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Abstract

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Multi-Parametric Extremum Seeking-based Learning Control for Electromagnetic Actuators

Mouhacine Benosman and Gökhan M. Atınç

***Abstract*— We study in this paper the problem of adaptive robust control of electromagnetic actuators. We first design a nonlinear controller that stabilizes locally the error dynamics. Next, we complement this nonlinear controller with a *multi-parametric extremum seeking* control to tune the feedback gains of the nonlinear controller. We use numerical tests to demonstrate the performance of this controller in dealing with model uncertainties.**

I. Introduction

Electromagnetic actuators can be used in many practical applications, e.g. electromagnetic valve actuators of combustion engines, artificial heart actuators etc. This system requires accurate control of the moving armature between two desired positions. The main objective, known as 'soft landing' of the moving armature, is to assure small contact velocity. In industrial applications of electromagnetic actuators it is necessary to avoid high velocity impacts between the moving armature and the fixed parts of the actuator. Furthermore, the 'soft-landing' requirement has to be guaranteed over a long period of time during which the actuator's components may age. Due to these practical constraints we have developed a robust control algorithm that aims for a zero impact velocity, and also adapts to the actuator aging via a learning-based adaptive algorithm. We present here the results of this study.

Many papers have been dedicated to the soft-landing problem for electromagnetic actuators, e.g. [1], [2], [3], [4], [5], [6], [7], [8], [9]. In [2], the authors studied the problem of electromagnetic valve actuator control in an internal combustion engine. The solution proposed by the authors is based on iteratively solving a constrained nonlinear optimal problem using Nelder-Mead algorithm. The optimal solution defines a feedforward control signal. The robustness of this approach to the system's aging has neither been proven nor tested, and there is no feedback terms to robustify the feedforward control. Furthermore, solving in real-time a nonlinear constrained optimal problem can be computationally hard. In [6], the authors proposed a nonlinear controller to solve the problem of armature stabilization for an electromechanical valve actuator. The authors obtained a global asymptotic stability result using Sontag's nonlinear controller. However, this approach does not solve the problem of armature trajectory tracking and does not consider robustness of the controller with respect to system's uncertainties and parameters aging. In [8], a nonlinear sliding mode approach is used to solve the problem of trajectory tracking for an electromagnetic valve actuator. The authors used a nonlinear model to design the sliding mode control. The reported results showed good tracking performances, however, this sliding mode controller does not ensure robustness with respect to model uncertainties. In [3], the authors used a single parameter extremum seeking learning method to solve the problem of soft landing for an electromechanical valve actuator. The authors first designed a nonlinear controller based on a nonlinear model of the actuator and then used an extremum seeking algorithm to tune a scalar gain of the controller. Although the learning algorithm is not directly tailored to ensure robustness of the controller to model uncertainties or parameters drift over time, one could argue that this robustness is intrinsic due to the iterative nature of the learning process. However, in this controller only one gain is tuned online.

In this work we use a nonlinear model of the electromagnetic actuator to design a controller that ensures trajectory tracking for the nominal system, i.e. assuming first that there are no uncertain parameters in the model. Subsequently, the controller is robustified by a *multi-parametric* extremum seeking algorithm that is used to *tune the feedback parameters online*, allowing the controller to adapt to the system aging over time. Notice that contrary to [3], we are using a *multi-parametric extremum seeking* approach to learn *a vector of feedback gains*.

This paper is organized as follows: We first present in Section II, a nonlinear model of the electromagnetic actuator. Then, in Section III, we report the main result of this work, namely the nominal controller and its learning-based adaptive version together with some stability discussion. Numerical validation of the proposed controller is given in Section IV, and finally the paper ends with a summarizing conclusion in Section V.

II. System modelling

Following [7], [3], we consider the following nonlinear model for electromagnetic actuators

$$\begin{aligned} m\tfrac{d^2x}{dt^2} &= k(x_0-x)-\eta\tfrac{dx}{dt}-\tfrac{ai^2}{2(b+x)^2}+f_d \\ u &= Ri+\tfrac{a}{b+x}\tfrac{di}{dt}-\tfrac{ai}{(b+x)^2}\tfrac{dx}{dt},\ 0\leq x\leq x_f, \end{aligned} \tag{1}$$

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where, x represents the armature position physically constrained between the initial position of the armature 0, and the maximal position of the armature x_f, $\frac{dx}{dt}$ represents the armature velocity, m is the armature mass, k the spring constant, x_0 the initial spring length, η the damping coefficient (assumed to be constant), $\frac{ai^2}{2(b+x)^2}$ represents the electromagnetic force (EMF) generated by the coil, a, b are two constant parameters of the coil, f_d a constant term modelling unknown disturbance force, e.g. static friction, R the resistance of the coil, $L = \frac{a}{b+x}$ the coil inductance, $\frac{ai}{(b+x)^2}\frac{dx}{dt}$ represents the back EMF. Finally, i denotes the coil current, $\frac{di}{dt}$ its time derivative and u represents the control voltage applied to the coil. In this model we do not consider the saturation region of the flux linkage in the magnetic field generated by the coil, since we assume a current and armature motion ranges within the linear region of the flux.

Based on this well known nonlinear model of electromagnetic actuators we develop in the next section a nonlinear controller and then we robustify this controller using a multi-parametric extremum seeking algorithm.

III. Learning-based control

A. Nominal control

In this section we first design a nonlinear nominal control, assuming that all the coefficients of the model are known. We then prove the stability of the equilibrium point for the closed-loop equations, i.e. model plus feedback controller. Next, we robustify this controller by using a model-free learning technique based on extremum seeking theory to tune the controller's feedback gains online.

Consider the dynamical system (1). The objective of the control is to make the variable x track a desired smooth time-varying position trajectory $x_{ref}(t)$ satisfying the following assumption.

Assumption 1: The desired trajectory is a smooth function satisfying the initial/final constraints: $x_{ref}(0) = 0$, $x_{ref}(t_f) = x_f$, $\dot{x}_{ref}(0) = 0$, $\dot{x}_{ref}(t_f) = 0$, where t_f is a desired finite motion time and x_f is a desired final position.

We define the tracking error vector $\mathbf{z} := [z_1 \ \ z_2 \ \ z_3]^T = [x - x_{ref} \ \ \dot{x} - \dot{x}_{ref} \ \ i - i_{ref}]^T$, where $\dot{x}_{ref} = \frac{dx_{ref}(t)}{dt}$, and i_{ref} is the reference current associated with the desired trajectory x_{ref}, obtained from the model (1), where we assume for the time being that $f_d = 0$

$$i_{ref} = \sqrt{\frac{2(b+x_{ref})^2}{a}\left(k(x_0 - x_{ref}) - \eta\frac{dx_{ref}}{dt} - m\frac{d^2x_{ref}}{dt^2}\right)} \quad (2)$$

Let us now define the control voltage u as

$$u(t,z) = u_{ff}(t) + v(z), \quad (3)$$

where $v(z)$ is a feedback term and u_{ff} is a feedforward term given by

$$u_{ff}(t) = Ri_{ref} + \frac{a}{b+x_{ref}}\frac{di_{ref}}{dt} - \frac{ai_{ref}}{(b+x_{ref})^2}\frac{dx_{ref}}{dt} \quad (4)$$

Using (1), (3) and (4) we can write the following error dynamics

$$\begin{aligned} m\dot{z}_2 &= -kz_1 - \eta z_2 - \tfrac{a}{2(b+z_1+x_{ref}(t))^2}(z_3 + i_{ref})^2 \\ &\qquad + \tfrac{ai_{ref}^2}{2(b+x_{ref})^2} + f_d \\ v &= Rz_3 + L(z_1)(\dot{z}_3 + \tfrac{di_{ref}}{dt}) - \tfrac{a}{b+x_{ref}}\tfrac{di_{ref}}{dt} \\ &\quad - \tfrac{a(z_3+i_{ref})}{(b+z_1+x_{ref})^2}(z_2 + \dot{x}_{ref}) + \tfrac{ai_{ref}}{(b+x_{ref})^2}\dot{x}_{ref}, \end{aligned} \quad (5)$$

where $L(z_1) = \frac{a}{b+z_1+x_{ref}}$ denotes the nonlinear inductance of the coil.

To simplify the control problem we consider a linearization of the inductance around the point $z_1 = 0$, which is equivalent to linearizing the inductance term along the desired trajectory $x_{ref}(t)$. In this case equations (5) writes as

$$\begin{aligned} m\dot{z}_2 &= -kz_1 - \eta z_2 - \tfrac{a}{2(b+z_1+x_{ref}(t))^2}(z_3 + i_{ref})^2 \\ &\qquad + \tfrac{ai_{ref}^2}{2(b+x_{ref})^2} + f_d \\ v &= Rz_3 + (L(0) + \tfrac{dL(z_1)}{dz_1}|_{z_1=0} z_1 + o(z_1))(\dot{z}_3 + \tfrac{di_{ref}}{dt}) \\ &\quad - \tfrac{a}{b+x_{ref}}\tfrac{di_{ref}}{dt} + \tfrac{ai_{ref}}{(b+x_{ref})^2}\dot{x}_{ref} \\ &\quad - \tfrac{a(z_3+i_{ref})}{(b+z_1+x_{ref})^2}(z_2 + \dot{x}_{ref}), \end{aligned} \quad (6)$$

where $o(z_1)$ denotes a function vanishing for small values of z_1, i.e. small tracking errors along the desired trajectory. Eventually, in a small neighborhood of $z_1 = 0$, equations (6) writes as

$$\begin{aligned} m\dot{z}_2 &= -kz_1 - \eta z_2 - \frac{a}{2(b+z_1+x_{ref}(t))^2}(z_3 + i_{ref})^2 \\ &\qquad + \frac{ai_{ref}^2}{2(b+x_{ref})^2} + \frac{a}{2}z_3 - \frac{a}{2}z_3 + f_d \\ v &= Rz_3 + l(t)\dot{z}_3 - \frac{a}{(b+x_{ref})^2}z_1(\dot{z}_3 + \frac{di_{ref}}{dt}) \\ &\quad - \frac{a(z_3+i_{ref})}{(b+z_1+x_{ref})^2}(z_2 + \dot{x}_{ref}) + \frac{ai_{ref}}{(b+x_{ref})^2}\dot{x}_{ref}, \end{aligned} \quad (7)$$

where $l(t) = \frac{a}{b+x_{ref}(t)}$ is the time-varying inductance along the desired trajectory $x_{ref}(t)$. We now rewrite equations (7) in the standard control form

$$\begin{aligned} \dot{z}_2 &= -\tfrac{k}{m}z_1 - \tfrac{\eta}{m}z_2 + \tfrac{a}{2m}z_3 + F_1(t,z) \\ \dot{z}_3 &= -\tfrac{R}{l(t)}z_3 + \tfrac{v}{l(t)} + F_2(t,z), \end{aligned} \quad (8)$$

where $F_1 = -\frac{a}{2m(b+z_1+x_{ref}(t))^2}(z_3+i_{ref})^2 + \frac{ai_{ref}^2}{2m(b+x_{ref})^2} - \frac{a}{2m}z_3 + f_d$, and $F_2 = \frac{a}{l(t)(b+x_{ref})^2}z_1(\dot{z}_3 + \frac{di_{ref}}{dt}) + \frac{a(z_3+i_{ref})}{l(t)(b+z_1+x_{ref})^2}(z_2+\dot{x}_{ref}) - \frac{ai_{ref}}{l(t)(b+x_{ref})^2}\dot{x}_{ref}$. Note that we added and subtracted the term $\frac{a}{2}z_3$ in the first equation to ensure the controllability of the linear part of the system. The price to pay in doing so is the additional control effort that will be needed to compensate for the effect of the nonlinear part of the system. All the above algebraic manipulations of the model equations were used to transform the error dynamics in the form of a controllable linear (time-varying) part and a nonlinear part. To write the nominal controller we first need to introduce the following assumptions.

Assumption 2: The disturbance term f_d satisfies $|f_d| \leq f_{d_{max}}$.

Assumption 3: The time-varying inductance $l(t)$ is bounded and varies slowly along the desired trajectory $x_{ref}(t)$; i.e. $|\frac{dl(t)}{dt}| \leq \epsilon_1$, with $\epsilon_1 > 0$.

Assumption 4: There exist at least one set of constant gains $k_p,\ k_d,\ k_i$ such that the matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ \frac{-k}{m} & \frac{-\eta}{m} & \frac{a}{2m} \\ 0 & 0 & \frac{-R}{l(t)} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{l(t)} \end{bmatrix} . \begin{bmatrix} k_p & k_d & k_i \end{bmatrix}$$

is Hurwitz $\forall\, t \in [0,\ t_f]$, uniformly in t.

Remark 1: Assumptions 3, 4 might seem restrictive, but for many practical applications, the time-varying inductance $l(t)$ changes slightly along the desired trajectory, which makes this assumption easier to satisfy.

Remark 2: We underline here that the linearization approximation of the inductance is done along the desired trajectories, which is different from a linearization at a given fixed point. Indeed, in this case, as long as the controller tracks the desired trajectories with small errors, the linearization assumption remains valid, even if the armature travels a non-negligible distance, i.e. the linearization validity is independent of x_f.

We can now state the following practical stability result.

Theorem 1: Consider the system (1), where f_d satisfies Assumption 2, under the feedback control

$$\begin{aligned} u(z) = u_{ff}(t) &+ Kz \\ &- \tilde{k}l(t)sgn(z_3)[|z_2|F_{1_{max}} + |z_3|F_{2_{max}}], \end{aligned} \quad (9)$$

where u_{ff} is given by (4), $K = [k_p, k_d, k_i]$ such that $l(t)$ satisfies Assumption 3, k_p, k_d, k_i satisfy Assumption 4, $F_{1_{max}},\ F_{2_{max}}$ are upper bounds of $F_1,\ F_2$ for $x \in [0,\ x_f]$, and $sgn(.)$, $|.|$ denotes the sign function and the L_2 vector norm, respectively. Then, starting from a non-zero small initial condition $z(0)$, the error dynamic $z(t)$ asymptotically converges to the ball $B_z = \{z \in \mathbb{R}^3,\ s.t.\ |z| \leq \tilde{\epsilon}\}$, for any chosen radius $\tilde{\epsilon}$, such that, the feedback gain $\tilde{k}$ satisfies the inequality $\tilde{k} > \frac{1}{|z_3(0)|}(1 - \frac{(0.5\tilde{\epsilon}^2 - 0.5|z(0)|^2)/t_f}{|z_2(0)|F_{1_{max}} + z_3(0)F_{2_{max}}})$.

Proof: We saw earlier that the system (1) with the control (3) leads locally, i.e. for small z in the neighborhood of 0, to the error dynamics (8). Let us first consider the linear time-varying part of the system (8)

$$\begin{aligned} \dot{z}_2 &= -\frac{k}{m}z_1 - \frac{\eta}{m}z_2 + \frac{a}{2m}z_3 \\ \dot{z}_3 &= -\frac{R}{l(t)}z_3 + \frac{v}{l(t)} \end{aligned} \quad (10)$$

Under Assumption 4, there exists $K = [k_p, k_d, k_i]$, s.t. $\begin{bmatrix} 0 & 1 & 0 \\ \frac{-k}{m} & \frac{-\eta}{m} & \frac{a}{2m} \\ \frac{k_p}{l(t)} & \frac{k_d}{l(t)} & \frac{-R}{l(t)} + \frac{k_i}{l(t)} \end{bmatrix}$ is Hurwitz uniformly in t. Next, under Assumption 3 and using the results of Lemma 9, ([10], p.370), we conclude about the existence of a unique positive definite Q, such that $A(t) + A^T(t) = -Q(t),\ \forall t$, with $A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-k}{m} & \frac{-\eta}{m} & \frac{a}{2m} \\ 0 & 0 & \frac{-R}{l(t)} \end{bmatrix}$, and the associated Lyapunov function $V = 0.5z^T z$, satisfying the inequality

$$\dot{V} \leq -c_1 z^T z,\ c_1 > 0,$$

along the dynamics (10). We use now the technique of Lyapunov reconstruction from nonlinear robust control, e.g.[11], to construct the third term of the controller (9). We compute the derivative of V along the full dynamics (8), where we choose $v = Kz + v_{nl}$

$$\begin{aligned} \dot{V} &\leq -c_1|z|^2 + z_3 v_{nl}/l(t) + F_1 z_2 + F_2 z_3 \\ &\leq -c_1|z|^2 + z_3 v_{nl}/l(t) + |F_1||z_2| + |F_2||z_3| \end{aligned} \quad (11)$$

Let us examine the bounds of $F_1,\ F_2$: From the structure of $F_1,\ F_2$ and the fact that $x_{ref},\ i_{ref},\ l(t),\ |f_d|$ are bounded , i.e. Assumptions 1, 2, 3, and since z_1 is assumed to be very small around the origin, i.e. local stability analysis (refer to the linearization step at equation (6)), we can write the upper bounds for $F_1,\ F_2$ as follows

$$\begin{aligned} |F_1(t,z)| &\leq c_2 z_3^2 + c_3 z_3 + c_4 \\ |F_2(t,z)| &\leq c_5 z_2 z_3 + c_6, \end{aligned}$$

with $c_i > 0\ \forall i \in \{2, ...6\}$. This shows that, locally, the nonlinearities $F_1,\ F_2$ are upper-bounded with decreasing functions of z. Thus, if we select two upper-bounds $F_{1_{max}},\ F_{2_{max}}$, s.t. $c_2 z_3^2(0) + c_3 z_3(0) + c_4(0) \leq F_{1_{max}}$, $c_5 z_2(0) z_3(0) + c_6 \leq F_{2_{max}}$, and if we select v_{nl} such that, $\dot{V} < 0,\ \forall t$, we impose that the states can only decrease starting from $z(0)$ and thus the upper-bounds $F_{1_{max}},\ F_{2_{max}}$ remains valid for all z. Now to impose the negativeness of $\dot{V}$, we choose v_{nl} as

$$v_{nl} = -l(t)\tilde{k}sgn(z_3)[|z_2|F_{1_{max}} + |z_3|F_{2_{max}}],\ \tilde{k} > 0,$$

this leads to

$$\begin{aligned} \dot{V} &< -\tilde{k}|z_3|(|z_2|F_{1_{max}} + |z_3|F_{2_{max}}) + F_{1_{max}}|z_2| \\ &\qquad + |F_{2_{max}}||z_3| \\ &< (1 - \tilde{k}|z_3|)(|z_2|F_{1_{max}} + |z_3|F_{2_{max}}), \end{aligned} \quad (12)$$

which ensures that $\dot{V} < 0$ if we choose a high gain $\tilde{k}$, i.e. $\tilde{k}$, s.t. $\tilde{k} > 1/|z_3|$. This implies that the norm of the error decreases until it enters the invariant set $\{z \in \mathbb{R}^3,\ s.t.\ 1 - \tilde{k}|z_3| > 0\}$. Next, we obtain a better characterization of the state vector norm as function of the gain $\tilde{k}$. First, we can write the following

$$\begin{aligned} &\dot{V} < (1 - \tilde{k}|z_3|)(|z_2|F_{1_{max}} + |z_3|F_{2_{max}}) < -\epsilon < 0 \\ &\Rightarrow V(t) - V(0) < -\epsilon t,\ 0 \leq t \leq t_f \\ &\Rightarrow V(t) < -\epsilon t + V(0), \end{aligned}$$

we choose now to drive the states to the ball $B_z = \{z \in \mathbb{R}^3,\ s.t.\ |z| \leq \tilde{\epsilon}\}$. To this purpose, we write

$$\begin{aligned} &V(t) < -\epsilon t + V(0) < 0.5\tilde{\epsilon}^2 \\ &\Rightarrow \epsilon > \frac{V(0) - 0.5\tilde{\epsilon}^2}{t} > \frac{V(0) - 0.5\tilde{\epsilon}^2}{t_f} \\ &\Rightarrow -\epsilon < \frac{0.5\tilde{\epsilon}^2 - V(0)}{t_f}, \end{aligned}$$

finally, we can write

$$\begin{aligned} &(1 - \tilde{k}|z_3|)(|z_2|F_{1_{max}} + |z_3|F_{2_{max}}) < \frac{0.5\tilde{\epsilon}^2 - V(0)}{t_f} \\ &\Rightarrow \tilde{k} > \frac{1 - \frac{(0.5\tilde{\epsilon}^2 - V(0))/t_f}{|z_2|F_{1_{max}} + z_3 F_{2_{max}}}}{|z_3|} \\ &\Rightarrow \tilde{k} > \frac{1 - \frac{(0.5\tilde{\epsilon}^2 - V(0))/t_f}{|z_2(0)|F_{1_{max}} + z_3(0)F_{2_{max}}}}{|z_3(0)|}. \end{aligned}$$

■

Remark 3: The lower-bound inequality of $\tilde{k}$ in Theorem 1 involves $|z_3(0)|$, and this leads to the legitimate question of division by 0. However, for the real practical case, we are never in this situation. Indeed, in real systems, the term f_d in (1) is never zero and cannot be identified precisely, which implies that the desired ideal current initial value $i_{ref}(0)$ obtained by (2), where we assumed $f_d = 0$, is never equal to the real current $i(0)$, which discards the case $z_3(0) = 0$.

Remark 4: We underline here that in the design of the controller (9), we have used a simple quadratic Lyapunov function, however, more general Lyapunov functions can be used. This will not change the design procedure (based on Lyapunov reconstruction technique), and will not alter the conclusion of this work, which is mainly about the combination of a model-based nonlinear controller and a model-free learning algorithm to obtain a robust adaptive controller.

In the next section, we extend this controller to its adaptive version, using multi-variable extremum seeking.

B. Learning-based robustification

We first define the cost function to be minimized as

$$Q(z(\beta)) = C_1 z_1(t_f)^2 + C_2 z_2(t_f)^2, \tag{13}$$

where $C_1,\ C_2 > 0$, and $\beta = (\delta\hat{k}_p,\ \delta\hat{k}_d,\ \delta\hat{k}_i, \delta\hat{\tilde{k}})'$ represents the variations of the learned gains $(\hat{k}_p, \hat{k}_d, \hat{k}_i, \hat{\tilde{k}})$ defined as

$$\begin{aligned} \hat{k}_p &= k_{p_{nominal}} + \delta\hat{k}_p \\ \hat{k}_d &= k_{d_{nominal}} + \delta\hat{k}_d \\ \hat{k}_i &= k_{i_{nominal}} + \delta\hat{k}_i \\ \hat{\tilde{k}} &= \tilde{k}_{nominal} + \delta\hat{\tilde{k}}, \end{aligned} \tag{14}$$

where $k_{p_{nominal}},\ k_{d_{nominal}},\ k_{i_{nominal}},\ \tilde{k}_{nominal}$ are the nominal initial values of the feedback gains in (9).

Folowing multi-parametric extremum seeking theory [12], the variations of the estimated parameters are defined as

$$\begin{aligned} \dot{x}_{\hat{k}_p} &= a_{\hat{k}_p} sin(\omega_1 t + \tfrac{\pi}{2}) Q(z(\beta)) \\ \delta\hat{k}_p(t) &= x_{\hat{k}_p}(t) + a_{\hat{k}_p} sin(\omega_1 t + \tfrac{\pi}{2}) \\ \dot{x}_{\hat{k}_d} &= a_{\hat{k}_d} sin(\omega_2 t + \tfrac{\pi}{2}) Q(z(\beta)) \\ \delta\hat{k}_d(t) &= x_{\hat{k}_d}(t) + a_{\hat{k}_d} sin(\omega_2 t + \tfrac{\pi}{2}) \\ \dot{x}_{\hat{k}_i} &= a_{\hat{k}_i} sin(\omega_3 t + \tfrac{\pi}{2}) Q(z(\beta)) \\ \delta\hat{k}_i(t) &= x_{\hat{k}_i}(t) + a_{\hat{k}_i} sin(\omega_3 t + \tfrac{\pi}{2}) \\ \dot{x}_{\hat{\tilde{k}}} &= a_{\hat{\tilde{k}}} sin(\omega_4 t + \tfrac{\pi}{2}) Q(z(\beta)) \\ \delta\hat{\tilde{k}}(t) &= x_{\hat{\tilde{k}}}(t) + a_{\hat{\tilde{k}}} sin(\omega_4 t + \tfrac{\pi}{2}), \end{aligned} \tag{15}$$

where $a_{\hat{k}_p},\ a_{\hat{k}_d},\ a_{\hat{k}_i},\ a_{\hat{\tilde{k}}}$ are positive tuning parameters, $\pi/2$ phase is introduced to search for a minima (vs. maxima for a zero phase) and

$$\omega_p + \omega_q \neq \omega_r,\ p, q, r \in \{1, 2, 3, 4\},\ \text{for } p \neq q \neq r. \tag{16}$$

1) Stability discussion: Under the constraints (16), the convergence of the previous learning algorithm has been proven, e.g. [12]. However, proving the stability of the combined controller (9) and the learning algorithm (14) and (15), is more challenging. Due to paper-length constraints we will not report here the detailed analysis, however, we can give simple guidelines to ensure that practically the learning-based controller is stable. Indeed, for real-case examples we noticed that the inductance changes slowly and slightly along the desired trajectories. Thus, after choosing nominal gains $k_{p_{nominal}},\ k_{d_{nominal}},\ k_{i_{nominal}}$ satisfying Assumption 4 for the nominal model, we can choose the parameters $a_{\hat{k}_p},\ a_{\hat{k}_d},\ a_{\hat{k}_i}$ small enough to make sure that the new gains $\hat{k}_p, \hat{k}_d, \hat{k}_i$ still satisfy Assumption 4. Next, one can choose $\tilde{k}_{nominal}$ satisfying the condition of Theorem 1. The nominal gain can be selected to satisfy this condition with some margin, i.e. high gain, and the learning parameter $a_{\hat{\tilde{k}}}$ can be selected small enough to still satisfy this condition during the learning process. Now, one can argue that even if we can have a stable feedback controller for each value of the gains during the learning, we still need to worry about the stability of the switching between these stable closed-loop plants. Indeed, to make sure that the switching during the learning process does not lead to instability, we use a dwelling-time argument, e.g. [13]. In practical cases, it is sometimes possible to restrict the switching time, when the new learned gains are used in the feedback loop, it is sufficient to wait long enough to make sure that the effect of the previous gains dissipates, which ensures that the switching process remains stable. We underline that the above discussion does not pretend to be a proof of stability of the learning-based controller, a more rigorous proof is under developmental and, due to paper-length constraints, will be presented in a journal version of this work.

Parameter	Value
m	$0.27\ [kg]$
R	$6\ [\Omega]$
η	$7.53\ [kg/sec]$
x_0	$8\ [mm]$
k	$158\ [N/mm]$
a	$14.96 \times 10^{-6}\ [Nm^2/A^2]$
b	$4 \times 10^{-5}\ [m]$

TABLE I

NUMERICAL VALUES OF THE MECHANICAL PARAMETERS

IV. NUMERICAL RESULTS

We show here the behavior of the proposed approach on the electromagnetic actuator example presented in [14], where the model (1) is used with the numerical values of Table I. The desired trajectory has been selected as the $5th$ order polynomial $x_{ref}(t) = \sum_{i=0}^{5} a_i (t/t_f)^i$, where the a_i's have been computed to satisfy the boundary constraints $x_{ref}(0) = 0.2, x_{ref}(t_f) = x_f, \dot{x}_{ref}(0) = \dot{x}_{ref}(t_f) = 0, \ddot{x}_{ref}(0) = \ddot{x}_{ref}(t_f) = 0$, with $t_f = 0.5\ sec$, $x_f = 0.7\ mm$.

We assume in these numerical tests that the disturbance value is $f_d = 10\ N$, which is negligible compared to the spring force, but it has to be accounted for since it is never zero in real systems. However, since it is difficult to measure this disturbance beforehand, its value is assumed to be zero in the design of the controller, i.e. in the generation of the desired current trajectory (2), which induces an initial error on the current: $z_3(0) = 0.0895A$. Furthermore, to make the simulation case more challenging we assume an initial error both on the position and the velocity $z_1(0) = 0.01\ mm$, $z_2(0) = 0.1\ mm/sec$. Note that these values may seem small, but for this type of actuators it is usually the case that the armature starts form a predefined static position constrained mechanically, so we know that the initial velocity is zero and we know in advance very precisely the initial position of the armature. However, we want to show the performances of the controller on some challenging cases, to confirm the performance expected from the theoretical analysis. Next, based on this initial error vector, we compute the lower bound of the feedback gain $\tilde{k}$ using Theorem 1. Using the formula in Theorem 1, choosing a desired error ball-radius $\tilde{\epsilon} = 0.1$, we obtain a lower bound for $\tilde{k}$ equal to 13.5, and we choose a larger value $\tilde{k} = 17$ (refer to the stability discussion Section III-B.1). We also select the linear feedback gains $k_p = 500,\ k_d = 80,\ k_i = 30$ satisfying Assumption 4, and we select $F_{1_{max}} = F_{2_{max}} = 20$ (of course a more conservative, i.e. larger, upper bound of the nonlinearities can be chosen, if we assume a higher bound for $|f_d|$, but this will lead to higher control voltages).

First we show on figures 1, 2 the effect of the controller (9) with nominal gains on the nominal plant, i.e. with initial stats errors but without model uncertainties. We see that the controller has no problem following the desired trajectories in this case. Next, we teste a more realistic case where we assume an error on the spring constant coefficient of 3% and in the damping coefficient η of 20%, together with the position and the velocity initial errors mentioned above. The learning algorithm (14), (15) is implemented with the cost function (13) where $C_1 = 10,\ C_2 = 50$, and the learning coefficients for each feedback gain are $a_{\hat{k}_p} = 0.56$, $\omega_1 = 7.5\ rad/sec$, $a_{\hat{k}_d} = 0.28$, $\omega_2 = 5.3\ rad/sec$, $a_{\hat{k}_i} = 0.06$, $\omega_3 = 5.1\ rad/sec$, $a_{\hat{\tilde{k}}} = 0.35$, $\omega_4 = 6.1\ rad/sec$. We point out here that one way to choose the learning angular frequencies $\omega_i,\ i = 1, 2, 3, 4$, is to select these values in such a way to have slow learning dynamics relatively to the system dynamics, i.e. time-scale separation, which can ensure, via a dwelling-time argument, the stability of the switching between different gains during the learning. We show on figures 3, 4 the results of the nominal controller (9), with and without the learning algorithm. We see clearly the effect of the learning algorithm that corrects the tracking overall and also makes the landing velocity closer to the desired zero landing velocity, i.e. $z_2(t_f) = 0.07mm/sec$ without learning, and $z_2(t_f) = 0.0053mm/sec$ after learning the new gains. We also report on figure 5, the cost function value along the learning iterations. We see a clear decrease of the cost function. We stopped the learning after 1000 iterations since the trend of the learning was clearly towards decreasing the cost function and stabilizes after the first 1000 iterations. Finally, the variable parts of the feedback gains $\delta\hat{k}_p,\ \delta\hat{k}_d,\ \delta\hat{k}_i, \delta\hat{\tilde{k}}$ are given in figures 6, 7, 8, and 9, respectively. They also show a trend of convergence of the learning algorithm.

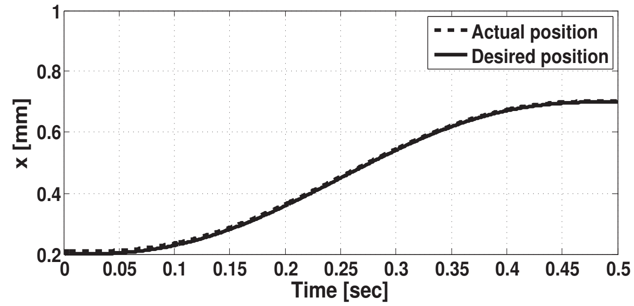

Fig. 1. Obtained armature position vs. reference trajectory - Nominal case

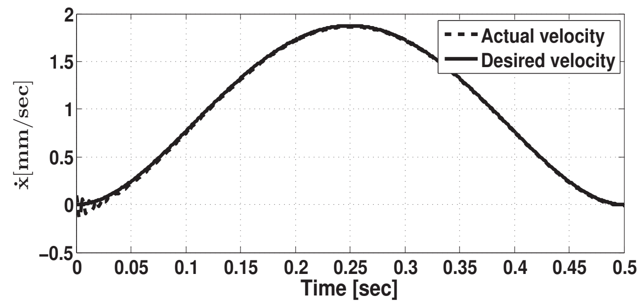

Fig. 2. Obtained armature velocity vs. reference trajectory - Nominal case

V. CONCLUSION

We have presented in this paper some preliminary results about multi-parametric extremum seeking learning control of electromagnetic actuators. First, we have proposed a nominal nonlinear controller that stabilizes locally the armature position tracking error and velocity tracking error to a desired small amplitude. Next, we have complemented this controller with a learning algorithm which uses multi-parametric extremum seeking to tune the feedback gains of the nominal controller. Finally, We have reported some numerical results that demonstrated the performance of this learning-based controller.

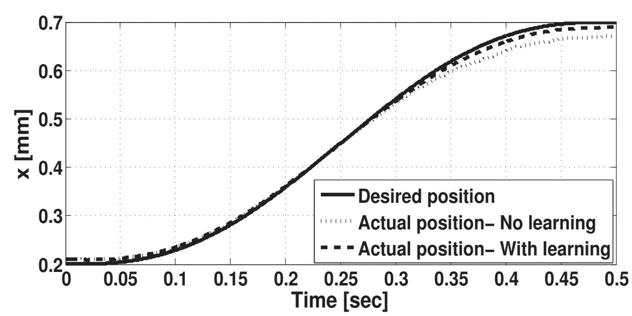

Fig. 3. Obtained armature position vs. reference trajectory - Uncertain case

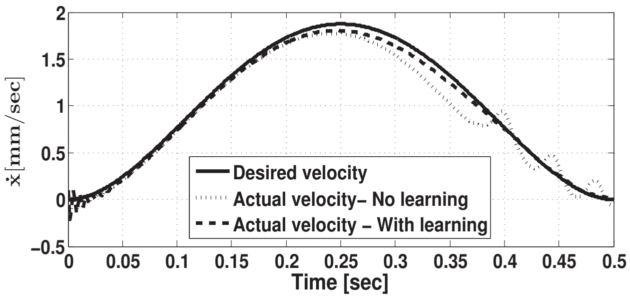

Fig. 4. Obtained armature velocity vs. reference trajectory - Uncertain case

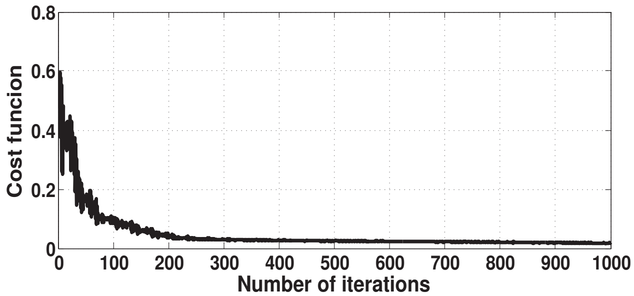

Fig. 5. Cost function vs. learning iterations

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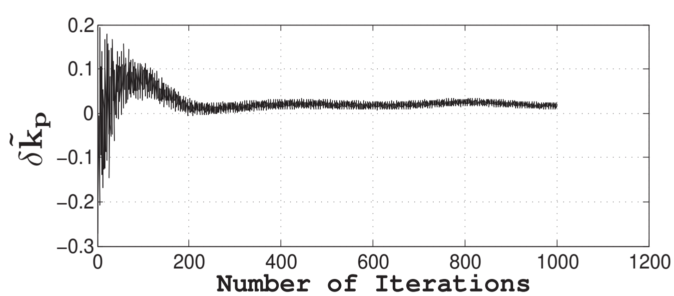

Fig. 6. $\delta\hat{k}_p$ vs. learning iterations

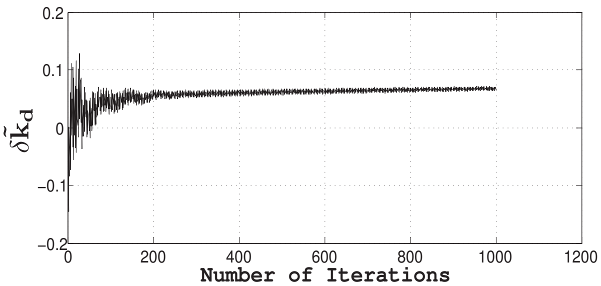

Fig. 7. $\delta\hat{k}_d$ vs. learning iterations

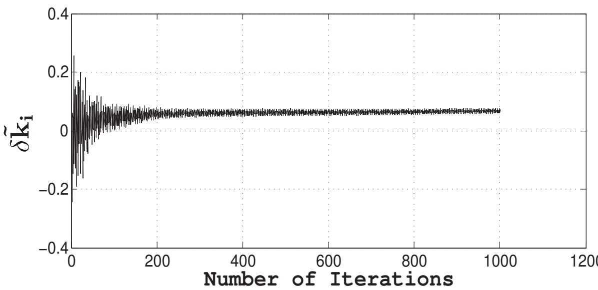

Fig. 8. $\delta\hat{k}_i$ vs. learning iterations

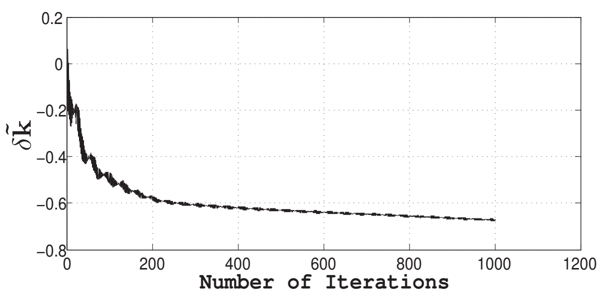

Fig. 9. $\delta\hat{\tilde{k}}$ vs. learning iterations

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