing synchronization schemes in the presence of unbalance [4]–[7], only limited attention has been paid in the problem of unbalance detection.

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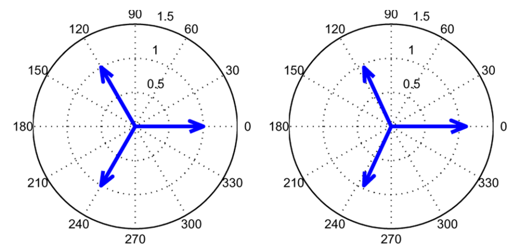


(a) (b)

Fig. 1. Illustration of two three-phase systems with zero negative sequence voltages (V_n). (a) A balanced 3-phase system. (b) An unbalanced 3-phase system.

The relationship between three phase line voltages and symmetrical components are given by

$$\begin{bmatrix} \mathbf{v_0} \\ \mathbf{v_p} \\ \mathbf{v_n} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \mathbf{v_a} \\ \mathbf{v_b} \\ \mathbf{v_c} \end{bmatrix} \tag{1}$$

where $a = e^{j(2\pi/3)}$, $\mathbf{v_a}$, $\mathbf{v_b}$ and $\mathbf{v_c}$ are three phase line voltage phasors; and $\mathbf{v_0}$, $\mathbf{v_p}$ and $\mathbf{v_n}$ are zero, positive and negative sequence phasors, respectively [5], [8]. In [12], [13], a ratio of the magnitudes of negative (V_n) and positive (V_p) sequence voltages with a multiplicative constant is used as a measure of unbalance. However any detector that relies on only a subset of the positive, negative and zero sequence amplitudes can be shown to fail under certain unbalance conditions. More specifically an unbalance condition may alter the amplitude of only one of the positive, negative or zero voltage sequences and not affect the remaining two amplitudes. One such case is when there is a disturbance of the form $[\Delta\mathbf{v}, \Delta\mathbf{v}, \Delta\mathbf{v}]^T$ to the three phase voltage vector $[\Delta\mathbf{v_a}, \Delta\mathbf{v_b}, \Delta\mathbf{v_c}]^T$. Fig. 1 illustrates the phasor diagram for $\Delta\mathbf{v} = 0.1e^{j0}$. Equation (1) implies that this will only trigger a change in $\Delta\mathbf{v_0}$ by $\Delta\mathbf{v} = 0.1e^{j0}$ and no change in $\Delta\mathbf{v_p}$ or $\Delta\mathbf{v_n}$. Hence unbalance detectors based on only (V_p), (V_n) or both will miss the unbalance condition. A detector with a good performance for both amplitude and phase unbalance has yet to be designed.

In this letter, a fast novel detection algorithm is developed for detection of voltage and phase unbalance in three phase systems that is suitable for real time applications since the required observation length is one cycle. The detection problem is formulated as a hypothesis test. It is then transformed to a parameter test and solved by generalized likelihood ratio test (GLRT) under the framework of detection theory. Besides the unknown amplitudes and initial phases, the grid frequency could also be an unknown parameter. If this is the case, an approximate maximum likelihood estimate (MLE) of the grid frequency is computed and used to replace the true unknown grid frequency in the GLRT.

II. Signal Model

The problem of interest is to detect whether there is any unbalance in an observed three-phase voltage signal of a utility network. Mathematically, suppose that the following three-phase voltage signal in abc natural reference frame over a certain time period $n = 0, 1, 2, \ldots, N-1$ is observed,

$$v_i(n) = V_i \cos(n\omega + \varphi_i) + e_i(n), i = a, b, c \tag{2}$$

where V_i and φ_i are the unknown amplitude and initial phase angle of the phase i, and ω is the grid frequency. The additive noise vector at time instant n is $\mathbf{e}(n) = [e_a(n), e_b(n), e_c(n)]^T$, and it is modeled as a zero-mean Gaussian random vector with a covariance matrix $\mathbf{Q} = \sigma^2 \mathbf{I}_3$, where σ^2 is the noise power and $\mathbf{I}_3$ is an identity matrix with size 3×3. Moreover, we assume that the noise vectors at different time instants are uncorrelated, i.e., $E[\mathbf{e}(n)\mathbf{e}(m)^T] = \mathbf{O}$, $n \neq m$ and $E[\cdot]$ is the expectation operation. Given the observed signal in (2), we would like to decide which one of the following two hypotheses is true:

$$\begin{aligned} H_0: &\quad V_a = V_b = V_c, \varphi_a = \varphi_b + \frac{2\pi}{3} = \varphi_c - \frac{2\pi}{3}, \\ H_1: &\quad \text{Otherwise.} \end{aligned} \tag{3}$$

Hypothesis H_0 represents the normal condition, and H_1 the entire set of unbalance conditions.

III. GLRT Based Unbalance Detector Algorithm

The hypothesis test in (2) is very difficult to solve directly. Instead, we resort to an equivalent hypothesis test by reformulating the detection problem as a parameter test in the $\alpha\beta$ stationary reference frame and solve it by a generalized likelihood ratio test (GLRT). We assume that the grid frequency is unknown.

Applying the Clarke transformation [15] to the observations in (2) yields the signal in $\alpha\beta$ stationary reference frame

$$\mathbf{v}_t(n) = \mathbf{T}\mathbf{v}(n) \tag{4}$$

where $\mathbf{v}_t(n) = [v_0(n), v_\alpha(n), v_\beta(n)]^T$ and $\mathbf{v}(n) = [v_a(n), v_b(n), v_c(n)]^T$ are the observations in $\alpha\beta$ and abc domains respectively. The transformation matrix $\mathbf{T}$ is given by

$$\mathbf{T} = \frac{2}{3} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}. \tag{5}$$

According to the Fortescue theorem [8], the unbalanced voltage signal is composed of positive, negative and zero sequences, i.e., and given by

$$\begin{aligned} v_a(n) &= V_p \cos(n\omega + \varphi_p) + V_n \cos(n\omega + \varphi_n) \\ &\quad + V_0 \cos(n\omega + \varphi_0) + e_a(n) \\ v_b(n) &= V_p \cos(n\omega + \varphi_p - \frac{2\pi}{3}) + V_n \cos(n\omega + \varphi_n + \frac{2\pi}{3}) \\ &\quad + V_0 \cos(n\omega + \varphi_0) + e_b(n), \\ v_c(n) &= V_p \cos(n\omega + \varphi_p + \frac{2\pi}{3}) + V_n \cos(n\omega + \varphi_n - \frac{2\pi}{3}) \\ &\quad + V_0 \cos(n\omega + \varphi_0) + e_c(n) \end{aligned}$$

where the subscript $p, n, 0$ represent positive, negative and zero sequences, respectively. V_i and φ_i, $i = p, n, 0$ are the amplitude and initial phase angle of each sequence. In a balanced system, $V_n = 0$, $V_0 = 0$ and there remain only the positive sequence related terms.

As a result, under H_0 the (4) can be rewritten as

$$\begin{aligned} v_0(n) &= e_0(n) \\ v_\alpha(n) &= V_p \cos(n\omega + \varphi_p) + e_\alpha(n) \\ v_\beta(n) &= V_p \sin(n\omega + \varphi_p) + e_\beta(n) \end{aligned} \tag{6}$$

Similarly, under H_1 we have

$$\begin{aligned} v_0(n) &= \sqrt{2} V_0 \cos(n\omega + \varphi_0) + e_0(n) \\ v_\alpha(n) &= V_p \cos(n\omega + \varphi_p) + V_n \cos(n\omega + \varphi_n) + e_\alpha(n) \\ v_\beta(n) &= V_p \sin(n\omega + \varphi_p) - V_n \sin(n\omega + \varphi_n) + e_\beta(n) \end{aligned} \tag{7}$$

where $\mathbf{e}_t(n) = [e_0(n), e_\alpha(n), e_\beta(n)]^T$ is a transformed noise vector at time index n, i.e., $\mathbf{e}_t(n) = \mathbf{T}\mathbf{e}(n)$. Note that, $\mathbf{e}_t(n)$ has a covariance matrix $\mathbf{Q}_t = \mathbf{T}\mathbf{Q}\mathbf{T}^T = (2/3)\sigma^2 \mathbf{I}_3$.

Let $\boldsymbol{\theta}$ denote a vector of unknown parameters given by $\boldsymbol{\theta} = [\boldsymbol{\theta}_r^T, \boldsymbol{\theta}_s^T]^T$, where $\boldsymbol{\theta}_r$ is the parameters of interest defined as

$$\boldsymbol{\theta}_r = [\, V_0 \cos\varphi_0 \quad V_0 \sin\varphi_0 \quad V_n \cos\varphi_n \quad V_n \sin\varphi_n \,]^T, \tag{8}$$

and $\boldsymbol{\theta}_s$ is a vector of nuisance parameters given by

$$\boldsymbol{\theta}_s = [\, V_p \cos\varphi_p \quad V_p \sin\varphi_p \,]^T. \tag{9}$$

Given the observation data $\mathbf{v}_t = [\mathbf{v}_t^T(0), \mathbf{v}_t^T(1), \ldots, \mathbf{v}_t^T(N-1)]^T$ and an estimate of the grid frequency $\hat{\omega}$ (derived in Appendix), the hypothesis test now becomes a parameter test,

$$\begin{aligned} H_0: &\quad \boldsymbol{\theta}_r = \mathbf{0}, \boldsymbol{\theta}_s, \\ H_1: &\quad \boldsymbol{\theta}_r \neq \mathbf{0}, \boldsymbol{\theta}_s. \end{aligned} \tag{10}$$

Note that the parameters in $\boldsymbol{\theta}_s$ are unknown, but we assume that the change in these parameters are negligible, and therefore we model them the same under both hypotheses.

The GLRT for this problem has a form [9]

$$L_G(\mathbf{v}_t) = \frac{p_1(\mathbf{v}_t; \hat{\boldsymbol{\theta}}_{r1}, \hat{\boldsymbol{\theta}}_s)}{p_0(\mathbf{v}_t; \hat{\boldsymbol{\theta}}_s)}, \tag{11}$$

where p_i, $i = 0, 1$ are the likelihood functions under H_0 and H_1. $\hat{\boldsymbol{\theta}}_1 = [\hat{\boldsymbol{\theta}}_{r1}^T, \hat{\boldsymbol{\theta}}_{s1}^T]^T$ is the maximum likelihood estimate (MLE) of $\boldsymbol{\theta}$ under H_1. The $\hat{\omega}$ is assumed to be the same under both H_0 and H_1. Conceptually, $\hat{\omega}$ should be computed separately for H_0 and H_1. Specifically, under H_0, the observations in the second and third lines of (6) are used to obtain $\hat{\omega}$ since the first line of (6) only contains a noise term. Under H_1, $\hat{\omega}$ is computed by using all the observations in (7). However, note that (6) and (7) are both linearly transformed from (2) and the transformation matrix $\mathbf{T}$ is invertible. Hence, there is no information loss with respect to the same unknown parameter ω. As a result, the $\hat{\omega}$ can be assumed unchanged.

It is easy to see from (7) that we have a linear model with respect to the unknown vector $\boldsymbol{\theta}$, given $\hat{\omega}$.

$$\mathbf{v}_t = \mathbf{G}\boldsymbol{\theta} + \mathbf{e}_t \tag{12}$$

where $\mathbf{e}_t = [\mathbf{e}_t^T(0), \mathbf{e}_t^T(1), \ldots, \mathbf{e}_t^T(N-1)]^T$ is a composite noise vector with covariance matrix $(2/3)\sigma^2 \mathbf{I}_{3N}$. $\mathbf{G} = [\mathbf{G}_0^T, \mathbf{G}_1^T, \ldots, \mathbf{G}_{N-1}^T]^T$ and $\mathbf{G}_n$ is a block diagonal matrix $\mathbf{G}_n = \text{diag}(\mathbf{G}_{n,1}, \mathbf{G}_{n,2})$ where

$$\mathbf{G}_{n,1} = [\sqrt{2}\cos(n\hat{\omega}) \quad -\sqrt{2}\sin(n\hat{\omega})]$$
$$\mathbf{G}_{n,2} = \begin{bmatrix} \cos(n\hat{\omega}) & -\sin(n\hat{\omega}) & \cos(n\hat{\omega}) & -\sin(n\hat{\omega}) \\ -\sin(n\hat{\omega}) & -\cos(n\hat{\omega}) & \sin(n\hat{\omega}) & \cos(n\hat{\omega}) \end{bmatrix}.$$

Therefore, the original detection problem can be recast as

$$H_0: \quad \mathbf{A}\boldsymbol{\theta} = \mathbf{0}, \qquad H_1: \quad \mathbf{A}\boldsymbol{\theta} \neq \mathbf{0}, \tag{13}$$

where $\mathbf{A} = [\mathbf{I}_4, \mathbf{O}_{4\times 2}]$. The GLRT in (11), after using (13) with Theorem 7.1 in [9], becomes

$$T(\mathbf{v}_t) = 2\ln L_G(\mathbf{v}_t) = \frac{\hat{\boldsymbol{\theta}}_1^T \mathbf{A}^T \left(\mathbf{A}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{A}^T\right)^{-1} \mathbf{A}\hat{\boldsymbol{\theta}}_1}{\frac{2\sigma^2}{3}} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \tag{14}$$

where γ is a threshold corresponding to a probability of false alarm and

$$\hat{\boldsymbol{\theta}}_1 = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{v}_t \tag{15}$$

is the MLE of $\boldsymbol{\theta}$ under H_1.

1) Detector Characteristics: The exact detection performance of a GLRT for a classical linear problem is given in [9] by

$$P_{fa} = Q_{\chi_r^2}(\gamma), \; P_d = Q_{\chi_r'^2(\lambda)}(\gamma) \tag{16}$$

where $Q_{\chi_r^2}(\gamma)$ denotes the right-tail probability for a chi-squared random variable with r degrees of freedom, and $Q_{\chi_r'^2(\lambda)}(\gamma)$ denotes the right tail probability for a non-central chi-squared random variable with r degrees of freedom and a non-centrality parameter λ which is given by

$$\lambda = \frac{3}{2\sigma^2}\boldsymbol{\theta}_1^T \mathbf{A}^T \left(\mathbf{A}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{A}^T\right)^{-1} \mathbf{A}\boldsymbol{\theta}_1, \tag{17}$$

The exact expression for P_{fa} is given by

$$P_{fa} = \left(1 + \frac{\gamma}{2}\right) e^{-(\gamma/2)} \tag{18}$$

In addition, the probability of detection P_d is given by

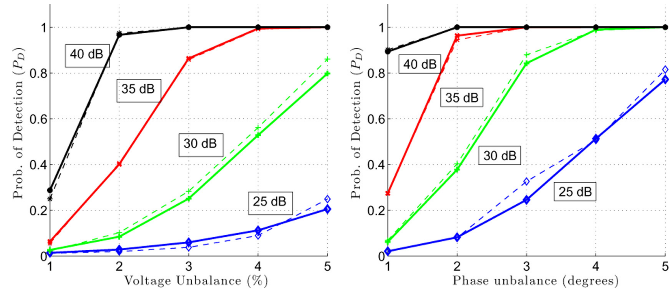
$$P_d = \sum_{k=0}^{\infty} \frac{e^{-(\lambda/2)}(\frac{\lambda}{2})^k}{k!} [e^{-(\gamma/2)} \sum_{s=0}^{k+1} \frac{(\frac{\gamma}{2})^s}{s!}] \tag{19}$$

This is a constant false alarm rate (CFAR) detector.

IV. Simulations

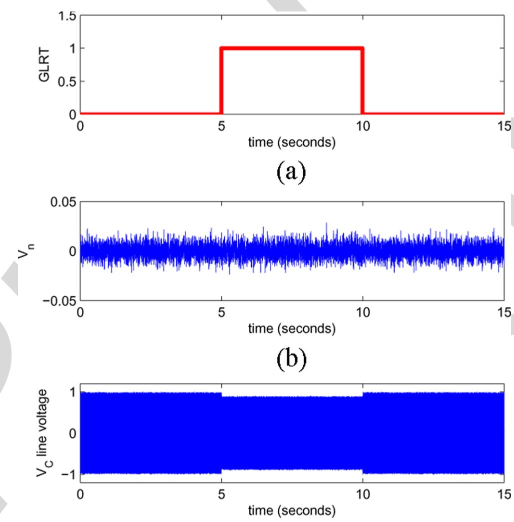
In the following simulations, the balanced amplitudes and initial phase angles of three phase voltage sequences are set to $V_a = 1.0$, $V_b = 1.0$, $V_c = 1.0$ and $\varphi_a = 0$, $\varphi_b = -2\pi/3$, $\varphi_c = 2\pi/3$, respectively. The grid frequency is $\omega = 2\pi * 60/f_s$ where the sampling frequency is set as $f_s = 600$ Hz and the length of the observation vector for each phase is $N = 10$ samples, corresponding to a one-cycle observation length. The probability of false alarm is set to $P_f = 0.01$. The balanced three phase waveforms were followed by unbalanced three phase waveforms. Unbalance is introduced in only one of the phases in the form of a voltage sag varying from 1% to 5% and a phase shift varying from 1 degree to 5 degrees.

Fig. 2(a) shows the probability of detection versus the level of voltage unbalance under different known SNR values. Even when the voltage unbalance occurs on a single phase, the new GLRT based algorithm can detect a voltage unbalance as low



(a) (b)

Fig. 2. Unbalance versus probability of detection at SNR levels of 25 dB, 30 dB, 35 dB and 40 dB. Note: theoretical (dashed),simulation (solid), and σ^2 is known. (a) Voltage unbalance. (b) Phase unbalance.



(a) (b) (c)

Fig. 3. Comparison of the GLRT based unbalance detector to a V_N based unbalance detector at $SNR = 40$ dB and voltage unbalance of 1% on line V_C of a three phase system a) Output of the GLRT based unbalance detector b) Negative sequence voltage c) Line voltage that goes through a voltage sag between $t = 5$ s and $t = 10$ s.

as 2.5% at 40 dB SNR and 4% at 35 dB SNR with 99% probability of detection. Each detection probability is evaluated by the relative number of detections in ten thousand Monte Carlo simulations. The simulation results are consistent with (19) as illustrated with the dashed lines.

Fig. 2(b) illustrates the performance of the new GLRT based algorithm in detecting the phase unbalance at various SNR and unbalance levels. The algorithm detects a phase shift on a single line as low as 2° at 40 dB SNR and 3° at 35 dB 99% probability of detection. The results are obtained from ten thousand Monte Carlo simulations for each case and they are consistent with the theory.

Fig. 3 shows the performance comparison of the proposed GLRT based voltage unbalance detector and an unbalance detector based on V_n, V_p or both [12], [13]. For $t \in [5, 10]$ seconds, the condition in Fig. 1(b) is simulated, where the phasors in abc domain experience an additive disturbance by $0.1e^{j0}$. In the other time periods, the system is balanced. Both V_p and V_n remain unchanged under such an unbalance condition. Hence, an unbalance detector based on these two figures of merit fails to detect the unbalance. On the other hand, the GLRT based detector fires immediately at the beginning of the voltage sag and remains high during the abnormal condition and goes back to normal after $t = 10$ s. The detection latency is one cycle in this setting. However, it can be reduced further. Numerous unbalance conditions exist that would have a canceling effect and fail the unbalance detectors that are based on a subset of V_p, V_n and V_0, whereas the GLRT based method would successfully detect such unbalance conditions.

V. Conclusion

This letter formulated the unbalance detection problem as a parameter test under the framework of detection theory and solved the parameter test by applying the GLRT. When the grid frequency is known, the data have the linear model and the GLRT has an exact expression. In the case of unknown grid frequency, an approximate MLE of the grid frequency was developed and used to replace the true value in the GLRT. Simulation results show that the proposed algorithm can detect both small phase and voltage unbalance conditions with greater than 5% probability at or above 30 dB SNR, even under the conditions that lead to zero negative voltage sequence. Therefore, the new GLRT based detector is a powerful tool not only to detect change points, but also to detect whether an abnormal condition is present throughout an observation window.

Appendix

An approximate MLE of ω is computed in abc natural reference frame. It is known that a sinusoidal signal $x(n) = V\cos(\omega n + \phi)$ satisfies [14]

$$x(n) - 2\cos\omega x(n-1) + x(n-2) = 0. \tag{20}$$

Equation (20) is also referred to as a discrete oscillator equation. It can be applied to a sinusoidal signal to eliminate the unknown parameters except the grid frequency and to yield a linear equation regarding to a function of ω. The weighted least-squares solution to this function and an estimate of the frequency can then obtained. In particular, after applying the discrete oscillator (20) to the three sinusoidal signals in (2) and taking the noise terms into account yield

$$\zeta_i(n) = v_i(n) + v_i(n-2) - 2\alpha v_i(n-1), i = a, b, c \tag{21}$$

where α is a function with respect to the grid frequency ω defined as $\alpha = \cos\omega$. The noise terms $\zeta_i(n)$ in (21) is given as $\zeta_i(n) = e_i(n) + e_i(n-2) - 2\alpha e_i(n-1), i = a, b, c$. Stacking (21) for $n = 0, 1, \ldots, N-1$ yields

$$\boldsymbol{\zeta} = \mathbf{g} - \mathbf{P}\alpha, \tag{22}$$

where $\boldsymbol{\zeta}$ is the noise vector given as $\boldsymbol{\zeta} = \mathbf{Be}$, $\mathbf{e} = [\mathbf{e}^T(0), \mathbf{e}^T(1), \ldots, \mathbf{e}^T(N-1)]^T$ and the matrix $\mathbf{B}$ is a $3(N-2) \times 3N$ matrix with $3 \times (i-2)$th to $3 \times i$th rows given as $[\mathbf{O}_{3\times 3(i-1)}, \mathbf{I}_3, -2\alpha\mathbf{I}_3, \mathbf{I}_3, \mathbf{O}_3 \times 3(N-i-2)]$, $i = 1, 2, \ldots, N-2$. On the right-hand side of (22), $\mathbf{g}$ and $\mathbf{P}$ are $3(N-2) \times 1$ vectors given as $\mathbf{g} = [v_a(2) + v_a(0), \ldots, v_c(N-1) + v_c(N-3)]^T$ and $\mathbf{P} = [2v_a(1), \ldots, 2v_c(N-2)]^T$.

It can be easily seen that (22) is a linear equation with respect to α and the weighted least-squares (WLS) solution is

$$\hat{\alpha} = (\mathbf{P}^T\mathbf{R}\mathbf{P})^{-1}\mathbf{P}^T\mathbf{R}\mathbf{g}, \tag{23}$$

where the weighting matrix $\mathbf{R}$ is defined as $\mathbf{R} = E[\boldsymbol{\zeta}\boldsymbol{\zeta}^T]^{-1} = (\mathbf{BQB}^T)^{-1}$. The estimate of ω can be obtained, from $\hat{\alpha}$, as

$$\hat{\omega} = \cos^{-1}\hat{\alpha}. \tag{24}$$

It should be emphasized that $\hat{\omega}$ is only an approximate MLE of ω since only a set of $3(N-2)$ linear equations is formed from $3N$ observations. The approximation becomes more accurate when the number of data samples N is larger. Note that to compute the WLS solution of ω, the true value of ω is needed to construct the matrices $\mathbf{B}$ and $\mathbf{R}$. However, in the three-phase voltage signal, the fundamental frequency $\omega_o = 2\pi f_o/f_s$ is usually known (f_o is 50 or 60 Hz and f_s is the sampling frequency) and can be treated as a nominal value of the actual frequency. Hence, $\alpha_o = \cos\omega_o$ can be used first to construct $\mathbf{R}$. Once an estimate of ω is found, it is used to obtain a more accurate $\mathbf{R}$ and then a more accurate estimate of ω.

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Joint Voltage and Phase Unbalance Detector for Three Phase Power Systems

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***Abstract*—This letter develops a fast detection algorithm for voltage and phase unbalance in three phase power systems. It is suitable for real time applications since the required observation length is one cycle. It is shown to successfully detect small unbalance conditions at low SNRs. Its detection performance is shown to outperform traditional detectors that rely on changes in only a subset of positive, negative and zero sequence voltages. Unbalance detection is formulated as a hypothesis test under a framework of detection theory and solved by applying a generalized likelihood ratio test (GLRT). We first obtain an approximate maximum likelihood estimate (MLE) of the system frequency and then use it to substitute the true unknown frequency in the GLRT. A closed form expression is provided to detect unbalance conditions. Theoretical derivations are supported by simulations.**

***Index Terms*—Frequency estimation, GLRT, phase unbalance, three-phase power systems, utility grid, voltage unbalance.**

I. Introduction

FOR the past several years, deployment of distributed and renewable power systems has been continuously growing. Connection of distributed generators to a power grid can lead to grid instability, if they are not properly operated. Synchronization is critical in controlling grid connected power converters by providing a reference phase signal synchronized with the grid voltage [1], [10]. The grid voltage signal often deviates from its ideal waveform due to various disturbances, resulting in unbalance. This degrades synchronization accuracy. Another important consequence of unbalance conditions is that they may generate overheating and mechanical stress on rotating machines. Therefore, unbalance needs to be detected and compensated to provision high power quality and maintain grid stability [11].

To address the grid synchronization problem, numerous techniques have been proposed, [2], [3]. The studies in [4]–[7] are all based on the separation of the positive and negative sequences through the application of symmetrical component transformation whose input signals are produced by adopting different techniques, such as all-pass filter, Kalman filter, enhanced PLL (EPLL), and adaptive notch filter (ANF). While extensive research effort has been put on designing synchronization schemes in the presence of unbalance [4]–[7], only limited attention has been paid in the problem of unbalance detection.

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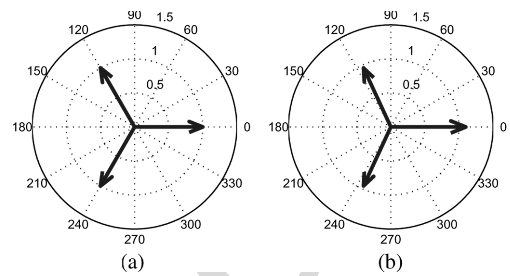


Fig. 1. Illustration of two three-phase systems with zero negative sequence voltages (V_n). (a) A balanced 3-phase system. (b) An unbalanced 3-phase system.

The relationship between three phase line voltages and symmetrical components are given by

$$\begin{bmatrix} \mathbf{v_0} \\ \mathbf{v_p} \\ \mathbf{v_n} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \mathbf{v_a} \\ \mathbf{v_b} \\ \mathbf{v_c} \end{bmatrix} \quad (1)$$

where $a = e^{j(2\pi/3)}$, $\mathbf{v_a}$, $\mathbf{v_b}$ and $\mathbf{v_c}$ are three phase line voltage phasors; and $\mathbf{v_0}$, $\mathbf{v_p}$ and $\mathbf{v_n}$ are zero, positive and negative sequence phasors, respectively [5], [8]. In [12], [13], a ratio of the magnitudes of negative (V_n) and positive (V_p) sequence voltages with a multiplicative constant is used as a measure of unbalance. However any detector that relies on only a subset of the positive, negative and zero sequence amplitudes can be shown to fail under certain unbalance conditions. More specifically an unbalance condition may alter the amplitude of only one of the positive, negative or zero voltage sequences and not affect the remaining two amplitudes. One such case is when there is a disturbance of the form $[\Delta\mathbf{v}, \Delta\mathbf{v}, \Delta\mathbf{v}]^T$ to the three phase voltage vector $[\Delta\mathbf{v_a}, \Delta\mathbf{v_b}, \Delta\mathbf{v_c}]^T$. Fig. 1 illustrates the phasor diagram for $\Delta\mathbf{v} = 0.1e^{j0}$. Equation (1) implies that this will only trigger a change in $\Delta\mathbf{v_0}$ by $\Delta\mathbf{v} = 0.1e^{j0}$ and no change in $\Delta\mathbf{v_p}$ or $\Delta\mathbf{v_n}$. Hence unbalance detectors based on only (V_p), (V_n) or both will miss the unbalance condition. A detector with a good performance for both amplitude and phase unbalance has yet to be designed.

In this letter, a fast novel detection algorithm is developed for detection of voltage and phase unbalance in three phase systems that is suitable for real time applications since the required observation length is one cycle. The detection problem is formulated as a hypothesis test. It is then transformed to a parameter test and solved by generalized likelihood ratio test (GLRT) under the framework of detection theory. Besides the unknown amplitudes and initial phases, the grid frequency could also be an unknown parameter. If this is the case, an approximate maximum likelihood estimate (MLE) of the grid frequency is computed and used to replace the true unknown grid frequency in the GLRT.

II. Signal Model

The problem of interest is to detect whether there is any unbalance in an observed three-phase voltage signal of a utility network. Mathematically, suppose that the following three-phase voltage signal in abc natural reference frame over a certain time period $n = 0, 1, 2, \ldots, N-1$ is observed,

$$v_i(n) = V_i \cos(n\omega + \varphi_i) + e_i(n), i = a, b, c \tag{2}$$

where V_i and φ_i are the unknown amplitude and initial phase angle of the phase i, and ω is the grid frequency. The additive noise vector at time instant n is $\mathbf{e}(n) = [e_a(n), e_b(n), e_c(n)]^T$, and it is modeled as a zero-mean Gaussian random vector with a covariance matrix $\mathbf{Q} = \sigma^2 \mathbf{I}_3$, where σ^2 is the noise power and $\mathbf{I}_3$ is an identity matrix with size 3×3. Moreover, we assume that the noise vectors at different time instants are uncorrelated, i.e., $E[\mathbf{e}(n)\mathbf{e}(m)^T] = \mathbf{O}$, $n \neq m$ and $E[\cdot]$ is the expectation operation. Given the observed signal in (2), we would like to decide which one of the following two hypotheses is true:

$$\begin{aligned} H_0: &\quad V_a = V_b = V_c, \varphi_a = \varphi_b + \frac{2\pi}{3} = \varphi_c - \frac{2\pi}{3}, \\ H_1: &\quad \text{Otherwise.} \end{aligned} \tag{3}$$

Hypothesis H_0 represents the normal condition, and H_1 the entire set of unbalance conditions.

III. GLRT Based Unbalance Detector Algorithm

The hypothesis test in (2) is very difficult to solve directly. Instead, we resort to an equivalent hypothesis test by reformulating the detection problem as a parameter test in the $\alpha\beta$ stationary reference frame and solve it by a generalized likelihood ratio test (GLRT). We assume that the grid frequency is unknown.

Applying the Clarke transformation [15] to the observations in (2) yields the signal in $\alpha\beta$ stationary reference frame

$$\mathbf{v}_t(n) = \mathbf{T}\mathbf{v}(n) \tag{4}$$

where $\mathbf{v}_t(n) = [v_0(n), v_\alpha(n), v_\beta(n)]^T$ and $\mathbf{v}(n) = [v_a(n), v_b(n), v_c(n)]^T$ are the observations in $\alpha\beta$ and abc domains respectively. The transformation matrix $\mathbf{T}$ is given by

$$\mathbf{T} = \frac{2}{3} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix}. \tag{5}$$

According to the Fortescue theorem [8], the unbalanced voltage signal is composed of positive, negative and zero sequences, i.e., and given by

$$\begin{aligned} v_a(n) &= V_p \cos(n\omega + \varphi_p) + V_n \cos(n\omega + \varphi_n) \\ &\quad + V_0 \cos(n\omega + \varphi_0) + e_a(n) \\ v_b(n) &= V_p \cos(n\omega + \varphi_p - \frac{2\pi}{3}) + V_n \cos(n\omega + \varphi_n + \frac{2\pi}{3}) \\ &\quad + V_0 \cos(n\omega + \varphi_0) + e_b(n), \\ v_c(n) &= V_p \cos(n\omega + \varphi_p + \frac{2\pi}{3}) + V_n \cos(n\omega + \varphi_n - \frac{2\pi}{3}) \\ &\quad + V_0 \cos(n\omega + \varphi_0) + e_c(n) \end{aligned}$$

where the subscript p, n, 0 represent positive, negative and zero sequences, respectively. V_i and φ_i, $i = p, n, 0$ are the amplitude and initial phase angle of each sequence. In a balanced system, $V_n = 0$, $V_0 = 0$ and there remain only the positive sequence related terms.

As a result, under H_0 the (4) can be rewritten as

$$\begin{aligned} v_0(n) &= e_0(n) \\ v_\alpha(n) &= V_p \cos(n\omega + \varphi_p) + e_\alpha(n) \\ v_\beta(n) &= V_p \sin(n\omega + \varphi_p) + e_\beta(n) \end{aligned} \tag{6}$$

Similarly, under H_1 we have

$$\begin{aligned} v_0(n) &= \sqrt{2} V_0 \cos(n\omega + \varphi_0) + e_0(n) \\ v_\alpha(n) &= V_p \cos(n\omega + \varphi_p) + V_n \cos(n\omega + \varphi_n) + e_\alpha(n) \\ v_\beta(n) &= V_p \sin(n\omega + \varphi_p) - V_n \sin(n\omega + \varphi_n) + e_\beta(n) \end{aligned} \tag{7}$$

where $\mathbf{e}_t(n) = [e_0(n), e_\alpha(n), e_\beta(n)]^T$ is a transformed noise vector at time index n, i.e., $\mathbf{e}_t(n) = \mathbf{T}\mathbf{e}(n)$. Note that, $\mathbf{e}_t(n)$ has a covariance matrix $\mathbf{Q}_t = \mathbf{T}\mathbf{Q}\mathbf{T}^T = (2/3)\sigma^2 \mathbf{I}_3$.

Let $\boldsymbol{\theta}$ denote a vector of unknown parameters given by $\boldsymbol{\theta} = [\boldsymbol{\theta}_r^T, \boldsymbol{\theta}_s^T]^T$, where $\boldsymbol{\theta}_r$ is the parameters of interest defined as

$$\boldsymbol{\theta}_r = [\, V_0 \cos\varphi_0 \quad V_0 \sin\varphi_0 \quad V_n \cos\varphi_n \quad V_n \sin\varphi_n \,]^T, \tag{8}$$

and $\boldsymbol{\theta}_s$ is a vector of nuisance parameters given by

$$\boldsymbol{\theta}_s = [\, V_p \cos\varphi_p \quad V_p \sin\varphi_p \,]^T. \tag{9}$$

Given the observation data $\mathbf{v}_t = [\mathbf{v}_t^T(0), \mathbf{v}_t^T(1), \ldots, \mathbf{v}_t^T(N-1)]^T$ and an estimate of the grid frequency $\hat{\omega}$ (derived in Appendix), the hypothesis test now becomes a parameter test,

$$\begin{aligned} H_0: &\quad \boldsymbol{\theta}_r = \mathbf{0}, \boldsymbol{\theta}_s, \\ H_1: &\quad \boldsymbol{\theta}_r \neq \mathbf{0}, \boldsymbol{\theta}_s. \end{aligned} \tag{10}$$

Note that the parameters in $\boldsymbol{\theta}_s$ are unknown, but we assume that the change in these parameters are negligible, and therefore we model them the same under both hypotheses.

The GLRT for this problem has a form [9]

$$L_G(\mathbf{v}_t) = \frac{p_1(\mathbf{v}_t; \hat{\boldsymbol{\theta}}_{r1}, \hat{\boldsymbol{\theta}}_s)}{p_0(\mathbf{v}_t; \hat{\boldsymbol{\theta}}_s)}, \tag{11}$$

where $p_i, i = 0, 1$ are the likelihood functions under H_0 and H_1. $\hat{\boldsymbol{\theta}}_1 = [\hat{\boldsymbol{\theta}}_{r1}^T, \hat{\boldsymbol{\theta}}_{s1}^T]^T$ is the maximum likelihood estimate (MLE) of $\boldsymbol{\theta}$ under H_1. The $\hat{\omega}$ is assumed to be the same under both H_0 and H_1. Conceptually, $\hat{\omega}$ should be computed separately for H_0 and H_1. Specifically, under H_0, the observations in the second and third lines of (6) are used to obtain $\hat{\omega}$ since the first line of (6) only contains a noise term. Under H_1, $\hat{\omega}$ is computed by using all the observations in (7). However, note that (6) and (7) are both linearly transformed from (2) and the transformation matrix $\mathbf{T}$ is invertible. Hence, there is no information loss with respect to the same unknown parameter ω. As a result, the $\hat{\omega}$ can be assumed unchanged.

It is easy to see from (7) that we have a linear model with respect to the unknown vector $\boldsymbol{\theta}$, given $\hat{\omega}$.

$$\mathbf{v}_t = \mathbf{G}\boldsymbol{\theta} + \mathbf{e}_t \tag{12}$$

where $\mathbf{e}_t = [\mathbf{e}_t^T(0), \mathbf{e}_t^T(1), \ldots, \mathbf{e}_t^T(N-1)]^T$ is a composite noise vector with covariance matrix $(2/3)\sigma^2 \mathbf{I}_{3N}$. $\mathbf{G} = [\mathbf{G}_0^T, \mathbf{G}_1^T, \ldots, \mathbf{G}_{N-1}^T]^T$ and $\mathbf{G}_n$ is a block diagonal matrix $\mathbf{G}_n = \text{diag}(\mathbf{G}_{n,1}, \mathbf{G}_{n,2})$ where

$$\mathbf{G}_{n,1} = \left[\, \sqrt{2}\cos(n\hat{\omega}) \quad -\sqrt{2}\sin(n\hat{\omega}) \,\right]$$
$$\mathbf{G}_{n,2} = \begin{bmatrix} \cos(n\hat{\omega}) & -\sin(n\hat{\omega}) & \cos(n\hat{\omega}) & -\sin(n\hat{\omega}) \\ -\sin(n\hat{\omega}) & -\cos(n\hat{\omega}) & \sin(n\hat{\omega}) & \cos(n\hat{\omega}) \end{bmatrix}.$$

Therefore, the original detection problem can be recast as

$$\begin{aligned} H_0: &\quad \mathbf{A}\boldsymbol{\theta} = \mathbf{0}, \\ H_1: &\quad \mathbf{A}\boldsymbol{\theta} \neq \mathbf{0}, \end{aligned} \tag{13}$$

where $\mathbf{A} = [\mathbf{I}_4, \mathbf{O}_{4\times 2}]$. The GLRT in (11), after using (13) with Theorem 7.1 in [9], becomes

$$\begin{aligned} T(\mathbf{v}_t) &= 2\ln L_G(\mathbf{v}_t) \\ &= \frac{\hat{\boldsymbol{\theta}}_1^T \mathbf{A}^T \left(\mathbf{A}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{A}^T\right)^{-1} \mathbf{A}\hat{\boldsymbol{\theta}}_1}{\frac{2\sigma^2}{3}} \underset{H_0}{\overset{H_1}{\gtrless}} \gamma \end{aligned} \tag{14}$$

where γ is a threshold corresponding to a probability of false alarm and

$$\hat{\boldsymbol{\theta}}_1 = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{v}_t \tag{15}$$

is the MLE of $\boldsymbol{\theta}$ under H_1.

1) Detector Characteristics: The exact detection performance of a GLRT for a classical linear problem is given in [9] by

$$P_{fa} = Q_{\chi_r^2}(\gamma),\ P_d = Q_{\chi_r'^2(\lambda)}(\gamma) \tag{16}$$

where $Q_{\chi_r^2}(\gamma)$ denotes the right-tail probability for a chi-squared random variable with r degrees of freedom, and $Q_{\chi_r'^2(\lambda)}(\gamma)$ denotes the right tail probability for a non-central chi-squared random variable with r degrees of freedom and a non-centrality parameter λ which is given by

$$\lambda = \frac{3}{2\sigma^2}\boldsymbol{\theta}_1^T \mathbf{A}^T \left(\mathbf{A}(\mathbf{G}^T\mathbf{G})^{-1}\mathbf{A}^T\right)^{-1} \mathbf{A}\boldsymbol{\theta}_1, \tag{17}$$

The exact expression for P_{fa} is given by

$$P_{fa} = \left(1 + \frac{\gamma}{2}\right) e^{-(\gamma/2)} \tag{18}$$

In addition, the probability of detection P_d is given by

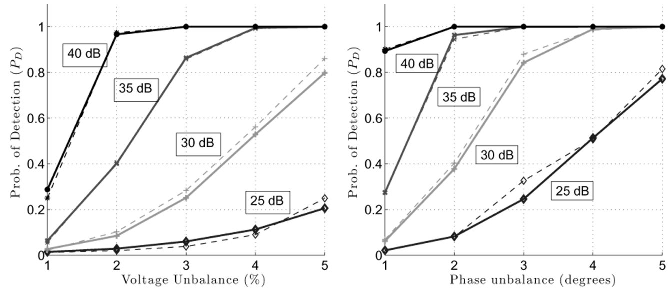
$$P_d = \sum_{k=0}^{\infty} \frac{e^{-(\lambda/2)}(\frac{\lambda}{2})^k}{k!}\left[e^{-(\gamma/2)} \sum_{s=0}^{k+1} \frac{(\frac{\gamma}{2})^s}{s!}\right] \tag{19}$$

This is a constant false alarm rate (CFAR) detector.

IV. Simulations

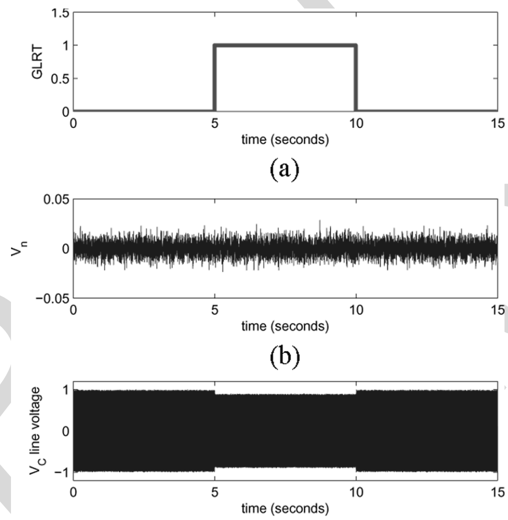
In the following simulations, the balanced amplitudes and initial phase angles of three phase voltage sequences are set to $V_a = 1.0$, $V_b = 1.0$, $V_c = 1.0$ and $\varphi_a = 0$, $\varphi_b = -2\pi/3$, $\varphi_c = 2\pi/3$, respectively. The grid frequency is $\omega = 2\pi * 60/f_s$ where the sampling frequency is set as $f_s = 600$ Hz and the length of the observation vector for each phase is $N = 10$ samples, corresponding to a one-cycle observation length. The probability of false alarm is set to $P_f = 0.01$. The balanced three phase waveforms were followed by unbalanced three phase waveforms. Unbalance is introduced in only one of the phases in the form of a voltage sag varying from 1% to 5% and a phase shift varying from 1 degree to 5 degrees.

Fig. 2(a) shows the probability of detection versus the level of voltage unbalance under different known SNR values. Even when the voltage unbalance occurs on a single phase, the new GLRT based algorithm can detect a voltage unbalance as low as 2.5% at 40 dB SNR and 4% at 35 dB SNR with 99% probability of detection. Each detection probability is evaluated by the relative number of detections in ten thousand Monte Carlo simulations. The simulation results are consistent with (19) as illustrated with the dashed lines.



(a) (b)

Fig. 2. Unbalance versus probability of detection at SNR levels of 25 dB, 30 dB, 35 dB and 40 dB. Note: theoretical (dashed),simulation (solid), and σ^2 is known. (a) Voltage unbalance. (b) Phase unbalance.



(a) (b) (c)

Fig. 3. Comparison of the GLRT based unbalance detector to a V_N based unbalance detector at $SNR = 40$ dB and voltage unbalance of 1% on line V_C of a three phase system a) Output of the GLRT based unbalance detector b) Negative sequence voltage c) Line voltage that goes through a voltage sag between $t = 5$ s and $t = 10$ s.

Fig. 2(b) illustrates the performance of the new GLRT based algorithm in detecting the phase unbalance at various SNR and unbalance levels. The algorithm detects a phase shift on a single line as low as 2° at 40 dB SNR and 3° at 35 dB 99% probability of detection. The results are obtained from ten thousand Monte Carlo simulations for each case and they are consistent with the theory.

Fig. 3 shows the performance comparison of the proposed GLRT based voltage unbalance detector and an unbalance detector based on V_n, V_p or both [12], [13]. For $t \in [5, 10]$ seconds, the condition in Fig. 1(b) is simulated, where the phasors in abc domain experience an additive disturbance by $0.1e^{j0}$. In the other time periods, the system is balanced. Both V_p and V_n remain unchanged under such an unbalance condition. Hence, an unbalance detector based on these two figures of merit fails to detect the unbalance. On the other hand, the GLRT based detector fires immediately at the beginning of the voltage sag and remains high during the abnormal condition and goes back to normal after $t = 10$ s. The detection latency is one cycle in this setting. However, it can be reduced further. Numerous unbalance conditions exist that would have a canceling effect and fail the unbalance detectors that are based on a subset of V_p, V_n and V_0, whereas the GLRT based method would successfully detect such unbalance conditions.

V. Conclusion

This letter formulated the unbalance detection problem as a parameter test under the framework of detection theory and solved the parameter test by applying the GLRT. When the grid frequency is known, the data have the linear model and the GLRT has an exact expression. In the case of unknown grid frequency, an approximate MLE of the grid frequency was developed and used to replace the true value in the GLRT. Simulation results show that the proposed algorithm can detect both small phase and voltage unbalance conditions with greater than 5% probability at or above 30 dB SNR, even under the conditions that lead to zero negative voltage sequence. Therefore, the new GLRT based detector is a powerful tool not only to detect change points, but also to detect whether an abnormal condition is present throughout an observation window.

Appendix

An approximate MLE of ω is computed in abc natural reference frame. It is known that a sinusoidal signal $x(n) = V\cos(\omega n + \phi)$ satisfies [14]

$$x(n) - 2\cos\omega x(n-1) + x(n-2) = 0. \tag{20}$$

Equation (20) is also referred to as a discrete oscillator equation. It can be applied to a sinusoidal signal to eliminate the unknown parameters except the grid frequency and to yield a linear equation regarding to a function of ω. The weighted least-squares solution to this function and an estimate of the frequency can then obtained. In particular, after applying the discrete oscillator (20) to the three sinusoidal signals in (2) and taking the noise terms into account yield

$$\zeta_i(n) = v_i(n) + v_i(n-2) - 2\alpha v_i(n-1), i = a, b, c \tag{21}$$

where α is a function with respect to the grid frequency ω defined as $\alpha = \cos\omega$. The noise terms $\zeta_i(n)$ in (21) is given as $\zeta_i(n) = e_i(n) + e_i(n-2) - 2\alpha e_i(n-1), i = a, b, c$. Stacking (21) for $n = 0, 1, \ldots, N-1$ yields

$$\boldsymbol{\zeta} = \mathbf{g} - \mathbf{P}\alpha, \tag{22}$$

where $\boldsymbol{\zeta}$ is the noise vector given as $\boldsymbol{\zeta} = \mathbf{Be}$, $\mathbf{e} = [\mathbf{e}^T(0), \mathbf{e}^T(1), \ldots, \mathbf{e}^T(N-1)]^T$ and the matrix $\mathbf{B}$ is a $3(N-2) \times 3N$ matrix with $3 \times (i-2)$th to $3 \times i$th rows given as $[\mathbf{O}_{3\times 3(i-1)}, \mathbf{I}_3, -2\alpha\mathbf{I}_3, \mathbf{I}_3, \mathbf{O}_3 \times 3(N-i-2)]$, $i = 1, 2, \ldots, N-2$. On the right-hand side of (22), $\mathbf{g}$ and $\mathbf{P}$ are $3(N-2) \times 1$ vectors given as $\mathbf{g} = [v_a(2) + v_a(0), \ldots, v_c(N-1) + v_c(N-3)]^T$ and $\mathbf{P} = [2v_a(1), \ldots, 2v_c(N-2)]^T$.

It can be easily seen that (22) is a linear equation with respect to α and the weighted least-squares (WLS) solution is

$$\hat{\alpha} = (\mathbf{P}^T\mathbf{R}\mathbf{P})^{-1}\mathbf{P}^T\mathbf{R}\mathbf{g}, \tag{23}$$

where the weighting matrix $\mathbf{R}$ is defined as $\mathbf{R} = E[\boldsymbol{\zeta}\boldsymbol{\zeta}^T]^{-1} = (\mathbf{BQB}^T)^{-1}$. The estimate of ω can be obtained, from $\hat{\alpha}$, as

$$\hat{\omega} = \cos^{-1}\hat{\alpha}. \tag{24}$$

It should be emphasized that $\hat{\omega}$ is only an approximate MLE of ω since only a set of $3(N-2)$ linear equations is formed from $3N$ observations. The approximation becomes more accurate when the number of data samples N is larger. Note that to compute the WLS solution of ω, the true value of ω is needed to construct the matrices $\mathbf{B}$ and $\mathbf{R}$. However, in the three-phase voltage signal, the fundamental frequency $\omega_o = 2\pi f_o/f_s$ is usually known (f_o is 50 or 60 Hz and f_s is the sampling frequency) and can be treated as a nominal value of the actual frequency. Hence, $\alpha_o = \cos\omega_o$ can be used first to construct $\mathbf{R}$. Once an estimate of ω is found, it is used to obtain a more accurate $\mathbf{R}$ and then a more accurate estimate of ω.

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