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Nonlinear Backstepping Learning-based Adaptive Control of Electromagnetic Actuators with Proof of Stability

Gökhan M. Atınç and Mouhacine Benosman

***Abstract*— In this paper we present a learning-based adaptive method to solve the problem of robust trajectory tracking for electromagnetic actuators. We propose a learning-based adaptive controller; we merge together a nonlinear backstepping controller that ensures bounded input/bounded states stability, with a model-free multiparameter extremum seeking to estimate online the uncertain parameters of the system. We present a proof of stability of this learning-based nonlinear controller. We show the efficiency of this approach on a numerical example.**

I. INTRODUCTION

This work deals with the "soft landing" problem of electromagnetic actuators. Soft landing requires accurate control of the moving element of the actuator between two desired positions. It refers to attaining small contact velocity, in turn ensuring a low-noise low-component-wear operation of the actuator. Furthermore, an actuator must achieve soft landing over long periods of time during which the components may age. To overcome these practical constraints, we developed a robust control algorithm that 1) aims for a virtually zero impact velocity, hence achieving soft landing, and 2) adapts to the system aging via a learning-based algorithm.

Many papers have been dedicated to the soft landing problem for electromagnetic actuators, e.g. [1], [2], [3] and [4]. Linear controllers have been proposed for example in [1]. Linear controllers are usually designed to operate in a small neighborhood of linearization points. To control the system over a larger operation space, in this paper, we consider the nonlinear dynamics of the system for control design. Various nonlinear controllers have been used in [2], [3] and [4]. In [3], the authors proposed a nonlinear control based on Sontag's feedback to solve the problem of armature stabilization for an electromechanical valve actuator. However, this approach does not solve the problem of trajectory tracking and does not consider robustness of the controller with respect to system uncertainties. In [5], authors designed a backstepping based controller for electromagnetic actuators . However, changes in system parameters are not considered in this paper. In [4], a nonlinear sliding mode approach was used to solve the problem of trajectory tracking for an electromagnetic valve actuator. Reported results show good tracking performance; however, robustness with respect to parametric uncertainties is not guaranteed. In [2], authors used a single parameter extremum seeking (ES) learning method to tune a scalar gain of the control online along with a nonlinear controller to solve the armature trajectory tracking problem for an electromechanical valve actuator. Due to the iterative nature of the learning process, the controller is intrinsically robust to model uncertainties and parameter drift over time. However, an explicit proof of robustness with respect to model uncertainties or system's aging is not provided. In [6], authors proposed a nonlinear controller based on Lyapunov redesign techniques for electromagnetic actuators. The controller is complemented by a multiparameter extremum seeking (MES) control for tuning *the feedback gains* in order to provide robustness. In [7], authors designed a backstepping based controller which was robustified by an ES algorithm to estimate *uncertain parameters*. In [6] and [7], the effectiveness of the proposed schemes was illustrated numerically; however, no rigorous stability analysis was presented. In this work we use a nonlinear model of the electromagnetic actuator to design a backstepping controller that ensures asymptotic trajectory tracking for the nominal system, i.e., with no model uncertainties. Subsequently, the controller is robustified by a *MES* algorithm that is used to *identify the model's uncertain parameters online*, including parameters that drift slowly over time due to aging. Notice that contrary to [2], [6], we use a MES approach to learn *a vector of parameters*, and not the gains of the controller. In this sense, we are proposing a new *learning-based adaptive control*. Furthermore, *we present a stability analysis of the overall control system.*

This paper is organized as follows: We first recall some useful definitions in Section II. Next, we present in Section III, a nonlinear model of electromagnetic actuators. Then, in Section IV, we report the main result of this work, namely, the learning-based adaptive nonlinear controller along with the stability analysis. Numerical validation of the proposed controller is given in Section V, and finally, concluding remarks are stated in Section VI.

II. PRELIMINARIES

Throughout the paper we use $\|.\|$ to denote the Euclidean norm. We denote an $n \times n$ diagonal matrix by $diag\{m_1, ..., m_n\}$ and k times differentiable functions by C^k. We now review some definitions that we will use later.

Definition 1 (Integral Input-to-State Stability [8]): Consider the system

$$\dot{x} = f(t, x, u), \tag{1}$$

where $x \in \mathcal{D} \subseteq \mathbb{R}^n$ such that $0 \in \mathcal{D}$, and $f : [0, \infty) \times \mathcal{D} \times \mathcal{D}_u \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz

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in x and u, uniformly in t. The inputs are assumed to be measurable and locally bounded functions $u : \mathbb{R}_{\geq 0} \to \mathcal{D}_u \subseteq \mathbb{R}^m$. Given any control $u \in \mathcal{D}_u$ and any $\xi \in \mathcal{D}_0 \subseteq \mathcal{D}$, there is a unique maximal solution of the initial value problem $\dot{x} = f(t, x, u)$, $x(t_0) = \xi$. Without loss of generality, assume $t_0 = 0$. The unique solution is defined on some maximal open interval, and it is denoted by $x(\cdot, \xi, u)$. System (1) is locally integral input-to-state stable (LiISS) if there exist functions α, $\gamma \in \mathcal{K}$ and $\beta \in \mathcal{KL}$ such that, for all $\xi \in \mathcal{D}_0$ and all $u \in \mathcal{D}_u$, the solution $x(t, \xi, u)$ is defined for all $t \geq 0$ and

$$\alpha(\|x(t, \xi, u)\|) \leq \beta(\|\xi\|, t) + \int_0^t \gamma(\|u(s)\|)ds \qquad (2)$$

for all $t \geq 0$. Equivalently, system (1) is LiISS if and only if there exist functions $\beta \in \mathcal{KL}$ and γ_1, $\gamma_2 \in \mathcal{K}$ such that

$$\|x(t, \xi, u)\| \leq \beta(\|\xi\|, t) + \gamma_1 \left(\int_0^t \gamma_2(\|u(s)\|)ds \right) \qquad (3)$$

for all $t \geq 0$, all $\xi \in \mathcal{D}_0$ and all $u \in \mathcal{D}_u$.

Definition 2 (iISS-Lyapunov [8], [9]): A C^1 function $V : \mathcal{D} \to \mathbb{R}$ is called an iISS-Lyapunov function for system (1) if there exist functions α_1, α_2, $\sigma \in \mathcal{K}$, and a continuous positive definite function α_3, such that

$$\alpha_1(\|x\|) \leq V(\|x\|) \leq \alpha_2(\|x\|) \qquad (4)$$

for all $x \in \mathcal{D}$ and

$$\dot{V} \leq -\alpha_3(\|x\|) + \sigma(\|u\|) \qquad (5)$$

for all $x \in \mathcal{D}$ and all $u \in \mathcal{D}_u$.

Definition 3 (Weakly Zero-Detectability [9]): Let $h : \mathcal{D} \to \mathbb{R}^p$ with with $h(0) = 0$ be the output for the system (1). For each initial state $\xi \in \mathcal{D}_0$, and each input $u \in \mathcal{D}_u$, let $y(t, \xi, u) = h(x(t, \xi, u))$ be the corresponding output function defined on some maximal interval $[0, T_{\xi,u})$. The system (1) with output h is said to be *weakly zero-detectable* if, for each ξ such that $T_{\xi,0} = \infty$ and $y(t, \xi, 0) \equiv 0$, it must be the case that $x(t, \xi, 0) \to 0$ as $t \to \infty$.

Definition 4 (Smooth Dissipativity [9]): The system (1) with output h is dissipative if there exists a C^1, proper and positive definite function V, together with a $\sigma \in \mathcal{K}$ and a continuous positive definite function α_4, such that

$$\dot{V} \leq -\alpha_4(\|h(x(t, \xi, u))\|) + \sigma(\|u\|) \qquad (6)$$

for all $x \in \mathcal{D}$ and all $u \in \mathcal{D}_u$. If this property holds with a V that is also smooth, system (1) with output h is said to be *smoothly dissipative*. Finally, if (6) holds with $h \equiv 0$, i.e., if there exists a smooth proper and positive definite V, and a $\sigma \in \mathcal{K}$, so that

$$\dot{V} \leq \sigma(\|u\|) \qquad (7)$$

holds for all $x \in \mathcal{D}$ and all $u \in \mathcal{D}_u$, the system (1) is said to be *zero-output smoothly dissipative*.

III. SYSTEM MODELLING

We recall below a nonlinear model of the electromagnetic actuator presented in [2]:

$$\begin{array}{l} m\frac{d^2x}{dt^2} = k(x_0 - x) - \eta\frac{dx}{dt} - \frac{ai^2}{2(b+x)^2} + f_d \\ u = Ri + \frac{a}{b+x}\frac{di}{dt} - \frac{ai}{(b+x)^2}\frac{dx}{dt},\ 0 \leq x \leq x_f \end{array} \qquad (8)$$

where, x represents the armature position physically constrained between the initial position of the armature 0, and the maximal position of the armature x_f, $\frac{dx}{dt}$ represents the armature velocity, m is the armature mass, k the spring constant, x_0 the initial spring length, η the damping coefficient (assumed to be constant), $\frac{ai^2}{2(b+x)^2}$ represents the electromagnetic force (EMF) generated by the coil, a, b being constant parameters of the coil, f_d a constant term modelling unknown disturbance force, e.g. static friction, R the resistance of the coil, $L = \frac{a}{b+x}$ the coil inductance (assumed to be armature-position dependent), and $\frac{ai}{(b+x)^2}\frac{dx}{dt}$ represents the back EMF. Finally, i denotes the coil current, $\frac{di}{dt}$ its time derivative and u represents the control voltage applied to the coil. In this model we do not consider the saturation region of the flux linkage in the magnetic field generated by the coil, since we assume a current and armature motion ranges within the linear region of the flux. Based on this well known nonlinear model of the electromagnetic actuator we will develop a backstepping nonlinear control and then we extend it to an adaptive version based on a MES algorithm.

IV. LEARNING-BASED ADAPTIVE NONLINEAR CONTROL

A. Backstepping Controller with Guaranteed Integral Input-to-State Stability

Consider the dynamical system (8). We define the state vector $\mathbf{z} := [z_1\ \ z_2\ \ z_3]^T = [x\ \ \dot{x}\ \ i]^T$. The objective of the control is to make the variables (z_1, z_2) track a sufficiently smooth (at least C^2) time-varying position and velocity trajectories $z_1^{ref}(t)$, $z_2^{ref}(t) = \frac{dz_1^{ref}(t)}{dt}$ that satisfy the following constraints: $z_1^{ref}(t_0) = z_{1_{int}}$, $z_1^{ref}(t_f) = z_{1_f}$, $\dot{z}_1^{ref}(t_0) = \dot{z}_1^{ref}(t_f) = 0$, $\ddot{z}_1^{ref}(t_0) = \ddot{z}_1^{ref}(t_f) = 0$, where t_0 is the starting time of the trajectory, t_f is the final time, $z_{1_{int}}$ is the initial position and z_{1_f} is the final position.

Let us first write the system (8) in the following form:

$$\begin{array}{rcl} \dot{z}_1 & = & z_2 \\ \dot{z}_2 & = & \frac{k}{m}(x_0 - z_1) - \frac{\eta}{m}z_2 - \frac{a}{2m(b+z_1)^2}z_3^2 + \frac{f_d}{m} \\ \dot{z}_3 & = & -\frac{R}{\frac{a}{b+z_1}}z_3 + \frac{z_3}{b+z_1}z_2 + \frac{u}{\frac{a}{b+z_1}}. \end{array} \qquad (9)$$

In this section, we will first state a result discussed in [9] for autonomous systems, and then show that the sufficiency part of these results also hold for non-autonomous systems. Subsequently, we will make use of these results to discuss the stability of the overall control system.

Theorem 1 (Equivalent Characterizations of iISS [9]): Consider the autonomous system

$$\dot{x} = f(x, u) \qquad (10)$$

where $x \in \mathbb{R}^n$, $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is locally Lipschitz and inputs are measurable and locally bounded functions u :

$\mathbb{R}_{\geq 0} \to \mathbb{R}^m$. The unique solution of the initial value problem $\dot{x} = f(x, u)$ with $x(0) = \xi$ defined on some maximal open interval is denoted by $x(\cdot, \xi, u)$. The following properties are equivalent for the system (10): [*1*] The system is iISS. [*2*] The system admits a smooth iISS-Lyapunov function. [*3*] There exists an output that makes the system smoothly dissipative and weakly zero-detectable. [*4*] The system is 0-GAS and zero-output smoothly dissipative.

Now we propose the following lemma.

Lemma 1: **(Result 1)** Consider the non-autonomous system (1). If there is some output that makes the system dissipative and weakly zero-detectable locally, then the system is LiISS.

Remark 1: Note that we will analyze the local stability properties of the electromagnetic actuator system, hence we do not require conditions that give global iISS properties. To this purpose, we will modify the 0-GAS condition to 0-LUAS for the non-autonomous system. Moreover, we only need sufficiency, hence smoothness of iISS Lyapunov functions is not required. Thus, we modify properties $1-4$ of Theorem 1 to the following ones for the non-autonomous system (1): [*1a*] The system is LiISS. [*2a*] The system admits a continuously differentiable iISS-Lyapunov function. [*3a*] There is some output that makes the system dissipative and weakly zero-detectable locally. [*4a*] The system is 0-LUAS and zero-output dissipative.

Remark 2: If we interpret Lemma 1 in terms of the modified conditions of Remark 1, then Lemma 1 states that for non-autonomous systems, if $3a$ holds, then $1a$ is true. To prove this lemma, we will first show that $3a \implies 4a$; then, we will show $4a \implies 2a$, and finally we will prove that $2a \implies 1a$.
Now we proceed with the proof of Lemma 1.

Proof: $3a \implies 4a$: Assume that there is some output $h(\cdot)$ that makes the system weakly-zero detectable locally, and there exist a C^1 positive definite function V, functions $\alpha_1,\ \alpha_2 \in \mathcal{K}$, $\sigma \in \mathcal{K}$ and a continuous positive definite function α_4 such that

$$\alpha_1(\|x\|) \leq V(t,x) \leq \alpha_2(\|x\|) \quad (11)$$
$$\dot{V} \leq -\alpha_4(\|h(x)\|) + \sigma(\|u\|) \quad (12)$$

hold for all $x \in \mathcal{D}$ and all $u \in \mathcal{D}_u$. With $u = 0$, we have $\dot{V} \leq -\alpha_4(\|h(\xi)\|)$, and since the system is weakly-zero detectable, by LaSalle-Yoshizawa Theorem [10], we conclude that the system (10) is 0-LUAS. Also, we have $\dot{V} \leq \sigma(\|u\|)$ from (12), implying, by Definition 4, that system is zero-output dissipative.

$4a \implies 2a$: Assume $4a$ holds, and let V and σ be so that (7) holds. Since the system is 0-LUAS, by a converse Lyapunov theorem (e.g., [10]), there exists a C^1 function V_0 for the system (1) such that

$$\alpha_1(x) \leq V_0(x) \leq \alpha_2(x) \quad (13)$$
$$\tfrac{\partial V_0}{\partial t} + \tfrac{\partial V_0}{\partial x} f(t,x,0) \leq -\alpha_0(\|x\|), \qquad \forall x \in \mathcal{D} \quad (14)$$

holds for some continuous positive definite functions α_1, α_2, $\alpha_0 \in \mathcal{K}$. If we take the derivative of V_0 along the trajectories of the whole system (1), we have

$$\tfrac{\partial V_0}{\partial t} + \tfrac{\partial V_0}{\partial x} f(t,x,u) = \tfrac{\partial V_0}{\partial t} + \tfrac{\partial V_0}{\partial x} f(t,x,0) + \tfrac{\partial V_0}{\partial x}[f(t,x,u) - f(t,x,0)]. \quad (15)$$

Since V_0 is continuously differentiable and we consider x in a compact subset D, there exists a positive constant K_{V_0} such that

$$\left\| \frac{\partial V_0}{\partial x} \right\| \leq K_{V_0}, \quad \forall x \in \mathcal{D}. \quad (16)$$

Moreover, system (1) is locally Lipschitz in x and u, uniformly in t. This implies that there exists a positive constant $L_u(x)$ such that

$$\left\| f(t,x,u) - f(t,x,0) \right\| \leq L_u(x)\|u\|, \quad (17)$$

$\forall x \in \mathcal{D}$, $\forall u \in \mathcal{D}_u$, $\forall t \geq 0$. Since $x \in \mathcal{D}$, where $\mathcal{D}$ is compact, $L_{u_{max}} := \max_{x \in \mathcal{D}} L_u(x)$ exists. Thus, using the inequality (14), and the definitions for K_{V_0} and $L_{u_{max}}$, we have

$$\begin{aligned} &\tfrac{\partial V_0}{\partial t} + \tfrac{\partial V_0}{\partial x} f(t,x,0) + \tfrac{\partial V_0}{\partial x}[f(t,x,u) - f(t,x,0)] \\ &\leq -\alpha_0(\|x\|) + K_{V_0} L_{u_{max}} \|u\|. \end{aligned} \quad (18)$$

After defining the $\mathcal{K}$-function $\sigma_0(s) = K_{V_0} L_{u_{max}} s$ for $s \in \mathbb{R}_{\geq 0}$, we rewrite (18) as

$$\dot{V}_0 \leq -\alpha_0(\|x\|) + \sigma_0(\|u\|). \quad (19)$$

Thus, by Definition 2, V_0 is an iISS Lyapunov function for the system (1).

$2a \implies 1a$: Consider the iISS Lyapunov function V_0 for system (1) satisfying (13) and (17). Then, by sufficiency discussion in [8] and [9], system (1) is LiISS. ■

We now address the control problem of the dynamic system with uncertain parameters. Uncertain parameters of the system (8) are the spring constant k , the damping coefficient η, and the additive disturbance f_d. To take into account the uncertainty, the backstepping controller is defined as

$$\begin{aligned} u=&\tfrac{a}{b+z_1}\Bigg(\tfrac{R(b+z_1)}{a} z_3 - \tfrac{z_2 z_3}{(b+z_1)} + \tfrac{1}{2z_3}\Big(\tfrac{a}{2m(b+z_1)^2}(z_2 - z_2^{ref}) - c_2(z_3^2 - \tilde{u})\Big)\Bigg) \\ &+ \tfrac{2mz_2}{z_3}\Bigg(\tfrac{\hat{k}}{m}(x_0 - z_1) - \tfrac{\hat{\eta}}{m} z_2 + \tfrac{\hat{f}_d}{m} + c_3(z_1 - z_1^{ref}) + c_1(z_2 - z_2^{ref}) - \dot{z}_2^{ref} \\ &+ \kappa_1(z_2 - z_2^{ref})\|\psi\|_2^2\Bigg) + \tfrac{m(b+z_1)}{z_3}\Bigg(\Big(\tfrac{\hat{k}}{m}(x_0 - z_1) - \tfrac{\hat{\eta}}{m} z_2 + \tfrac{\hat{f}_d}{m} - \tfrac{a}{2m(b+z_1)^2} z_3^2 \\ &- \dot{z}_2^{ref}\Big)\Big(c_1 + \kappa_1\|\psi\|_2^2 - \tfrac{\hat{\eta}}{m}\Big) - \tfrac{\hat{\eta}}{m}\dot{z}_2^{ref}\Bigg) + \tfrac{m(b+z_1)}{z_3}\Big(2\kappa_1(z_2 - z_2^{ref})\Big) \\ &\Bigg(\frac{(x_0 - z_1)(-z_2)}{m^2} + \frac{z_2\Big(\tfrac{\hat{k}}{m}(x_0 - z_1) - \tfrac{\hat{\eta}}{m} z_2 + \tfrac{\hat{f}_d}{m} - \tfrac{a z_3^2}{2m(b+z_1)^2}\Big)}{m^2}\Bigg) \\ &- \kappa_2(z_3^2 - \tilde{u})\left|\tfrac{m(b+z_1)}{z_3}\right|^2\Bigg[\left|c_1 + \kappa_1\|\psi\|_2^2 - \tfrac{\hat{\eta}}{m}\right|^2 + \left|2\kappa_1(z_2 - z_2^{ref})\right|^2\left|\tfrac{z_2}{m^2}\right|^2\Bigg]\|\psi\|_2^2 \\ &- \kappa_3(z_3^2 - \tilde{u})\left|\tfrac{m(b+z_1)}{z_3}\right|^2\|\psi\|_2^2 + \tfrac{m(b+z_1)}{z_3}\Big(-\tfrac{\hat{k}}{m} z_2 - \ddot{z}_2^{ref} + c_3(z_2 - z_2^{ref})\Big), \end{aligned} \quad (20)$$

with

$$\begin{aligned} \tilde{u}=&\tfrac{2m(b+z_1)^2}{a}\Bigg(\tfrac{\hat{k}}{m}(x_0 - z_1) - \tfrac{\hat{\eta}}{m} z_2 + \tfrac{\hat{f}_d}{m} + c_3(z_1 - z_1^{ref}) \\ &+ c_1(z_2 - z_2^{ref}) - \dot{z}_2^{ref}\Bigg) + \tfrac{2m(b+z_1)^2}{a}\Big(\kappa_1(z_2 - z_2^{ref})\|\psi\|_2^2\Big), \end{aligned} \quad (21)$$

where $\hat{k}$, $\hat{\eta}$, $\hat{f}_d$ are the system parameter estimates, and $\psi \triangleq \begin{bmatrix} \frac{x_0 - z_1}{m} & \frac{z_2}{m} & \frac{1}{m} \end{bmatrix}^T$. We can now state the following lemma.

Lemma 2: **(Result 2)** Consider the closed-loop dynamics given by (9), (20) and (21), with constant unknown parameters k, η, f_d and consider the parameter error vector

$\Delta \triangleq \left[\begin{array}{ccc} k-\hat{k} & \eta-\hat{\eta} & f_d-\hat{f}_d \end{array} \right]^T$. Then, there exist positive gains c_1, c_2, c_3, κ_1, κ_2 and κ_3 such that $(z_1(t), z_2(t))$ are uniformly bounded and the system (9) is locally integral input-to state stable (LiISS) with respect to $(\Delta, \dot{\Delta})$.

Proof: Consider the full mechanical subsystem that consists of only the first two equations with the virtual control input $\tilde{u} := z_3^2$:

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \frac{k}{m}(x_0-z_1) - \frac{\eta}{m}z_2 + \frac{f_d}{m} - \frac{a}{2m(b+z_1)^2}\tilde{u}. \end{aligned} \quad (22)$$

Defining the Lyapunov function $V_{sub} = \frac{c_3}{2}(z_1 - z_1^{ref})^2 + \frac{1}{2}(z_2 - z_2^{ref})^2$, with $c_3 > 0$, we would like to design $\tilde{u}$ so that $\dot{V}_{sub} = -c_1(z_2 - z_2^{ref})^2$ along the trajectories of (22), but since the system parameters k, η and f_d are unknown, we design the virtual input to be $\tilde{u}$ given by (21). Substituting $\tilde{u}$ into $\dot{V}_{sub}$, leads to

$$\begin{aligned} \dot{V}_{sub} =& -c_1(z_2-z_2^{ref})^2+(z_2-z_2^{ref})\left(\frac{(k-\hat{k})(x_0-z_1)}{m}-\frac{(\eta-\hat{\eta})z_2}{m}+\frac{f_d-\hat{f}_d}{m}\right) \\ & -\kappa_1(z_2-z_2^{ref})^2\|\psi\|_2^2 \end{aligned} \quad (23)$$

Using the definitions of ψ and Δ, we can show the following:

$$\begin{aligned} \dot{V}_{sub} &\leq -c_1(z_2-z_2^{ref})^2-\kappa_1\left[|z_2-z_2^{ref}|\|\psi\|_2-\frac{\|\Delta\|_2}{2\kappa_1}\right]^2+\frac{\|\Delta\|_2^2}{4\kappa_1} \\ &\leq -c_1(z_2-z_2^{ref})^2+\frac{\|\Delta\|_2^2}{4\kappa_1}. \end{aligned} \quad (24)$$

Note that we have made use of the nonlinear damping term $-\kappa_1(z_2-z_2^{ref})^2\|\psi\|_2^2$ ([12]) to attain a negative quadratic term of ψ and Δ $\left(\text{i.e.,} -\kappa_1\left[|z_2-z_2^{ref}|\|\psi\|_2-\frac{\|\Delta\|_2}{2\kappa_1}\right]^2\right)$ and a positive term that is function of Δ only $\left(\frac{\|\Delta\|_2^2}{4\kappa_1}\right)$, hence rendering V_{sub} an iISS-Lyapunov function for the mechanical subsystem. Since we cannot directly control z_3^2, we use backstepping to design the control input $u(t)$ so that z_3^2 converges to $\tilde{u}$, which in turn will render the mechanical subsystem LiISS. Unfortunately, because of the uncertainty in system parameters, the best that can be done is to have z_3^2 follow $\tilde{u}$ with an error that is function of the uncertainty vector Δ; i.e., achieving input-to-state stability. To this purpose, we define the Lyapunov function for the full system: $V_{aug} = V_{sub} + \frac{(z_3^2-\tilde{u})^2}{2}$. Taking the derivative of V_{aug} along the trajectories of the full system, leads to the following inequality:

$$\begin{aligned} \dot{V}_{aug} \leq& -c_1(z_2-z_2^{ref})^2+\frac{\|\Delta\|_2^2}{4\kappa_1}+(z_3^2-\tilde{u})\left(-\frac{a(z_2-z_2^{ref})}{2m(b+z_1)^2}-\dot{\tilde{u}}\right) \\ & +(z_3^2-\tilde{u})\left(2z_3\left(-\frac{R(b+z_1)}{a}z_3+\frac{z_2z_3}{(b+z_1)}+\frac{b+z_1}{a}u\right)\right), \end{aligned} \quad (25)$$

where $\dot{\tilde{u}}$ is the time derivative of (21). By substituting the control input (20) into (25), using the aforementioned definitions of ψ and Δ (note that $\dot{\Delta}=\left[-\dot{\hat{k}} \quad -\dot{\hat{\eta}} \quad -\dot{\hat{f}}_d\right]^T$) and by making use of the quadratic damping terms, e.g. $-\kappa_1(z_2 - z_2^{ref})^2\|\psi\|_2^2$, we can show that $\dot{V}_{aug}$ satisfies the following inequality:

$$\begin{aligned} \dot{V}_{aug} \leq& -c_1(z_2-z_2^{ref})^2+\frac{\|\Delta\|_2^2}{4\kappa_1}-c_2(z_3^2-\tilde{u})^2 \\ & -\kappa_2\left[\left|z_3^2-\tilde{u}\right|\left|\frac{m(b+z_1)}{z_3}\right|\left|c_1+\kappa_1\|\psi\|_2^2-\frac{\hat{\eta}}{m}\right|\|\psi\|_2-\frac{\|\Delta\|_2}{2\kappa_2}\right]^2 \\ & +\frac{\|\Delta\|_2^2}{4\kappa_2}-\kappa_2\left[\left|z_3^2-\tilde{u}\right|\left|2\kappa_1(z_2-z_2^{ref})\right|\left|\frac{z_2(b+z_1)}{mz_3}\right|\|\psi\|_2-\frac{\|\Delta\|_2}{2\kappa_2}\right]^2 \\ & +\frac{\|\Delta\|_2^2}{4\kappa_2}-\kappa_3\left[\left|z_3^2-\tilde{u}\right|\left|\frac{m(b+z_1)}{z_3}\right|\|\psi\|_2-\frac{\|\dot{\Delta}\|_2}{2\kappa_3}\right]^2+\frac{\|\dot{\Delta}\|_2^2}{4\kappa_3}. \end{aligned} \quad (26)$$

Finally, from (26), we deduce

$$\dot{V}_{aug} \leq -c_1(z_2-z_2^{ref})^2-c_2(z_3^2-\tilde{u})^2+\left(\frac{1}{4\kappa_1}+\frac{1}{2\kappa_2}\right)\|\Delta\|_2^2+\frac{\|\dot{\Delta}\|_2^2}{2\kappa_3}. \quad (27)$$

We now express the uncertain system in the following nonlinear time-varying form:

$$\dot{e} = f(t, e, \tilde{\Delta}), \quad (28)$$

with $e \in \mathcal{D}_e$, $\tilde{\Delta} \in \mathcal{D}_{\tilde{\Delta}}$, where $e := [z_1 - z_1^{ref} \quad z_2 - z_2^{ref} \quad z_3^2 - \tilde{u}]^T$ and $\tilde{\Delta} = \left[\Delta^T \quad \dot{\Delta}^T\right]^T$. By considering the output defined by $h = [z_2 - z_2^{ref} \quad z_3^2 - \tilde{u}]^T$, we can show that the system (28) with h is weakly zero-detectable (i.e. by analyzing the zero-dynamics of (28) with $h \equiv \tilde{\Delta} \equiv 0$). Furthermore, inequality (27) satisfies (6), meaning that property *3a* holds for (28). By the virtue of Lemma 1, we conclude that system (28) is LiISS with respect to the input $\tilde{\Delta}$, implying that there exist functions $\alpha \in \mathcal{K}$, $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$, such that, for all $e(0) \in \mathcal{D}_e$ and $\tilde{\Delta} \in \mathcal{D}_{\tilde{\Delta}}$, and

$$\|e(t)\| \leq \beta(\|e(0)\|, t) + \alpha(\int_0^t \gamma(\|\tilde{\Delta}\|))ds \quad (29)$$

for all $t \geq 0$. ■

Remark 3: In the rest of this paper, we will refer to the controller of Section IV-A as the *ISS-backstepping controller*.

Remark 4: In this work, we assume that the unknown parameters k, η, f_d are constant. This is a realistic assumption since we are targeting the problem of aging in real applications, and usually aging happens very slowly over long period of time. Hence, the slowly varying parameters can be approximated by constant uncertain parameters.

The analysis of the dynamical behavior of estimated parameters is done via MES theory [13]. The obtained controller is described in the next section.

B. Robustification of the ISS-backstepping Controller

We now discuss how MES scheme is utilized along with ISS-backstepping controller to render the control system robust to uncertainties in system parameters. To this purpose we make the following assumptions.

Assumption 1: Consider the cost function $Q=q_1(z_1(t_f)-z_1(t_f)^{ref})^2+q_2(z_2(t_f)-z_2^{ref}(t_f))^2$, $q_1,q_2>0$ for the dynamical system (9). Q has a local minimum at $\theta^* = \left[k \quad \eta \quad f_d\right]^T$.

Assumption 2: The initial error $\Delta(t_0)$ is sufficiently small, i.e., the original parameter estimates vector $\theta = \left[\hat{k} \quad \hat{\eta} \quad \hat{f}_d\right]^T$ is close enough to θ^*.

Assumption 3: Q is analytic and its variation with respect to the uncertain variables is bounded in the neighborhood of θ^*, i.e., $\|\frac{\partial Q}{\partial \theta}(\tilde{\theta})\| \leq \xi_2$, $\xi_2 > 0$, $\forall \tilde{\theta} \in \mathcal{V}(\theta^*)$, where $\mathcal{V}(\theta^*)$ denotes a compact neighborhood of θ^*.

Remark 5: Assumption 2 implies that our result will be of local nature, meaning that our analysis holds in a small neighborhood of the actual values of system parameters.

Following [14], [13], we propose the MES algorithm for the system (9):

$$\begin{aligned} \dot{x}_p &= a_p sin(\omega_p t + \tfrac{\pi}{2})Q[x_p + a_p sin(\omega_p t + \tfrac{\pi}{2})] \\ \theta_p &= x_p + a_p sin(\omega_p t + \tfrac{\pi}{2}), \end{aligned} \quad (30)$$

where $p = 1, 2, 3$ corresponds to k, η and f_d respectively. In addition, $\omega_p = \omega_0 p,\ \omega_0 > 0,\ p = 1, 2, 3$. Operation of the scheme given in (30) is a multi-parameter analog of the scheme proposed in [2]; here, the MES scheme is implemented throughout the travel of the moving part of the actuator, and at $t = t_f$, Q is updated, and the whole cycle is reiterated with the new measurements. The purpose of using MES scheme along with ISS-backstepping controller is to improve the performance of the ISS-backstepping controller by better estimating the system parameters over many cycles, hence decreasing the error in parameters over time to provide better trajectory following for the actuator.

Now we can state the main result on the proposed MES-based robust controller.

Lemma 3: **(Result 3)** Consider the dynamical system (9) with the ISS-backstepping controller given by (20) and the cycle-to-cycle MES algorithm given by (30) for estimation of system parameters k, η, and f_d under Assumptions 1, 2 and 3. Then, the following bound holds:

$$\|e(t)\|\leq\beta(\|e(0)\|,t)+\alpha\Big(\int_0^t\gamma\big(\tilde{\beta}(\|\Delta(0)\|,t)+\|\tilde{\Delta}\|_{max}\big)\Big)ds, \quad (31)$$

where $e := \left[z_1-z_1^{ref}\quad z_2-z_2^{ref}\quad z_3^2-\tilde{u}\right]^T$, $\|\tilde{\Delta}\|_{max} = \frac{\xi_1}{\omega_0} + 2\sum_{i=1}^3\sqrt{a_i^2} + \max_{i\in\{1,2,3\}} 0.5\xi_2 a_i^2$, $\xi_1, \xi_2 > 0$, $e(0) \in \mathcal{D}_e$, $\alpha \in \mathcal{K}$, $\beta \in \mathcal{KL}$, $\tilde{\beta} \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$.

Proof: Based on Lemma 2, we know that for the closed-loop dynamics given by (9) and (20), there exist functions $\alpha \in \mathcal{K}$, $\beta \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$, such that, for all $e(0) \in \mathcal{D}_e$ and $\tilde{\Delta} \in \mathcal{D}_{\tilde{\Delta}}$, such that the following inequality holds for all $t \geq 0$:

$$\|e(t)\| \leq \beta(\|e(0)\|, t) + \alpha(\int_0^t \gamma(\|\tilde{\Delta}\|))ds. \quad (32)$$

Now, in order to evaluate the bound on the estimation vector $\tilde{\Delta}$, we use the results presented in [14]. First, based on Assumption 3, the cost function is locally Lipschitz, i.e. $\exists\eta_1 > 0,\ s.t.\ |Q(\theta_1) - Q(\theta_2)| \leq \eta_1\|\theta_1 - \theta_2\|,\ \forall\theta_1, \theta_2 \in \mathcal{V}(\theta^*)$. Moreover, since Q is analytic, it can be approximated locally in $\mathcal{V}(\theta^*)$ by a second order Taylor series. Defining $d(t):=\left[a_1 sin(\omega_1 t+\beta_1+\frac{\pi}{2})\ a_2 sin(\omega_2 t+\beta_2+\frac{\pi}{2})\ a_3 sin(\omega_3 t+\beta_3+\frac{\pi}{2})\right]^T$, by the virtues of Assumptions 1 and 2, we can then write the following bound ([14]):

$$\|\Delta(t)\|-\|d(t)\|\leq\|\Delta(t)-d(t)\|\leq\tilde{\beta}(\|\Delta(0)\|,t)+\frac{\xi_1}{\omega_0}$$
$$\Rightarrow\|\Delta(t)\|\leq\tilde{\beta}(\|\Delta(0)\|,t)+\frac{\xi_1}{\omega_0}+\|d(t)\|\leq\tilde{\beta}(\|\Delta(0)\|,t)+\frac{\xi_1}{\omega_0}+\sum_{i=1}^3\sqrt{a_i^2},$$

with $\xi_1 > 0$, for all $t \geq 0$. Moreover, in [14], the MES algorithm is shown to be a gradient-based algorithm, such that the variation of θ is approximated by

$$\dot{\theta} \simeq -R\frac{\partial Q}{\partial\theta}(\theta) + \dot{d}(t),$$

with $R = lim_{T\to\infty}\int_0^T\|d(s)\|^2 ds = 0.5\ diag\{a_1^2, a_2^2, a_3^2\}$. Using Assumption 3, we can write

$$\|\dot{\theta}\| = \|\dot{\Delta}\| \leq 0.5\max_{i\in\{1,2,3\}} a_i^2\xi_2 + \sum_{i=1}^3\sqrt{a_i^2}.$$

Finally, noting that $\|\tilde{\Delta}\|\leq\|\Delta\|+\|\dot{\Delta}\|$, we have

$$\|\tilde{\Delta}\|\leq\tilde{\beta}(\|\Delta(0)\|,t)+\frac{\xi_1}{\omega_0}+2\sum_{i=1}^3\sqrt{a_i^2}+\max_{i\in\{1,2,3\}}0.5\xi_2 a_i^2, \quad (33)$$

which together with (32) completes the proof. ■

V. Simulations

In this section, we illustrate our approach for the nonlinear electromagnetic actuator given by (8), with the system parameters given by: $m = 0.27\ [kg]$, $R = 6\ [\Omega]$, $\eta = 7.53\ [kg/sec]$, $x_0 = 8\ [mm]$, $k = 158\ [N/mm]$, $a = 14.96 \times 10^{-6}\ [Nm^2/A^2]$, $b = 4 \times 10^{-5}\ [m]$. The reference trajectory is designed to be a 5^{th} order polynomial, $x^{ref}(t) = \sum_{i=0}^5 a_i(\frac{t}{t_f})^i$ where the coefficients a_i are selected such that the following conditions are satisfied: $x^{ref}(0) = 0.2,\ x^{ref}(0.5) = 0.7,\ \dot{x}^{ref}(0) = 0,\ \dot{x}^{ref}(0.5) = 0, \ddot{x}^{ref}(0) = 0,\ \dot{x}^{ref}(0.5) = 0$. Although several cases have been tested to validate the performance of the proposed approach, due to space constraints, we present here only two cases. **Case 1 [k, η]:** We consider the uncertainty in the mechanical parameters k and η. Uncertainties in k and η are given by $\Delta k = -10$ and $\Delta\eta = -1.2$. We set the parameters of the extremum seeking algorithm in the following way: $a_k = 1,\ \omega_k = 7.5,\ a_\eta = 0.2,\ \omega_\eta = 7.4,\ q_1 = q_2 = 50$. The results of this case are depicted in the figures 1, 2, 3 and 4. As can be seen in Figures 1 and 2, without the robustification of the backstepping control via extremum seeking, the errors at $t = t_f$ are quite large; around 0.3 mm and 0.3 $\frac{mm}{s}$. With the extremum seeking, the performance of the control is significantly improved. It can be seen in Figure 3 that after 5955 iterations, the cost decreases below a very small value $Q_{threshold} = 0.01$. Moreover, the estimated parametric uncertainties Δk and $\Delta\eta$ converge to regions around the actual uncertainty values; these regions are approximated by the extremum seeking algorithm parameters a_k and a_η as dictated by **Lemma 3**. The number of iterations for the cost to decrease below the threshold level may appear to be high; the reason is that the allowed uncertainties in the parameters are relatively large, hence the extremum seeking scheme requires a lot of iterations to improve performance. In real life applications, the uncertainty in parameters accumulate gradually over a long period of time, while the learning algorithm keeps tracking these changes continuously. Thus, the extremum seeking algorithm will be able to improve the controller performance relatively quickly, meaning that it will enhance the backstepping control in much fewer iterations. In this paper, we intentionally report on challenging cases to show the adaptive ability the proposed method.**Case 2 [b]:** The simulations discussed here are designed to show that our scheme works even for the situations where the system is not linear with respect to the uncertain parameters. To this purpose, we initially considered a case where the uncertainty is in the nonlinear parameter b, and the uncertainty is given by $\Delta b = 0.02$. Note that the backstepping control has not been explicitly designed to compensate for the uncertainty in b, but we wanted to test numerically the challenging case where the estimated parameters enters the model in a nonlinear term. We set the parameters of the extremum seeking algorithm in the following way: $a_b = 0.0005$, $\omega_b = 7.6$, $q_1 = q_2 = 250$. The results of this case are depicted in the figures 5, 6, 7 and 8. It can be seen in Figures 5, 6 that the extremum seeking compensates for the uncertainty and improves the performance of the backstepping control. Without the estimation scheme, since the parameter b appears nonlinearly in the model, the uncertainty in b deteriorates the position and velocity tracking of the system significantly. With the addition of the extremum seeking scheme to estimate b, the performance is improved immensely. It can be seen in Figure 7 that the cost starts at an initial value around 12, and after 6380 iterations, it decreases below the threshold $Q_{threshold} = 0.01$. Moreover, the estimated uncertainty in b is 0.0198 after 6380 iterations, which is in accordance with the result of **Lemma 3**.

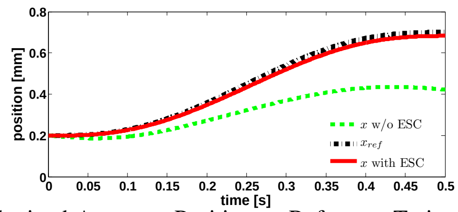

Fig. 1. Obtained Armature Position vs Reference Trajectory, a_k=1, a_η=0.2, ω_k=7.5, ω_η=7.4

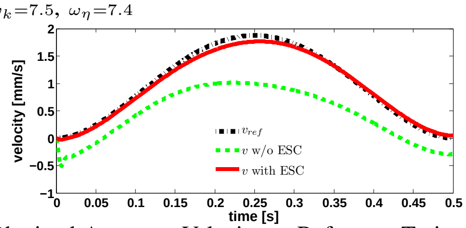

Fig. 2. Obtained Armature Velocity vs Reference Trajectory, a_k=1, a_η=0.2, ω_k=7.5, ω_η=7.4

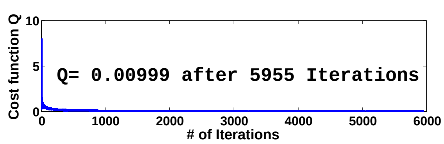

Fig. 3. Q=50$((x(t_f)-x_{ref}(t_f))^2$+50$((v(t_f)-v_{ref}(t_f))^2$, Q_{thres}=0.01

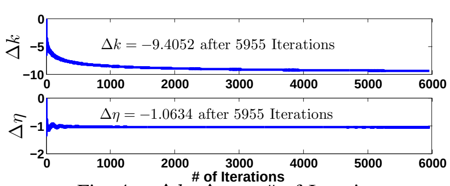

Fig. 4. Δk, $\Delta \eta$ vs # of Iterations

VI. Conclusion

In this paper, we have proposed an adaptive controller based on a nonlinear backstepping and a model-free MES algorithm. We have proved the bounded input/bounded states stability of backstepping control and the stability of the combined backstepping and MES controller. Future work will include taking explicitly into account nonlinear parameters in the control law design and in the stability analysis, as well as comparing the performance of this type of learning-based adaptive controllers to classical adaptive control methods.

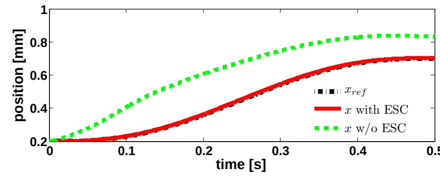

Fig. 5. Obtained Armature Position vs Reference Trajectory, a_b= 0.0005, ω_b=7.6

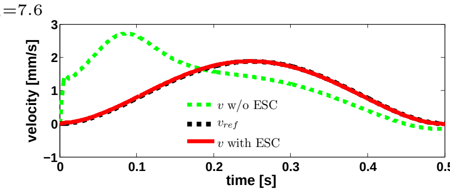

Fig. 6. Obtained Armature Velocity vs Reference Trajectory, a_b= 0.0005, ω_b=7.6

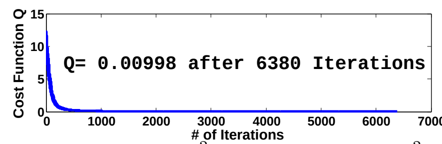

Fig. 7. Q=250$((x(t_f)-x_{ref}(t_f))^2$+250$((v(t_f)-v_{ref}(t_f))^2$, Q_{thres}=0.01

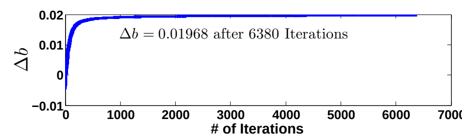

Fig. 8. Δb vs # of Iterations

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