

## A 24-Dimensional Modulation Format Achieving 6 dB Asymptotic Power Efficiency

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### Abstract

Although 4-dimensional (4D) modulation formats have been known to the optical communications community since the early days of coherent detection [1], the recent work of Agrell and Karlsson [2,3], has led to a great deal of interest in optimized constellations in 4D. These modulation formats can achieve substantial gains compared with conventional formats such as dual-polarization quaternary phase-shift keying (DP-QPSK) and 16-ary quadrature amplitude modulation (DP-16QAM). For example, polarization-switched QPSK (PS-QPSK) [4], and set-partitioned 128-ary QAM (SP-128QAM) [5], achieve 1.76 dB and 2.43 dB gains in asymptotic power efficiency, respectively. Experimental results have demonstrated gains of up to 1.5 dB in the forward-error correction (FEC) limit regimes [6, 7]. While higher-dimensional modulations have been investigated for many years [8,9], their application to optical communications has been limited to 4D case.

In this paper, we investigate the performance of a 24D format based on mapping the codewords of the extended Golay code [10], onto a 24D hypercube, achieving a spectral efficiency of 1 b/s/Hz/pol. We refer to this format as 12-bit 24D Golay coded hypercube (12b-24D-GCHC). Tolerance to additive white Gaussian noise (AWGN) is found to be better than dual-polarization binary PSK (DP-BPSK) by 3 dB at a BER of  $10^{-3}$ , and 1.9 dB at a BER of  $10^{-2}$ . Transmission performance is simulated over a link of 50 spans of standard single mode fiber (SSMF) with erbium doped fiber amplification (EDFA) both with and without inline optical dispersion compensation, and 12b-24D-GCHC is compared to DP-BPSK and DP-QPSK. We find an increase in maximum tolerable span loss of at least 4 dB compared to DP-BPSK and 8 dB compared to DP-QPSK.

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# A 24-Dimensional Modulation Format Achieving 6 dB Asymptotic Power Efficiency

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**Abstract:** We propose modulation using the extended Golay code over the 24D hypercube, achieving 6 dB asymptotic power efficiency with 1 b/s/Hz/pol spectral efficiency. Noise tolerance is improved by 3 dB over DP-BPSK at a BER of  $10^{-3}$ .

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**OCIS codes:** (060.4080) Modulation; (060.4510) Optical communications.

## 1. Introduction

Although 4-dimensional (4D) modulation formats have been known to the optical communications community since the early days of coherent detection [1], the recent work of Agrell and Karlsson [2,3], has led to a great deal of interest in optimized constellations in 4D. These modulation formats can achieve substantial gains compared with conventional formats such as dual-polarization quaternary phase-shift keying (DP-QPSK) and 16-ary quadrature amplitude modulation (DP-16QAM). For example, polarization-switched QPSK (PS-QPSK) [4], and set-partitioned 128-ary QAM (SP-128QAM) [5], achieve 1.76 dB and 2.43 dB gains in asymptotic power efficiency, respectively. Experimental results have demonstrated gains of up to 1.5 dB in the forward-error correction (FEC) limit regimes [6,7]. While higher-dimensional modulations have been investigated for many years [8,9], their application to optical communications has been limited to 4D case.

In this paper, we investigate the performance of a 24D format based on mapping the codewords of the extended Golay code [10], onto a 24D hypercube, achieving a spectral efficiency of 1 b/s/Hz/pol. We refer to this format as 12-bit 24D Golay coded hypercube (12b-24D-GCHC). Tolerance to additive white Gaussian noise (AWGN) is found to be better than dual-polarization binary PSK (DP-BPSK) by 3 dB at a BER of  $10^{-3}$ , and 1.9 dB at a BER of  $10^{-2}$ . Transmission performance is simulated over a link of 50 spans of standard single mode fiber (SSMF) with erbium doped fiber amplification (EDFA) both with and without inline optical dispersion compensation, and 12b-24D-GCHC is compared to DP-BPSK and DP-QPSK. We find an increase in maximum tolerable span loss of at least 4 dB compared to DP-BPSK and 8 dB compared to DP-QPSK.

## 2. 24D modulation using the Golay-coded hypercube

To transmit a 24D modulation format over a 4D channel, we first need to map the 24D orthogonal signal vector [8,9,11] onto the 4D optical carrier for practical use. We consider in-phase, quadrature, polarization, and time as orthogonal dimensions to do so. An example mapping is given in Fig. 1(a). It is of course possible to use other orthogonal bases, such as different frequency domain subcarriers or spatial modes, to map multi-dimensional constellations. Indeed, some recent research into spatial-division multiplexing for optical communication has focused on a 24D channel with 12 spatial and polarization modes [12], which would be suitable for 24D modulation. In this paper, we use a 24-dimensional basis consisting of 6 consecutive 4D symbols in time, as shown in Fig. 1(a). Many other mappings are possible, all of which have identical performance in the linear regime. While different mappings may have different performance over nonlinear channels, we did not consider these effects in this work, and leave exhaustive study of mapping effects for future research.

In a Gray-coded hypercube constellation (i.e. one where each dimension has a value  $\pm 1$  which is independent of all other dimensions, and every dimension is bit-labeled independently), squared Euclidean distance between constellation points is linearly proportional to Hamming distance. Therefore, we may use a code designed to increase Hamming distance to increase Euclidean distance between constellation points. Taking advantage of this effect, we use the extended Golay code to determine a subset of the 24D hypercube, which determines our constellation.

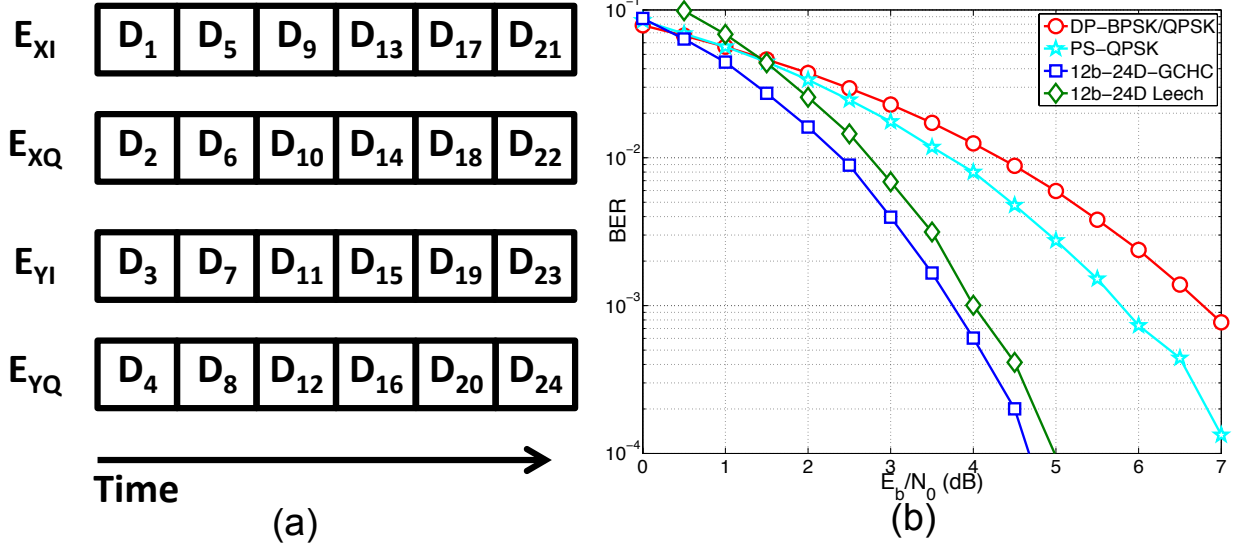


Fig. 1. (a) Example mapping of 24D basis to 4D carrier using time domain. (b) Noise sensitivity of 12b-24D-GCHC compared with DP-BPSK/QPSK, PS-QPSK and the 12b-24D Leech lattice.

The extended Golay code encodes 12 bits of information into a 24-bit word with a minimum Hamming distance of 8 [10]. While this code may be used with an appropriate decoding matrix to correct for errors, we have taken maximum-likelihood (ML) decisions in 24D to maintain soft information for a forward-error correction (FEC) decoder. Although conventional ML decisions for a 12 bit word in 24D are highly complex, we note that there exists a great deal of literature concerning low complexity demodulation of such formats [10], including multiplier free algorithms [13].

In 12b-24D-GCHC, the  $2^{12}$  points which correspond to valid extended Golay codewords are our constellation points, from a possible  $2^{24}$  points on the 24D hypercube. The minimum squared Euclidean distance increases by a factor of 8 compared with the 24D hypercube (which has identical performance to that of DP-QPSK), while the mean energy per bit is doubled. Asymptotic power efficiency [2], is therefore increased by 6 dB compared with the 24D hypercube. Since the constellation is simply a subset of a hypercube, photonic transmitter hardware required is identical to that of DP-QPSK, and receiver DSP should be similar.

The bit error rate (BER) performance over additive white Gaussian noise (AWGN) channels is plotted in Fig. 1(b). We note that noise sensitivity is reduced by 3 dB at a BER of  $10^{-3}$  and by 1.5 dB at a BER of  $10^{-2}$  against DP-BPSK/QPSK. This compares favorably to PS-QPSK, which is the optimal 4D format in terms of asymptotic power efficiency [3], which has a gain over DP-QPSK of 1 dB at a BER of  $10^{-3}$  and by 0.6 dB at a BER of  $10^{-2}$ . Interestingly, 12b-24D-GCHC has superior performance than a 12-bit 24D constellation based on spherical cutting of the Leech lattice [9], (which is the most dense lattice in 24D) shown in Fig. 1(b) as "12b-24D Leech". The performance gain of 12b-24D Leech compared to DP-QPSK is 2.8 dB at a BER of  $10^{-3}$  and 1.6 dB at a BER of  $10^{-2}$ . This implies that optimization of labeling and packing is difficult for such high-dimensional modulations, and that the proposed hypercube lattice with linear codes can resolve its difficulty.

### 3. Transmission simulation setup

For simulating transmission over fiber, we consider a 5-channel wavelength-division multiplexing (WDM) system with a channel spacing of 100 GHz and a data rate of 112 Gb/s per channel, leading to a signal bandwidth of 56 GHz for 12b-24D-GCHC and DP-BPSK, and a bandwidth of 28 GHz for DP-QPSK. Both transmitter and receiver use 5<sup>th</sup> order Bessel filters with 0.7 times signal bandwidth. No optical filtering was used. The transmission link is 50 spans of standard single mode fiber (SSMF) with parameters of  $D = 17$  ps/nm/km,  $\gamma = 1.2$ /W/km,  $\alpha = 0.2$  dB/km, and  $S = 0$ . Polarization mode dispersion was not considered. Nonlinear propagation was simulated using an adaptive step-size split-step Fourier method with the Manakov model. We assume either no inline optical dispersion compensation, or 95% compensation per span before amplification using an ideal linear and lossless compensator. Noise loading was performed at the receiver assuming 50 inline amplifiers each with a noise figure of 5 dB. We use a data-directed adaptive equalizer with least-mean-square updating.

For analysis of transmission results, we used a performance metric of span-loss budget inspired by Poggiolini

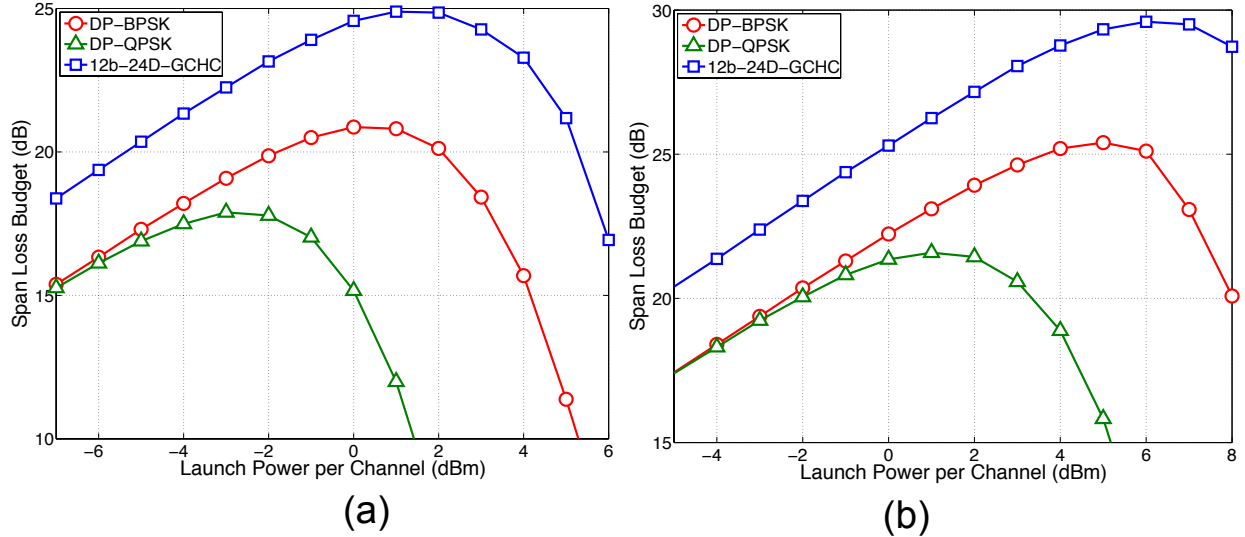


Fig. 2. Span loss budget for 50-span transmission link with a target BER of  $10^{-3}$  (a) with 95% inline dispersion compensation per span, and (b) without inline dispersion compensation.

*et al.* [4] since error counting at the received optical signal-to-noise ratios would be impractical due to the high margin for a 50-span system. Span-loss budget is calculated by performing a linear fit of Q-factor vs. span loss, and using this fit to calculate the maximum permissible span loss for a target BER of  $10^{-3}$ .

#### 4. Results and discussion

Figs. 2(a) and (b) show the performance of the 5-channel WDM system over a 50-span link with (a) and without (b) inline dispersion compensation. In the linear region, we note a 3 dB increase in span loss budget for 12b-24D-GCHC compared with both DP-BPSK and DP-QPSK. In the nonlinear region for the 95% dispersion compensated map shown in Fig. 2(a), DP-QPSK has an optimum launch power of  $-3$  dBm, compared with 0 dBm for DP-BPSK and 1 dBm for 12b-24D-GCHC. The superior performance of DP-BPSK compared to DP-QPSK is due to both the increased phase margin of DP-BPSK, and the higher signal bandwidth, which reduces signal auto-correlation and mitigates self-phase modulation [14]. Maximum permissible span loss is 17.9 dB for DP-QPSK; 21.9 dB for DP-BPSK and 24.9 dB for 12b-24D-GCHC. As expected, the non dispersion compensated link exhibits better performance, due to the reduction fiber nonlinearity in highly dispersed signals. The optimum launch power is increased to 1 dBm for DP-QPSK, 5 dBm for DP-BPSK and 6 dBm for 12b-24D-GCHC. Maximum permissible span loss is 21.6 dB for DP-QPSK; 25.4 dB for DP-BPSK and 29.6 dB for 12b-24D-GCHC.

#### 5. Conclusions

We have proposed a 24-dimensional modulation format achieving 6 dB asymptotic power efficiency with 1 b/s/Hz/pol spectral efficiency, using extended Golay codewords over the 24D hypercube. Noise tolerance was 3 dB better than DP-QPSK at a BER of  $10^{-3}$ , and 1.9 dB better at a BER of  $10^{-2}$ . Through fiber transmission simulation over 50 spans of SSMF with and without 95% inline optical dispersion compensation, span loss margin was found to be 8 dB better than DP-QPSK and 4 dB better than DP-BPSK with inline dispersion compensation, and 8 dB better than DP-QPSK and 4.2 dB better than DP-BPSK without inline dispersion compensation.

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