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Line Fault Analysis of Ungrounded Distribution Systems

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Abstract—**This paper proposes a new method for line fault analysis of ungrounded distribution systems. The fault condition of a line fault is integrated into the nodal admittance matrix of the faulted line to be modeled. The zero-impedance branch is merged into adjacent impedance branches to be taken into account, and one of its terminal buses with zero neutral-to-ground voltage is chosen as a slave bus when it is an ideal transformer or a voltage regulator with ungrounded winding connection. The three-phase jointly regulation of a distributed generation source is embedded into nodal admittance model of its internal impedance branch by combining three phases of its internal bus into one equivalent phase. The distribution system is partitioned into a main network and a set of lateral networks to be solved. The main network is formed by the connected paths between the terminal buses of the faulted line, and generation sources, and solved by a Gauss-Seidel method. A lateral network is formed by one of the buses of main network and all buses and branches downstream to the bus, and solved by a backward and forward sweep method. The numerical examples are provided to prove the effectiveness of proposed method.**

I. INTRODUCTION

Ungrounded distribution systems are widely used, especially at medium voltage levels. It adopts three phase three wire configuration. The windings of three-phase transformers and voltage regulators are either WYE connected or DELTA connected. The three-phase loads are DELTA connected as well.

A short circuit fault on a line is usually converted into a fault at a bus to be analyzed by modeling the fault location as an independent bus. There are many techniques published for analyzing the short circuit faults, including symmetrical component methods [1], time simulation based methods [1], and phase frame based methods [2],[3]. The phase frame based methods can be further classified into nodal admittance/impedance matrix based methods [2], and topology based methods [3]. All those methods have their own limitations when applying to real time analysis of ungrounded distribution systems either in modeling accuracy, or solution efficiency. The symmetrical component methods are designed for balanced systems, and not well suited for unbalanced systems like distribution systems. The time simulation methods are good at modeling capability and accuracy, but quite time consuming for practical size systems. The nodal admittance/impedance matrix based methods are suitable for modeling of most distribution systems, but have difficulties in modeling zero-impedance components, and usually take longer time for a converged solution. The topology based methods are designed for radial distribution systems, mostly for grounded systems, but the computation performance will be heavily impacted by the number of loops, and generation sources.

This paper proposes a new method for line fault analysis of ungrounded distribution systems. Unlike the conventional methods that modeled the location of a fault along a line as an independent bus, the proposed method has integrated the fault condition of a line fault into the nodal admittance matrix of the fault line, such that the dimension of nodal admittance matrix of the system, and complexity of constructing and factorizing the nodal admittance matrix of the system under a fault will not be affected by the type of fault to be analyzed. The zero-impedance branches such as voltage regulators have been merged with adjacent impedance branches to be modeled, and the inaccuracy or divergence problem introduced by adding small impedance into those branches that used by conventional methods have been avoided. During the merging, for an ideal transformer or a voltage regulator, one of its terminal bus that having zero neutral-to-ground voltage is chosen as a slave bus to be removed. For a three-phase jointly-regulated distributed generation source, an equivalent nodal admittance model associated one equivalent phase of its internal bus, and three phases of its external bus is used to avoid the additional coordination between three-phases if full-phase internal bus model is used.

Based on the location of the fault and the topology connectivity of the system, the proposed method has partitioned the distribution system into a main network and a set of lateral networks to solve iteratively. The main network is formed by the connected paths between the terminal bus of faulted line and generation sources, and solved by a bus admittance matrix based method, such as Gauss-Seidel method. A lateral network is formed by one of the buses of main network and all buses and branches downstream to the bus, and solved by a backward and forward sweep method.

Such partitioning can take advantage of matrix based methods for handling the various regulation patterns of generation sources, and complicated voltage inter-dependence between buses, and the computation efficiency provided by topology based method for radial systems. To further reduce the required iterations for a converged solution, the proposed method also initializes the voltage of bus based on its corresponding control zone. The voltages of buses within a fault controlled zone are set as the voltages of fault point, and the voltages of buses of generation source controlled zones are set as the generation source voltages multiplying by the voltage amplifying factors introduced by the regulators or transformers on the paths of generation source and the bus under consideration. Test results of a sample system are given to demonstrate the effectiveness of proposed method.

II. PROPOSED METHOD

A. Modeling of line faults of distribution systems

A short circuit may occur at any location along a line segment of a distribution system. Unlike the conventional methods that modeled the fault point as an independent bus, the fault condition of a line fault has been embedded into the nodal admittance matrix of the faulted line to be modeled. By doing so, when using nodal admittance matrix based methods to analyze a line fault, the system to be analyzed keeps the same topology as the system constructed under normal states, and the nodal admittance matrix of the system are also almost the same as ones constructed under normal states, and only minor changes are needed for those elements associated with the terminal buses of the fault line. In addition, the system under a fault can be solved with less effort and without factorization of the system admittance matrix if the factorized triangular matrices of the system constructed under normal states are available by using numerical methods, such as matrix inversion lemma. In comparison, for a conventional method, the dimension of nodal admittance matrix of the system and complexity of constructing and factorizing the nodal admittance matrix of the system under a fault are varied with the type of the fault to be analyzed.

Fig. 1 gives a model of line segment between an upstream bus p and a downstream bus s with a fault at the location f within the segment. The line segment is divided into two sub-segments according to the location of the fault, one is between bus p and the location of fault f, and the other is between location of the fault f and bus s. Assumed d is the ratio of distance between the fault location f and the upstream bus p over total length of the line segment between bus p and bus s, the sub-segment between bus p and fault location f is modeled with a series impedance matrix dZ_{ps}^{se}, and a shunt admittance matrix dY_{ps}^{sh} split into two terminal buses, p and f, and the sub-segment between fault location f and bus s is modeled with a series impedance matrix $(1-d)Z_{ps}^{se}$, and a shunt admittance matrix $(1-d)Y_{ps}^{sh}$ split into two terminal buses, f and s. Z_{ps}^{se} and Y_{ps}^{sh} are the series impedance matrix and shunt admittance matrix of line segment between bus p and bus s. The fault at location f can be either a bolted fault, or a fault with impedance.

For a fault at a location f within a line segment between bus p and bus s, its branch currents and terminal voltages can be related as:

$$\begin{bmatrix} I_{ps} \\ I_{sp} \end{bmatrix} = \begin{bmatrix} Y_{pp} & Y_{ps} \\ Y_{sp} & Y_{ss} \end{bmatrix} \begin{bmatrix} V_p \\ V_s \end{bmatrix} \quad (1)$$

where, I_{ps} and I_{sp} are the vectors of phase currents flowing on the branch from bus p to bus s, and from bus s to bus p. V_p and V_s are the vectors of the phase-to-ground voltages of bus p, and bus s. Y_{pp} and Y_{ss} are the self-admittance matrices at bus p and bus s, and Y_{ps} and Y_{sp} are the mutual admittance matrices between bus p and bus s, and bus s and bus p, respectively.

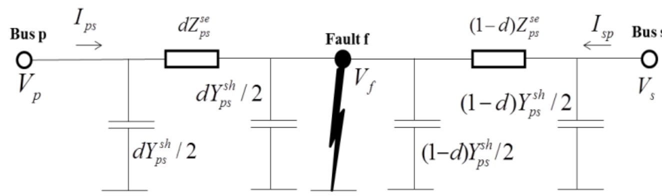

Figure 1. Model of a line segment with a fault

The self and mutual admittance matrices for the fault line segment are defined as:

$$Y_{pp} = \frac{Y_{ps}^{se}}{d} + \frac{dY_{ps}^{sh}}{2} - \frac{Y_{ps}^{se}T^T}{d}\left(\frac{TY_{ps}^{se}T^T}{d(1-d)} + \frac{TY_{ps}^{sh}T^T}{2} + Y_f\right)^{-1}\frac{TY_{ps}^{se}}{d} \quad (2)$$

$$Y_{ps} = Y_{sp} = -\frac{Y_{ps}^{se}T^T}{d}\left(\frac{TY_{ps}^{se}T^T}{d(1-d)} + \frac{TY_{ps}^{sh}T^T}{2} + Y_f\right)^{-1}\frac{TY_{ps}^{se}}{1-d} \quad (3)$$

$$Y_{ss} = \frac{Y_{ps}^{se}}{1-d} + \frac{(1-d)Y_{ps}^{sh}}{2} - \frac{Y_{ps}^{se}T^T}{1-d}\left(\frac{TY_{ps}^{se}T^T}{d(1-d)} + \frac{TY_{ps}^{sh}T^T}{2} + Y_f\right)^{-1}\frac{TY_{ps}^{se}}{1-d} \quad (4)$$

where, T is a transformation matrix used for modeling the impacts of fault. Y_f is a shunt admittance matrix of the fault point f.

The transformation matrix, T is defined according to the type of fault as follows:

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ for a low/high impedance fault} \quad (5)$$

$$T = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ for a bolted phase } a \text{ to ground fault} \quad (6)$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ for a bolted phase } b \text{ to ground fault} \quad (7)$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ for a bolted phase } c \text{ to ground fault} \quad (8)$$

$$T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ for a bolted phase } a \text{ to phase } b \text{ fault} \quad (9)$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \text{ for a bolted phase } b \text{ to phase } c \text{ fault} \quad (10)$$

$$T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \text{ for a bolted phase } c \text{ to phase } a \text{ fault} \quad (11)$$

$$T = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \text{ for a bolted phase } a \text{, and } b \text{ to ground fault} \quad (12)$$

$$T = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \text{ for a bolted phase } b \text{, and } c \text{ to ground fault} \quad (13)$$

$T = [0 \quad 1 \quad 0]$ for a bolted phase c, and a to ground fault (14)

$T = [1 \quad 1 \quad 1]$ for a bolted phase a to b to c fault (15)

$T = [0]$ for a bolted phase a, b, and c to ground fault (16)

The shunt admittance matrix of the fault point, Y_f is a 3-by-3 matrix, and determined according to the impedances between the fault point and the ground, and the faulted phases of the fault point. For a bolted fault, Y_f is a zero matrix.

B. Modeling of generation sources

The generation sources include an equivalent generation source that represents the transmission systems fed the distribution system, and the distributed generation sources that represent the generators residing dispersedly within the distribution system. Loads with large motors can also be considered as distributed generation sources with negative injected power during a severe fault condition. During line fault analysis, a generation source is modeled as a constant voltage source behind an equivalent impedance as shown in Fig.2.

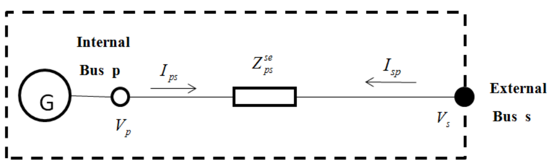

Figure 2. Model of a distributuon generator

Fig. 2 gives a generation source with an equivalent impedance branch between its internal bus p and external bus s. The impedance branch is represented by a series impedance matrix Z_{ps}^{se}. During the fault analysis, the internal bus p is modeled as a swing bus, or a constant active power and voltage magnitude (PV) bus. The external bus s is modeled as a constant active power and reactive power (PQ) bus. The scheduled voltages or powers of internal buses are given or determined based on a pre-fault load flow analysis.

When applying nodal admittance matrix based methods to simulate the system under a fault, the internal and external buses are modeled as a set of nodes for all modeled phases. The external bus can be represented by three individual nodes. The internal bus may be represented as a single node or three nodes determined according to the regulation between three phases of the generation source. If three phases are regulated independently, the internal bus is modeled as three individual nodes. If three phases are regulated jointly, a single-node model is used to model the generation source, such that additional efforts for coordination between three phases can be avoided compared with using three-node model.

For a jointly-regulated generator, its joint-regulation requirements can be merged with the nodal admittance matrix model of its internal impedance branch. For example, if the regulation requirement for a distributed generation source is maintaining its three-phase internal voltages balanced and the magnitude of voltages and total active power of three-phases constant, the generator can be modeled as impedance branch between an equivalent phase of its internal bus p, and three phases of its external bus s as:

$$\begin{bmatrix} I_{ps,e} \\ I_{sp} \end{bmatrix} = \begin{bmatrix} R^T Z_{ps}^{se^{-1}} R^* & -R^T Z_{ps}^{se^{-1}} \\ -Z_{ps}^{se^{-1}} R^* & Z_{ps}^{se^{-1}} \end{bmatrix} \begin{bmatrix} V_{p,e} \\ V_s \end{bmatrix} \tag{17}$$

where, $I_{ps,e}$ is the equivalent phase current flowing on the branch from bus p to bus s, $V_{p,e}$ is the phase-to-ground voltage at equivalent phase e of bus p, R is a rotation vector to rotate all phases to the selected equivalence phase e, R^T and R^* are the transpose and conjugate of rotation vector R respectively. If phase a is chosen as the equivalent phase, we have:

$$V_{p,e} = V_{p,a} \tag{18}$$

$$I_{ps,e} = R^T I_{ps} \tag{19}$$

$$S_{ps} = V_{p,e} I_{ps,e}^* \tag{20}$$

$$R = [1 \quad e^{j120°} \quad e^{-j120°}]^T \tag{21}$$

where, $V_{p,a}$ is the phase-to-ground voltage at phase a of bus p, and S_{ps} is the total power of generation output from bus p to bus s.

C. Modeling of zero-impedance branches

Many branches in a distribution system can be regarded as zero-impedance branches, such as step voltage regulators, ideal transformers, switches, jumpers and very short lines. In order to use admittance matrix based approaches, conventional methods have arbitrarily assigned small non-zero impedances to those branches. However, assigning such small impedances makes the analysis ill-conditioned, and difficult to converge. In the proposed method, those zero-impedance branches are merged with adjacent impedance branches into new non-zero impedance branches to be modeled. For each zero-impedance branch, one of its terminal buses is chosen to be a slave bus to be removed. In order to simplify the equivalent model, the slave bus has to be selected as the terminal bus that has a zero neutral-to-ground voltage, when the zero-impedance branch is a voltage regulator or an ideal transformer with ungrounded winding connections.

For an ungrounded ideal transformer or voltage regulator, it has to be merged with the branches connected with one of its terminal buses that having a zero neutral-to-ground voltage. Assumed bus s is a terminal bus of ungrounded transformer or regulator, its phase-to-ground voltages, V_s can be determined from its phase-to-phase voltages, V_s^{LL} according to:

$$V_s = C_V^{PL} V_s^{LL} + [1 \quad 1 \quad 1]^T v_{s,neutral} \tag{22}$$

where, C_V^{PL} is a conversion factor matrix for converting voltages from phase-to-phase form into phase-to-ground one, and defined as:

$$C_V^{PL} = \begin{bmatrix} 1/3 & 0 & 1/3 \\ -1/3 & 1/3 & 0 \\ 0 & -1/3 & 1/3 \end{bmatrix} \tag{23}$$

$v_{s,neutral}$ is the neutral-to-ground voltage of floating neutral, or fictitious neutral of bus s, and calculated as:

$$v_{s,neutral} = [1/3 \quad 1/3 \quad 1/3] V_s \tag{24}$$

Similarly, the phase-to-phase voltages of bus s, V_s^{LL} can be determined from its phase-to-ground voltages, V_s according to:

$$V_s^{LL} = C_V^{LP} V_s \tag{25}$$

where, C_V^{LP} is a conversion factor matrix for converting voltages from phase-to-ground form into phase-to-phase one, and defined as:

$$C_V^{LP} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix} \quad (26)$$

Obviously, only when the neutral-to-ground voltage is zero, the phase-to-phase voltages and phase-to-ground voltage can be transformed from each other uniquely. Thus for a transformer or regulator branch, the slave bus has to be chosen as the bus with zero neutral-to-ground voltage. For a fault condition, the slave bus is usually the terminal bus that far from to the fault, compared with other terminal bus of the transformer or regulator branch.

Fig.3 gives an example of a generalized zero-impedance branch between bus *s* and bus *r*. Bus *r* is the master bus, and bus *s* is the slave bus. The buses are connected by an ideal transformer. The slave bus *s* is connected with a load current I_s. The phase-to-ground voltages at its two terminal buses, V_s and V_r are related to each other with the voltage amplifying factor matrices, $A_{V_{sr}}$ and $A_{V_{rs}}$, $V_s = A_{V_{sr}} V_r$, and $V_r = A_{V_{rs}} V_s$. The currents flowing from bus *s* to bus *r*, and from bus *r* to bus *s*, I_{sr} and I_{rs} are related to each other with the current amplifying factor matrices, $A_{I_{sr}}$ and $A_{I_{rs}}$, $I_{sr} = A_{I_{sr}} I_{rs}$, and $I_{rs} = A_{I_{rs}} I_{sr}$. These amplifying factor matrices are determined according to the winding connection and tap positions for a transformer or a voltage regulator, and the phase connection for a switch, a short line or a jumper.

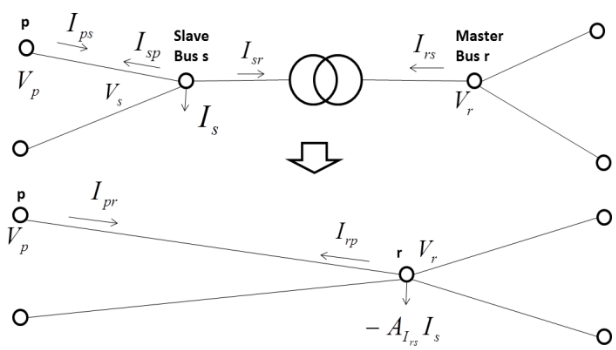

Figure 3. Model of a zero-impedance branch

As shown in Fig. 3, the zero-impedance branch is merged into adjacent impedance branches, such that the slave bus is not considered in the analysis of the model. In the example, the zero-impedance branch is connected to two branches by the slave bus *s*, and to another two branches by the master bus *r*. Taking one adjacent branch between bus *p* and the slave bus *s* as an example, the relationship between the branch currents and the terminal bus voltages for the equivalent branch between bus *p* and bus *r* can be described using Eq. (27) when the voltage amplifying factors are given in term of phase-to-ground voltages, and Eq. (28) when the amplifying factors are in phase-to-phase voltages. In the equivalent model, the zero-impedance branch and the slave bus *s* are removed. There are no changes for the branches connected to the master bus *r*. The branches connected to the slave bus *s* are reconnected to bus *r*, and the nodal admittance matrices of the branch and the current injections at the master bus *r* are modified accordingly. The load current I_s at bus *s* is modeled as an equivalent current at bus *r*, as $-A_{I_{rs}} I_s$. The branch between bus *p* and bus *s* is replaced with a new branch directly between bus *p* and bus *r*, and the branch currents, I_{pr} and I_{rp}, and the nodal voltages, V_p and V_r, are related as:

$$\begin{bmatrix} I_{pr} \\ I_{rp} \end{bmatrix} = \begin{bmatrix} Y_{pp} & Y_{ps} A_{V_{sr}} \\ -A_{I_{rs}} Y_{sp} & -A_{I_{rs}} Y_{ss} A_{V_{sr}} \end{bmatrix} \begin{bmatrix} V_p \\ V_r \end{bmatrix} \quad (27)$$

$$\begin{bmatrix} I_{pr} \\ I_{rp} \end{bmatrix} = \begin{bmatrix} Y_{pp} & Y_{ps} C_V^{PL} A_{V_{sr}}^{LL} C_V^{LP} \\ -A_{I_{rs}} Y_{sp} & -A_{I_{rs}} Y_{ss} C_V^{PL} A_{V_{sr}}^{LL} C_V^{LP} \end{bmatrix} \begin{bmatrix} V_p \\ V_r \end{bmatrix} \quad (28)$$

where, $A_{V_{sr}}^{LL}$ and $A_{V_{rs}}^{LL}$ are the voltage amplifying factor matrices for the branch between bus *s* to bus *r* written in terms of phase-to-phase voltages, and phase-to-phase voltages at bus *s*, and bus *r*, V_s^{LL} and V_r^{LL} are related to each other with those matrices, $V_s^{LL} = A_{V_{sr}}^{LL} V_r^{LL}$, and $V_r^{LL} = A_{V_{rs}}^{LL} V_s^{LL}$.

D. Partitioning the distribution system into main network and lateral networks

Based on the location of fault and generation sources, the system is partitioned into a main network, and a set of lateral networks. The main network is formed by buses and devices residing on the shortest paths between the terminal buses of faulted line, and the generation sources. A lateral network is formed by one of buses of main network as its root bus, and all buses and devices downstream and fed by the bus. Such partitioning enables analyzing the main and lateral networks separately by using a nodal-admittance matrix based method, and a topology based method respectively. Thus the proposed method can take advantage of matrix based methods for handling the various regulation patterns of generation sources, and complicated voltage inter-dependence between buses, and the computation efficiency provided by topology based method for radial systems.

Fig. 4 gives an example of distribution system. The system has one equivalent source, and one distribution generation source. A short circuit fault is occurring at a line segment, and the fault spot is located at the point *f*.

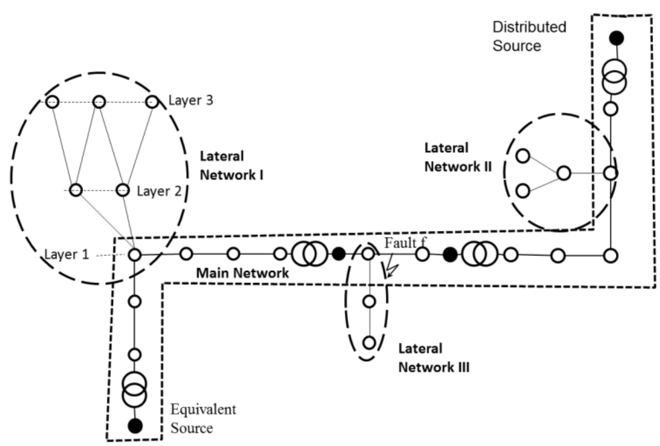

Figure 4. A sample of ungrounded distribution system

As shown in Fig. 4, the system has partitioned into one main network, and three lateral networks. The main network is formed by the connectivity paths between the fault point *f* and the equivalent source, and the distributed source. Each lateral network is formed by one of the buses in the main network, and all buses and devices fed by the bus. The buses of each lateral network can be further divided into several layers based on the number of connected devices between the bus located at the main network, and the bus under consideration. Taken lateral network I as example, its buses can be divided into 3 layers. Layer 1 has one bus, layer 2 has two buses, and layer 3 has three buses.

E. Solving the main network and lateral networks

The main network is solved using the Gauss-Seidel method. The nodal current injection equations are used to simulate the fault behaviors:

$$I = YV \tag{29}$$

where, I and V are the vector of equivalent current injections, and voltages for all nodes, and Y is the nodal admittance matrix of the main network that constructed based on the nodal admittance model for each impedance branch in the network.

The modeled buses and phases in the main network are converted into nodes, and the number of nodes for each bus is equal to the number of modeled or available phases at the bus. The impacts of line fault, zero-impedance branches, and three-phase joint-regulation of generation sources are embedded into the nodal admittance matrix of the main network using models discussed above.

The equivalent current injections at each bus are determined by the connected loads, generation sources and downstream branches, if it is a root bus for a lateral network. After a converged solution is obtained, the equivalent node currents are updated and a new solution may be triggered if the convergence criterion is not met.

A new equation (29) is formulated when a new fault needs to be analyzed. However, if a solution has already obtained for a specific fault, and the new fault is on the same line segment but at different location, or with different fault type, there is no need to factorize the new formulated admittance. The solution for the new fault can be obtained based on available factorized admittance matrices, and admittance changes between the previous fault and the new fault. For example, a solution has already obtained for $I = YV$, and a solution is wanted for $I = \hat{Y}V$, where the difference between Y and $\hat{Y}$ is a low rank change, that is only a 6 by 6 block corresponding to the terminal buses of fault line segment need to be modified as described in Eq.(30):

$$\hat{Y} = (Y + M\Delta Y M^T) \tag{30}$$

where, ΔY is a 6-by-6 matrix containing the nodal admittance change for the terminal buses of the line segment having fault, M is an n-by-6 connection matrix, n is the total number of nodes, and M_{ij} is 1.0 when i is the row corresponding to the node of the element changes for the modification of Y caused by a fault within the line segment, and j is the column corresponding to the phase of the terminal buses of faulted line. Based on the matrix inversion lemma, a new solution can be obtained according to:

$$(Y + M\Delta Y M^T)^{-1} = Y^{-1} - Y^{-1}M(\Delta Y^{-1} + M^T Y^{-1} M)^{-1} M^T Y^{-1} \tag{31}$$

The new solution can be obtained based on existing lower and upper triangular factorization matrices, and forward and backward substitutions.

The lateral networks are solved using a backward/forward sweep with loop breakpoint compensation scheme similar as the one used in [4]. The loops within the networks are broken into radial paths, and the downstream loads of loops are initially allocated between its radial paths based on the path impedances, and adjusted iteratively to maintain identical voltages at the breakpoints. To avoid the inversion of singular matrices, the phase-to-phase voltages are used for calculation of transformers and voltage regulators, and then converted into phase-to-ground voltages.

F. Initializing voltages of buses

To reduce the required iterations for line fault analysis, the voltages of buses are initialized based on the control zone that the bus under consideration is located.

Fig. 5 gives an example of ungrounded distribution system with one equivalent generation source, one distributed generation source, and a line fault at point *f*. As shown in Fig. 5, the control zones include generation source controlled zones, and fault controlled zone.

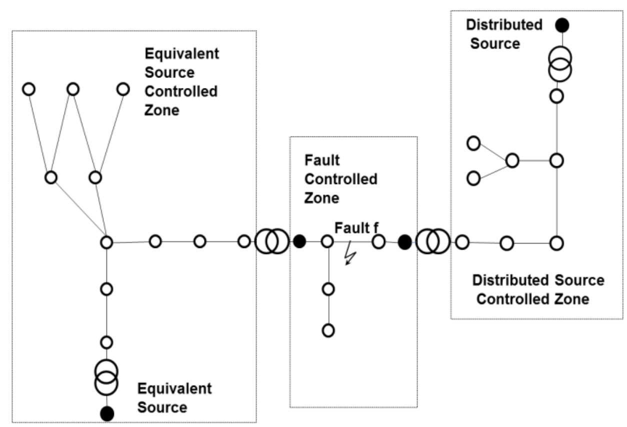

Figure 5. A sample of ungrounded distribution system

The fault controlled zone covers all buses that have connected with the terminal buses of faulted line without passing through any ungrounded transformers or voltage regulators. It is formed by executing an all-connected tracing starting at the buses of fault line and stopping at the terminal buses of ungrounded transformers or voltage regulators.

The generation source controlled zone for each generation source can be formed by executing an all-connected tracing starting at the generation source and stopping at the terminal buses of ungrounded transformers or voltage regulators connected to the boundaries of fault controlled zone.

All devices and buses not covered by the fault controlled zone, and distributed generation source controlled zone are of the equivalent generation source controlled zone.

Fig. 5 includes three control zones, including fault controlled zone, distribution source controlled zone, and equivalent source controlled zone.

The voltages of buses within the fault controlled zone are initialized with the initial voltage of fault point. The initial voltage of fault point is determined based on the fault type. For example, the initial voltage of faulted phase is zero, and two un-faulted phases are set as 1.732 per unit, if it is a single-phase-to-ground. For a double-phase-to-ground, the initial voltages of faulty phases are zero, and the un-faulted phases are set as 1.732 per unit. For a three-phase-to-ground, the initial voltages of all phases are set to be zero.

The initial voltages of buses within the equivalent or distributed generation source controlled zones are set to the values at the internal bus of the equivalent or distributed generation source multiplied with the aggregated voltage amplifying factor matrix introduced by the transformers or voltage regulators along the shortest path between the equivalent or distributed generation source and the bus under consideration:

$$V_p^{(0)} = \prod_{sr} A_{V_{sr}} V_{src} \tag{32}$$

where, $V_p^{(0)}$ is the vector of initial voltages of bus p, V_{src} is the voltage of the internal bus of generation source, $A_{V_{sr}}$ is the voltage amplifying factor matrix of a voltage regulator or transformer between two buses, bus s and bus r residing on the shortest path from the internal bus of generation source and the bus under consideration.

III. NUMERICAL EXAMPLES

The proposed method has been tested with several sample ungrounded systems, and satisfactory results are obtained.

Fig.6 gives an example of test systems. The system includes 1 three-phase transformer, 3 feeders, 10 breakers and switches, 54 three-phase line segments, and 66 three-phase buses.

Figure 6. A sample of ungrounded distribution system

Table I gives the test results on two fault cases with different fault conditions. Case I is a bolted single-phase-to-ground fault. Case II is a single-phase-to-ground fault with impedance, and the fault impedance is 25 Ohms. The fault is located at the middle of line segment between bus 2720 and 2725. Two different methods are used. The first method is the Gauss-Seidel method that solves the whole system based on nodal admittance matrix. The second method is the proposed one that divides the system into main network and lateral networks to solve iteratively.

TABLE I. TEST RESULTS ON A SAMPLE SYSTEM

Method	**Case**	**Substation Transformer Voltages (p.u.)**		**Solution Time (ms)**
		Primary Side	***Secondary Side***	
Gauss-Seidel Method	I	1.083/1.083/1.083	1.848/1.892/0.030	38.373
	II	1.083/1.083/1.083	1.065/1.071/1.067	41.519
The proposed method	I	1.083/1.083/1.083	1.848/1.892/0.030	2.476
	II	1.083/1.083/1.083	1.065/1.071/1.067	2.915

As shown in Table I, the computation speed of the proposed method is much faster than the Gauss-Seidel method while maintaining the same accuracy.

IV. CONCLUSIONS

This paper has proposed a new method for line fault analysis of ungrounded distribution systems. The fault condition of a line fault is integrated into the nodal admittance matrix of the fault line to be modeled. The zero-impedance branch is merged into adjacent impedance branches to be taken into account, and one of terminal buses with zero neutral-to-ground voltage are removed if ungrounded. The three phase jointly-regulation of distribution generation sources are embedded into nodal admittance model with one internal node and three external nodes.

The distribution system is partitioned into a main network and a set of lateral networks to be solved. The main network is formed by the connected paths between the terminal buses of fault line, and generation sources, and solved by a nodal admittance matrix based method. The lateral networks are formed by one of the buses of main network and all buses and branches downstream to the bus, and solved by a backward and forward sweep method. The test results on sample systems have proven the effectiveness of proposed method.

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