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State of Charge Estimation Based on a Real-time Battery Model and Iterative Smooth Variable Structure Filter

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***Abstract*—This paper proposes a novel real-time model-based state of charge (SOC) estimation method for lithium-ion batteries. The proposed method includes: 1) an electrical circuit battery model incorporating the hysteresis effect, 2) a fast upper-triangular and D-diagonal recursive least square (FUDRLS)-based online parameter identification algorithm for the electrical battery model, and 3) an iterated smooth variable structure filter (ISVSF) for SOC estimation. The proposed method enables an accurate and robust condition monitoring for lithium-ion batteries. Due to its low complexity, the proposed method is suitable for the real-time embedded battery management system (BMS) application. Simulation and experiment are performed to validate the proposed method for a polymer lithium-ion cell.**

***Index Terms*—Battery model, fast UD recursive least square (FUDRLS), hysteresis, iterative smooth variable structure filter (ISVSF), lithium-ion battery, state of charge (SOC) estimation**

I. Introduction

Lithium-ion batteries have gained more pervasive use in numerous applications from electronics to power tools owing to their high energy and power densities and long cycle life [1]. However, due to the low thermal stability and aging process, reliability and performance degradation are still the concerns when using lithium-ion batteries. Therefore, a battery management system (BMS) is required to monitor and control the conditions of batteries [2]. A key function of the BMS is to monitor the state of charge (SOC), state of health (SOH), instantaneous available power [i.e., state of power (SOP)], internal impedance, capacity, etc., during battery operation [3]. These parameters and states will offer the fault diagnostic and prognostic capability for the battery system [4]. It is well-understood that the parameters and states can only be obtained online, typically, from model-based estimation methods due to the absence of sensors for direct measurements of these quantities.

A variety of online battery SOC estimation methods have been developed, which, in general, can be classified into two categories: direct measurement methods (i.e., non-model-based methods) and model-based approaches. The direct measurement methods include voltage translation and Coulomb counting [5]. They are simple and easy to implement. However, both methods have limitations. For example, the former requires the battery to rest for a long period and cut off from the external circuit to measure the open circuit voltage (OCV); and the latter suffers from unrecoverable problems that might be caused by factors such as inaccurate initial SOC and maximum capacity values, cumulative integration errors, and noise corruption.

The recent effort on the SOC estimation has been focused on model-based methods with an improved accuracy. For example, extended Kalman filter (EKF) types of approaches [6], [7] have been proposed for real-time BMS applications. These methods provide accurate SOC estimations in general. However, they require an accurate electrical circuit battery model, whose parameters, e.g., resistances and capacitances, typically vary with the SOC, temperature, current, aging, etc., of the battery cell [8]. Therefore, additional online parameter estimation is usually needed to reduce the SOC estimation error. A joint EKF method has been proposed by combining SOC and parameter estimations in an EKF [9]. Some lithium-ion batteries have a relatively high nonlinearity of OCV, which is called the hysteresis effect [10]. To account for the time-varying model parameters and hysteresis effect, a dual EKF [11] and a dual sigma-point Kalman filter (SPKF) [12] which outperforms the EKF have been proposed to estimate the parameters and states of a battery simultaneously. Nevertheless, the SOC estimation error can be large when the process noise and the measurement noise are uncorrelated with zero mean Gaussian white noise and their covariance values are not properly defined. Moreover, the joint/dual EKF and dual SPKF SOC estimation methods have a high computational complexity. Other observer design methods,

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including a linear observer [13] and a sliding mode observer [14], [15], have been used for electrical circuit model-based SOC estimators with regression-based parameter estimation. The primary advantages of the observers with parameter update are their low computational complexity and the possibility to achieve robust convergence of the resultant estimation error dynamics. The observer-based approaches, however, perform moderately in terms of the estimation accuracy. Moreover, the sliding mode observer suffers a chattering problem [14].

This paper proposes a real-time model-based SOC estimation algorithm for lithium-ion batteries. The battery model consists of a conventional electrical circuit model and a hysteresis model. The fast UD recursive least square (FUDRLS) [16] method is applied to estimate the parameters of the electrical circuit model. Based on the identified model parameters, an iterated smooth variable structure filter (ISVSF) is designed to perform the SOC estimation. The proposed algorithm leads to an accurate and robust SOC estimation for lithium-ion batteries and is suitable for real-time embedded BMS due to its low complexity and easy implementation. The proposed method is validated by simulation and experimental results for a polymer lithium-ion battery cell.

II. Real-Time Battery Model

It is well-understood that an accurate battery model is important to obtain a precise estimation of the states and parameters in a model-based SOC estimation method. In addition, a balance between the accuracy and the complexity of the battery model should be considered for real-time condition monitoring in embedded systems. In general, electrical circuit battery models are suitable for embedded system applications due to the low complexity and the ability of predicting the current-voltage (I-V) dynamics [17] of battery cells. The hysteresis effect [10] is a fundamental phenomenon of batteries which shows an equilibrium difference between battery charging and discharging. The equilibrium difference depends on the history of battery usage. For some lithium-ion batteries (e.g., LiFePo4) having relatively strong hysteresis, the SOC estimation accuracy will deteriorate if the battery model does not incorporate the hysteresis effect. It was also demonstrated that the first-order resistor-capacitor (RC) model with a hysteresis provided a good balance between model accuracy and complexity [18]. Therefore, this paper considers a real-time battery model comprising a first-order RC electrical circuit with a hysteresis voltage, as shown in Fig. 1.

In Fig 1, *VOC* (i.e., the open-circuit voltage OCV) includes two parts. The first part, denoted by $V_{oc}(SOC)$,

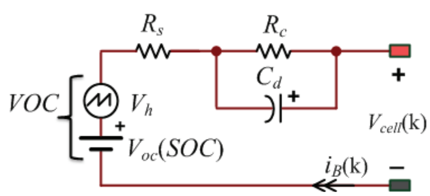


Fig. 1. The first-order RC model with a hysteresis.

represents the equilibrium OCV, which is used to bridge the SOC to the cell open-circuit voltage. The second part V_h is the hysteresis voltage to capture the nonlinearity of the OCV. The RC circuit models the I-V characteristics and the transient response of the battery cell. Particularly, the series resistance, R_s, is used to characterize the charge/discharge energy losses of the cell; the charge transfer resistance, R_c, and the double layer capacitance, C_d, are used to characterize the short-term diffusion voltage, V_d, of the cell; and V_{cell} represents the terminal voltage of the cell. Defining $H(i_B)=\exp(-\rho|i_B(k)|T_s)$, a discrete-time state-space version of the real-time battery model is expressed as follows:

$$x(k+1)=\begin{bmatrix} SOC(k+1) \\ V_d(k+1) \\ V_h(k+1) \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \exp(\frac{-T_s}{R_c\cdot C_d}) & 0 \\ 0 & 0 & H \end{bmatrix}\cdot\begin{bmatrix} SOC(k) \\ V_d(k) \\ V_h(k) \end{bmatrix} + \begin{bmatrix} -\eta T_s/C_{\max} & 0 \\ R_c(1-\exp(\frac{-T_s}{R_c\cdot C_d})) & 0 \\ 0 & (H-1)sign(i_B(k)) \end{bmatrix}\cdot\begin{bmatrix} i_B(k) \\ V_{h\max} \end{bmatrix} \tag{1}$$

$$y(k)=V_{cell}(k)=V_{oc}(SOC)-V_d(k)-R_s\cdot i_B(k)+V_h(k) \tag{2}$$

$$V_{oc}(SOC)=a_0\exp(-a_1SOC)+a_2+a_3SOC - a_4SOC^2+a_5SOC^3 \tag{3}$$

where k is the time index; $x(k)$ is the state vector; $y(k)$ is the measured output; η is the Coulomb efficiency (assuming η = 1); T_s is the sampling period; $i_B(k)$ is the instantaneous current of the battery at the time k; V_{hmax} is the maximum hysteresis voltage which may be a function of SOC; ρ is the hysteresis parameter, which represents the convergence rate; a_0~a_5 are the coefficients of the OCV curve. The state space model (1) can be written in a concise form as follows:

$$x(k+1)=f(x(k),i_B(k)) \tag{4}$$

where f is a smooth vector field denoting the right-hand side of (1).

Fig. 2 shows the two OCV curves as functions of SOC extracted for a polymer lithium-ion battery cell. Voc_{charge} and $Voc_{discharge}$ represent the major upper and lower hysteresis loops, respectively. $V_{oc}(SOC)$ is considered as an average voltage (i.e., $Voc_{average}$) of the charge and discharge open circuit voltage curves. *VOC* representing the trajectory of the instantaneous open circuit voltage whose boundary consists of the major loops. By subtracting $V_h(k)$ from *VOC*, the $V_{oc}(SOC)$ which has a one-to-one mapping to SOC will be extracted [19].

The first-order differential equation to model the hysteresis voltage V_h has been proposed as [19]:

$$\frac{\partial V_h}{\partial t}=-\rho(\eta i_{cell}-\upsilon S_D)[V_{h\max}+sign(i_{cell})V_h] \tag{5}$$

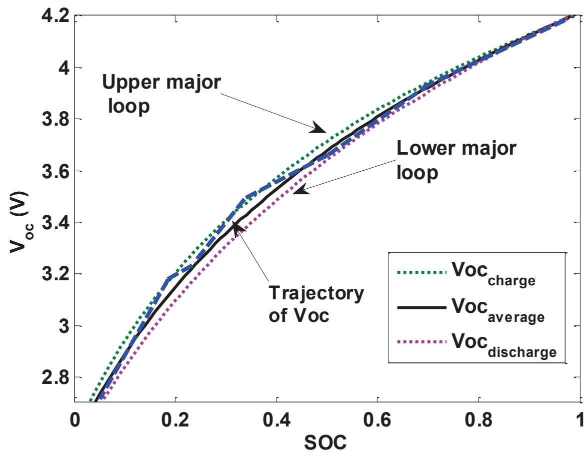


Fig. 2. OCV curves.

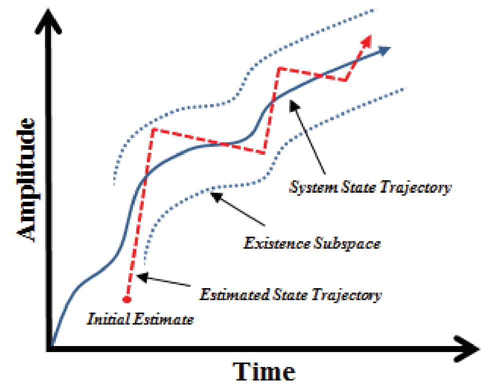


Fig. 3. The SVSF-based state estimation concept [21].

where υ is a self-discharge multiplier for hysteresis expression, S_D is a self-discharge rate. This hysteresis model describes the dependency of the hysteresis voltage V_h on the current rate, current direction, self-discharge, and hysteresis boundaries. For example, when a long-period charge current, or a short but very large charge current is applied, the hysteresis voltage will converge to V_{hmax} [19]. In other words, *VOC* will converge to the upper major loop. In the opposite case, *VOC* will converge to the lower major loop. V_{hmax} can be calculated by the difference between the Voc_{charge} and the $Voc_{average}$. The state, $V_h(k)$, in (1) is the discrete-time version of (5) using exact calculation [20]. The self-discharge effect is ignored in order to reduce the complexity of the battery model.

The parameter ρ is chosen to minimize the error between the simulation and experimental results of *VOC* versus SOC curves. The parameters ρ and V_{hmax} may depend on the SOC and the battery temperature [10], [19]. The coefficients a_0~a_5 can be extracted by pulsed current tests [17]. In this paper, the temperature dependency is ignored by testing the battery under the ambient temperature.

III. The Proposed Method

An adaptive ISVSF is proposed to estimate the battery SOC by using the state space model (1)-(2). The ISVSF, which is a modified version of the SVSF [21], speeds up the convergence of the SVSF by iteratively refining the state estimated around the current point at each time instant. The internal parameters R_s, R_c, and C_d of the state space model (1)-(2) are updated by the FUDRLS online parameter identification algorithm, which results in a more accurate SOC estimation [16].

The SVSF was introduced in 2007 [21] as a new predictor-corrector method based on the variable structure theory and the sliding mode concept for state and parameter estimation [21]. A switching gain is implemented to keep the estimated states to stay within a bounded domain, which is an invariant set containing the true states. The SVSF is relatively stable and robust to model uncertainties and noise, given that the uncertainties are upper-bounded. The basic concept of the SVSF-bases state estimation is shown in Fig. 3, where the solid line is the trajectory of a system state. The estimated state trajectory is forced towards the actual state trajectory until it reaches a subspace around the actual state trajectory, referred to as the existence subspace. Once the estimated state trajectory enters the existence subspace, it is pushed to remain within the existence subspace and switch along the actual system state trajectory [21]. The SVSF has been applied to estimate battery parameters and SOC and verified by using simulation results only [22].

In this paper, an ISVSF is designed based on the state space model (1)-(2) to perform the state estimation for battery cells. The dynamics of the proposed ISVSF are given by:

$$\hat{x}_{k+1|k} = \hat{f}(\hat{x}_{k|k}, i_B(k)) \tag{6}$$

where $\hat{x}_{k+1|k}$ is the predicted (or priori) state estimate, $\hat{x}_{k|k}$ is the previous state estimate, $\hat{f}$ is a vector field, and a predicted measurement $\hat{y}_{k+1|k}$ is written as follows:

$$\hat{y}_{k+1|k} = C\hat{x}_{k|k} \tag{7}$$

where C is the linearized measurement matrix and written as follows:

$$C = diag\left[\frac{\partial V_{OC}(SOC)}{\partial SOC}, \quad -1, \quad 1\right] \tag{8}$$

The measurement error $e_{y,k+1|k}$ may be calculated as follows:

$$e_{y,k+1|k} = y_{k+1} - C\hat{x}_{k+1|k} \tag{9}$$

The SVSF gain, K, is calculated as:

$$K_{k+1} = C^{-1}(|e_{y,k+1|k}| + \gamma |e_{y,k|k}|) \circ sat(e_{y,k+1|k}, \Psi) \tag{10}$$

where $e_{y,k|k}$ is a posteriori measurement error in the previous step; Ψ is the smoothing boundary layer width; γ (0< γ<1) is the SVSF convergence rate; ∘ denotes the Schur product. The value of C should be positive (i.e., C > 0) to ensure the numerical stability. The SVSF gain is used to correct the state estimate, $\hat{x}_{k+1|k}$ as follows:

$$\begin{cases} \hat{x}_{k+1|k+1} = \hat{x}_{k+1|k} + K_{k+1} \\ e_{y,k+1|k+1} = y_{k+1} - C\hat{x}_{k+1|k+1} \end{cases} \tag{11}$$

where $\hat{x}_{k+1|k+1}$ is the corrected (or posteriori) state estimate in the current time step.

In order to speed up the convergence rate of the SVSF, the iterated SVSF is applied. It consists of two procedures: prediction and update. The formulae of the ISVSF in the prediction procedure are the same as the original SVSF. If $e_{y,k+1|k+1}$ is larger than a prespecified error tolerance level, it will go to the update procedure, which is implemented iteratively as follows:

$$\begin{cases} C = diag\left[\dfrac{\partial V_{OC}(SOC^{(i)})}{\partial SOC^{(i)}}, \quad -1, \quad 1\right] \\ K_{k+1}{}^{(i)} = C^{-1(i)}(|e_{zk+1|k}{}^{(i-1)}| + \gamma |e_{z,k|k}|) \circ sat(e_{z,k+1|k}{}^{(i-1)}, \Psi) \\ \hat{x}_{k+1|k}{}^{(i)} = \hat{x}_{k+1|k}{}^{(i-1)} + K_{k+1}{}^{(i)} \\ e_{y,k+1|k}{}^{(i)} = y_{k+1} - C^{(i)}\hat{x}_{k+1|k}{}^{(i)} \end{cases} \tag{12}$$

The iteration process stops when the estimation error becomes less than the prespecified tolerance level ς or the value of i reaches the predefined maximum iteration number N_{max}. In the latter case, the estimated state $\hat{x}_{k+1|k}{}^{(i)}$ corresponding to the minimum error is set to be $\hat{x}_{k+1|k+1}$. Once the estimated state $\hat{x}_{k+1|k+1}$ converges, the iteration process will stop. It should be pointed out that the values of ς and N_{max} will affect the performance of the ISVSF.

IV. Validation

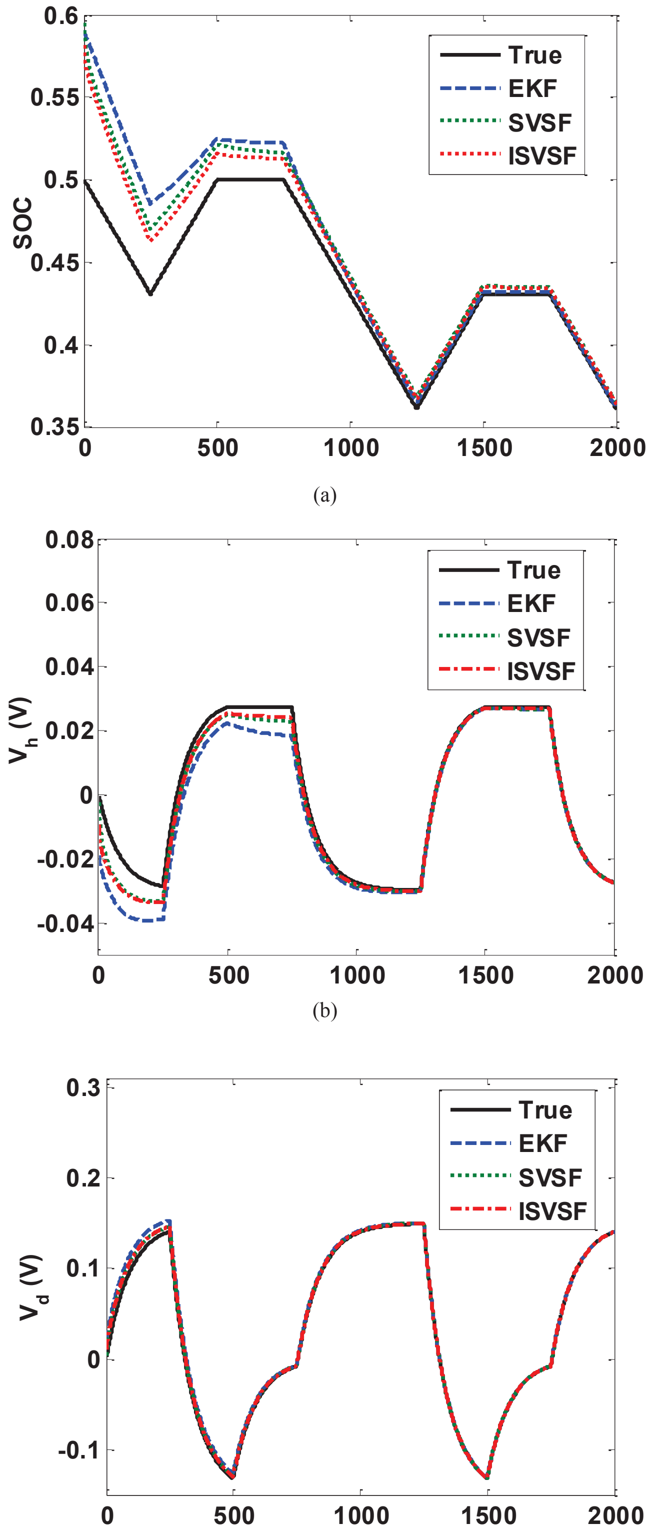
Simulation and experimental studies are carried out to validate the proposed SOC estimation algorithm for a polymer lithium-ion battery cell subject to various pulsed current operations. The nominal capacity, nominal voltage, and cutoff voltage of the battery cell are 5 Ah, 3.7 V, and 2.5 V, respectively. The proposed method is implemented in MATLAB on a computer. In the simulation study, the real-time battery model is given by (1)-(2), where the model parameters are listed in Table I. For the experimental study the experimental data of the cell voltage and current are collected from a battery tester under the ambient temperature. The measured data are then used by the proposed method for SOC estimation of the battery cell.

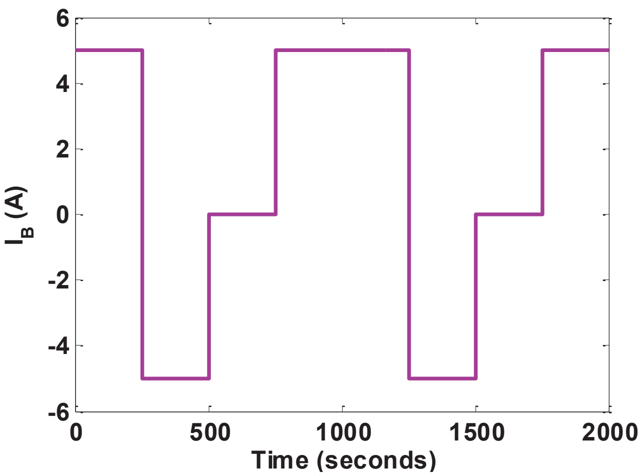
A. *Simulation Restults*

The proposed ISVSF-based SOC estimation algorithm is first validated by using the simulated data obtained from the developed real-time battery model. Comparisons with the existing methods, such as the traditional EKF and SVSF [22],

TABLE I: Battery Model Parameters

R_s	0.08	R_c	0.03
C_d	3000	ρ	$2.47\cdot10^{-3}$
V_{hmax}	0.03	a_0	-0.852
a_1	63.867	a_2	3.692
a_3	0.559	a_4	0.51
a_5	0.508		





(d)

Fig. 4. Comparison of true and estimated states of the simulated battery cell obtained from the online EKF, SVSF, and ISVSF algorithms: (a) SOC, (b) V_h, (c) V_d, and (d) the pulsed current cycles applied to the battery cell.

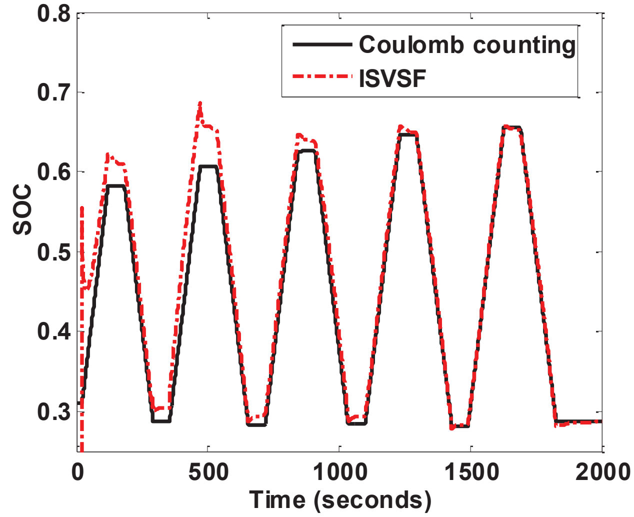
are performed using simulation results to show the superiority of the proposed ISVSF algorithm. The true initial SOC of the battery cell is set to be 0.5. However, the SOC estimation algorithms start from a wrong initial SOC of 0.6. In the EKF design, the system noise covariance matrix and the measurement noise covariance matrix are defined as 0.016 and 0.025, respectively. In the SVSF, the values of γ and Ψ are chosen to be 0.1 and 1. For the ISVSF, the values for γ and Ψ are the same as those in the SVSF; and ς and N_{max} are set to be 0.003 and 10, respectively. The battery model is subject to a pulsed current cycle shown in Fig. 4(d). Fig. 4(a)-(c) compares the true values of the states SOC, V_h, and V_d of the simulated battery cell and their estimated values obtained from different state estimation algorithms. Table II compares the performance of the state estimation algorithms in terms of accuracy using the root mean square error (RMSE) and computational cost using the simulation time on a Intel® Core™2 Duo CPU T6600@2.2GHz, 64-bit OS. The results show that the proposed ISVSF has the best estimation accuracy among the three estimation methods and a moderate computational cost. Furthermore, the parameters of the ISVSF are easier to tune compared with the EKF.

B. Experimental Results

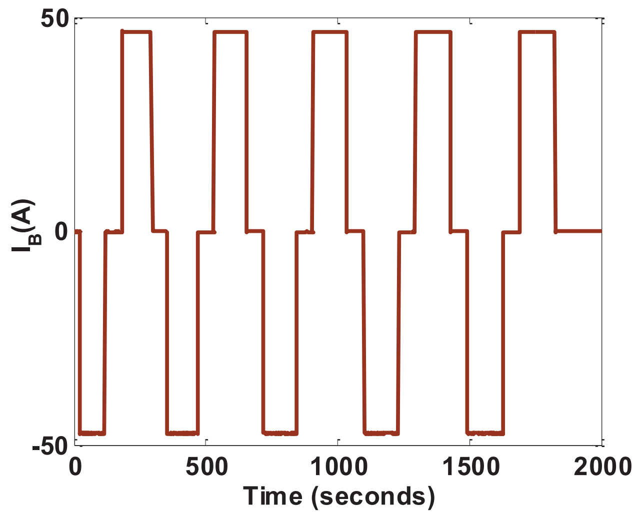
The proposed SOC estimation algorithm is further investigated using the measured data of a lithium-ion battery cell. The true SOC reference is obtained using the Coulomb counting method. In the SVSF and ISVSF, the values of γ and Ψ are set to be 0.1 and 1, respectively. In addition, the values of ς and N_{max} are set to be 0.01 and 10, respectively. The parameters of the OCV-SOC function of the battery cell are obtained under the ambient temperature. The initial SOC and maximum capacity are set to be 0.5 and 5 Ah, respectively, for the state space model (1). The true initial SOC and maximum capacity used in the Coulomb counting method are 0.31 and 4.732 Ah, respectively. To obtain the true initial SOC, the battery cell is first fully charged and rests for one hour, and then discharged using a small current (e.g., 0.2 A) to the desired initial SOC value. The true maximum capacity used in the Coulomb counting method is extracted offline from a full discharge test with a small current (e.g., 0.2 A) at the ambient temperature before testing the battery. The FUDRLS is first executed for 20 seconds and the ISVSF is executed with a constant sampling period (e.g., T_s = 1 second) to keep track of

TABLE II: SIMULATION TIME AND RMES REUSTLS FOR THE SOC ESTIMATION ALGORITHMS

Method	EKF	SVSF	ISVSF
Simulation time (seconds)	1.4752	0.8474	1.1783
SOC (RMSE)	0.1405	0.1325	0.1213
V_h (RMSE)	0.0668	0.0477	0.0466
V_d (RMSE)	0.0627	0.0441	0.0443



(a)



(b)

Fig. 5. Estimated SOC from the proposed ISVSF with FUDRLS algorithm on the experimental data: (a) SOC and (b) the a pulsed current cycle applied to the battery

the fast time varying electrical parameters and SOC. The battery cell was operated by a dynamic high pulsed current cycle (I_B = 10 C) shown in Fig. 5(b). Fig. 5(a) compares the estimated SOC obtained from the proposed ISVSF algorithm with that obtained from the Coulomb counting method. The SOC estimated by the ISVSF matches that obtained from Coulomb counting after a certain period due to the wrong initial SOC used in the proposed method. The error is less than 2% after 1000 seconds. The result clearly shows that the proposed algorithm is robust to the error of initial SOC by using the proposed real-time battery model, online parameter identification, and ISVSF state estimation algorithm together.

V. CONCLUSION

This paper has proposed a novel model-based SOC estimation algorithm. The proposed method has been implemented in MATLAB and validated by simulation and experimental results for a lithium-ion battery cell. The proposed model can be applied to any types of lithium-ion batteries, especially, the batteries having hysteresis effect. Due to low complexity and high accuracy, the proposed method can be used in real-time embedded battery management systems for various applications, such as EVs and PHEVs.

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