Soft-landing Control by Control Invariance and Receding Horizon Control

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I. Introduction

A significant number of control applications in the fields of automotive, aerospace, manufacturing, and transportation engineering involve the precise positioning of a moving object at a desired stopping location while ensuring that the velocity of the objects (or at least its allowed upper bound) is progressively reduced while approaching the stopping position. This results in soft landing (also called soft contact) of the object at the desired stopping position that induces robustness to disturbances and modeling errors. A classical example of soft landing control is the closing of a valve into its seating, which has found application in automotive actuators, especially for camless engines valve control [1] where high speed opening and closing of the valve is needed without rough impacts that reduce the components operating life. Besides valve control, some important problems that can be formulated as soft landing control are, among others, vehicle stopping [2] and spacecraft docking [3].

The soft landing requirements can be formulated in terms of constraints that relate the bounds on the allowed object velocity to the object position, and hence constrained control techniques can be used to design a controller that solves the soft landing problem. An early application of constrained control to soft landing control was based on reference governor [4], which can be guaranteed to satisfy constraints. On the other hand the reference governor has limited performance since it only manipulates the reference of a (linearly) pre-compensated system.

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More recently, approaches based on model predictive control (MPC) have been developed and applied to valve soft landing control in camless [5], [6] engines. Soft landing for spacecraft docking by MPC was proposed in [7], [8].

While MPC has increased capabilities when compared to reference governor, its design is significantly more complicated. This resulted in the approaches in [5], [7] to require prediction horizon and cost function tuning, and characterization of MPC recursive feasibility by extensive simulations. Also, the capabilities of MPC in dealing with uncertainty in the system parameters are still limited.

In this paper, we propose a soft landing control design based on model predictive control and control invariant sets which allows to characterize the regions where recursive feasibility is guaranteed regardless of the cost function and prediction horizon of MPC, and that can be applied also in the case of parameter uncertainties. In Section II we formulate the soft landing problem and we review some fundamental results in control invariant sets [9]. In Section III we show how by formulating soft landing constraints, the solution of the soft landing problem amounts to generating an infinite time admissible trajectories for a constrained system. In order to generate such trajectories, we compute a control invariant set and use it as additional constraints in the MPC optimal control problem. In Section IV we show how the approach can be extended to polytopic difference inclusions [10] to handle uncertainty in system dynamics. In Section V the approach is demonstrated through a test case involving an automated transportation vehicle stopping control. The conclusions and future directions are summarized in Section VI.

Notation: $\mathbb{R}$, $\mathbb{R}_{0+}$, $\mathbb{R}_+$ are the real, nonnegative real, positive real numbers, and $\mathbb{Z}$, $\mathbb{Z}_{0+}$, $\mathbb{Z}_+$ are the integer, nonnegative integer, positive integer numbers. By co and proj_x we denote the convex hull and the projection on the domain of vector x, respectively. Given sets $\mathcal{S}$, $\mathcal{P}$, $\mathrm{int}(\mathcal{S})$ denotes the interior, and $\mathcal{S} \oplus \mathcal{P}$ the Minkowski sum. For a discrete-time signal $x \in \mathbb{R}^n$ with sampling period T_s, $x(t)$ is the state at sampling instant t, i.e., at time $T_s t$. By $[x]_i$ we denote the i-th component of x, and by I and 0 the identity and the "zero" matrices of appropriate size.

II. Problem Formulation and Preliminaries

First we define the problem addressed in this paper and we review some preliminary results in control invariant sets that are used for the subsequent developments.

A. Problem definition

We consider a physical object moving along a one dimensional space towards a target position, where the object dynamics are defined by

$$\ddot{d}(t) = \dot{v}(t) = \frac{1}{m}\sum_{i=1}^{n_f} F_i(t) \tag{1}$$

where d is the position with respect to the target, v is the velocity, F_i, $i = 1, \ldots, n_f$ are the external forces acting on the object, including the controlled ones. We consider the dynamics (1) modeled as

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t), \vartheta(t)) && \text{(2a)} \\ y(t) &= h(x(t)), && \text{(2b)} \end{aligned}$$

where f and h are smooth functions, $x \in \mathcal{X} \subseteq \mathbb{R}^{n_x}$ is the state vector, $u \in \mathcal{U} \subseteq \mathbb{R}^{n_u}$ is the control input vector, $y = [d\ v]' \in \mathcal{Y} \subseteq \mathbb{R}^2$ is the (performance) output vector, and $\vartheta \in \mathcal{O} \subseteq \mathbb{R}^{n_\vartheta}$ is the disturbance vector which contains measured and unmeasured disturbances, and (possibly) parametric uncertainties. Note that (2) may include additional dynamics related to the forces $F_i(t)$, $i = 1, \ldots, n_f$, such as the ones due to actuator dynamics, dynamical friction models, and so on.

In what follows, we often separate two cases, one where $\vartheta(t) = 0$, for all $t \in \mathbb{R}_{0+}$, which means that there is no uncertainty, and one where uncertainty is present. For the simplicity of the subsequent discussion (1) represents a one-dimensional motion, but all the results can be easily extended to multi-dimensional motions.

The objective of the soft landing controller is to bring the system described by (1) to zero velocity in a pre-specified region $\mathcal{E}_{\rm tgt} \equiv [\varepsilon_{\min}, \varepsilon_{\max}] \subseteq \mathbb{R}$, $\varepsilon_{\min} \leq \varepsilon_{\max}$. Furthermore, it is required that the maximum velocity of the object towards the target decreases with the distance from the target. For simplicity, we assume without loss of generality that $0 \in {\rm int}(\mathcal{E}_{\rm tgt})$ and that $d(0) < 0$, $v(0) > 0$.

Problem 1 (Soft landing control): Given system (2) and an initial state $x(0)$ such that $d(0) < 0$, $v(0) > 0$, control the input u such that for all $t \in \mathbb{R}_{0+}$, $u(t) \in \mathcal{U}$, $x(t) \in \mathcal{X}$ and $v(t) \leq v_{\max}(|\xi(t)|)$ where $\xi(t) = d(t) - \arg\max_{z \in \mathcal{E}_{\rm tgt}} |z - d(0)|$ and $\frac{d}{d\xi} v_{\max}(|\xi|) < 0$. Also, there needs to be a time instant $\bar{t} \in \mathbb{R}_+$ when $v(\bar{t}) = 0$, $x(\bar{t}) \in \mathcal{E}_{\rm tgt}$. ■

With a little abuse of notation, by $\bar{t} = \infty$ we mean that the soft landing occurs asymptotically and that is an acceptable solution. Also, ξ represents the distance from the farthest point of the target set from the initial condition. Finally, the target set, $\mathcal{E}_{\rm tgt}$, is a generalization of the case where $\mathcal{E}_{\rm tgt}$ is a single point.

Problem 1 does not require the state to reach an equilibrium at the landing instant $\bar{t}$. This because in several practical problems at landing the dynamics of the system changes, such as the transition from dynamic to static friction in moving vehicles and valves, the engagement of parking brakes in elevators, or the engagement of clamps in spacecraft docking.

Finally, if (2) includes disturbances, Problem 1 implicitly requires the controller to robustly satisfy the constraints.

B. Preliminaries on control invariant sets and polytopic linear difference inclusions

Consider the discrete-time system

$$x(k+1) = f_d(x(k), u(k), \vartheta(k)), \tag{3}$$

where $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$ and $\vartheta \in \mathbb{R}^{n_\vartheta}$ are the state, input and disturbance vectors, respectively, subject to the constraints

$$x(k) \in \mathcal{X},\ u(k) \in \mathcal{U},\ \forall k \in \mathbb{Z}_{0+}. \tag{4}$$

Given the set of admissible disturbances. $\mathcal{O} \subseteq \mathbb{R}^{n_\vartheta}$, a *robust control invariant (RCI) set* is a set of states for which there exists a control law such that (3) never violates (4) for any admissible sequence of disturbances $\{\vartheta(k)\}_k$, where $\vartheta(k) \in \mathcal{O}$ for all $k \in \mathbb{Z}_{0+}$.

Definition 1: A set $\mathcal{C} \subseteq \mathcal{X}$ is said to be a robust control invariant set for (3) if

$$\begin{aligned} & x(k) \in \mathcal{C} \Rightarrow \exists u(k) \in \mathcal{U} : \\ & \qquad f_d(x(k), u(k), \vartheta(k)) \in \mathcal{C},\ \forall \vartheta(k) \in \mathcal{O},\ \forall k \in \mathbb{Z}_{0+}. \end{aligned} \tag{5}$$

The set $\mathcal{C}$ is said to be the *maximal* RCI (mRCI) set, denoted by $\mathcal{C}^\infty$, if it is a robust control invariant set and contains all the other RCI sets in $\mathcal{X}$.

The computation of RCI sets relies on the Pre-set operator

$$\begin{aligned} {\rm Pre}(\mathcal{S}, \mathcal{O}) \triangleq \{& x \in \mathcal{X} \ :\ \exists u \in \mathcal{U} \\ & f_d(x, u, \vartheta) \in \mathcal{S},\ \forall \vartheta \in \mathcal{O}\}, \end{aligned} \tag{6}$$

which computes the set of states for the system (3) that can be robustly driven to the target set $\mathcal{S} \subseteq \mathbb{R}^n$ in one step.

The procedure to compute the mRCI set for system (3), subject to constraints (4), and based on the operator (6) is described in Algorithm 1.

Algorithm 1 Computation of $\mathcal{C}^\infty$

1) $\Omega_0 \leftarrow \mathcal{X}$, $h \leftarrow 0$
2) $\Omega_{h+1} \leftarrow {\rm Pre}(\Omega_h, \mathcal{O}) \cap \Omega_h$
3) `If` $\Omega_{h+1} = \Omega_h$
   $\mathcal{C}^\infty \leftarrow \Omega_{h+1}$, `return`
4) $h \leftarrow h + 1$, `goto` 2.

Algorithm 1 generates the sequence of sets $\{\Omega_h\}_{h=0}^{\bar{h}}$, $\bar{h} \in \mathbb{Z}_{0+}$, satisfying $\Omega_{h+1} \subseteq \Omega_h$, for all $h \in \mathbb{Z}_{0+}$, $h \leq \bar{h}$. Algorithm 1 terminates if $\Omega_{h+1} = \Omega_h$, and in this case Ω_h is the mRCI set $\mathcal{C}^\infty$ for (3) subject to (4). We refer the reader to [9], [11] for details on the termination of Algorithm 1.

Definition 2 (Robustly admissible input set for $\mathcal{C}$): Given a robust control invariant set $\mathcal{C}$ for (3)-(4), the robustly admissible input (RAI) set for state $x \in \mathcal{C}$ is

$$\mathcal{C}_u(x) = \{u \in \mathcal{U} : f(x, u, \vartheta) \in \mathcal{C},\ \forall \vartheta \in \mathcal{O}\}.$$

For $x \in \mathcal{C}^\infty$, we denote the RAI set by $\mathcal{C}_u^\infty(x)$.

Definition 3: If system (3) is not subject to uncertainty (i.e., $\mathcal{O} = \{0\}$), the set $\mathcal{C}$ in Definition 1 is simply called *control invariant (CI) set*.

Definition 4: Given $\ell, \eta \in \mathbb{Z}_{0+}$, $A_i \in \mathbb{R}^{n_x \times n_x}$, $B_i \in \mathbb{R}^{n_x \times n_u}$, $i = 1, \ldots, \ell$ and a set $\mathcal{W} \equiv \text{co}\{w_i, \ldots, w_\eta\} \subset \mathbb{R}^{n_w}$ a (disturbed) polytopic linear difference inclusion (pLDI) is

$$x(k+1) \in (\text{co}\{A_i x(k) + B_i u(k)\}_{i=1}^{\ell} \oplus \text{co}\{B_w w_i\}_{i=1}^{\eta}) \quad (7)$$

where $x \in \mathcal{X} \subseteq \mathbb{R}^{n_x}$ is the state vector, $u \in \mathcal{U} \subseteq \mathbb{R}^{n_u}$ is the input vector, and $w \in \mathcal{W} \subseteq \mathbb{R}^{n_w}$ is the disturbance vector.

By convexity arguments, (7) can be written as

$$x(k+1) = \sum_{i=1}^{\ell} \lambda_i (A_i x(k) + B_i u(k)) + \sum_{i=1}^{\eta} \mu_i B_w w_i \quad (8a)$$

$$\sum_{i=1}^{\ell} \lambda_i = 1, \; \lambda_i \geq 0, \; i = 1, \ldots, \ell \quad (8b)$$

$$\sum_{i=1}^{\eta} \mu_i = 1, \; \mu_i \geq 0, \; i = 1, \ldots, \eta \quad (8c)$$

where $\Lambda = [\lambda_1, \ldots, \lambda_\ell]$, $M = [\mu_1, \ldots, \mu_\eta]$ are unknown (and possibly varying) but satisfy (8b), (8c), respectively.

Definition 5: For the pLDI (7) subject to the polytopic constraints $x \in \mathcal{X}$, $u \in \mathcal{U}$ and with a polytopic disturbance set $\mathcal{W}$, the RCI set (Definition 1) is

$$\begin{aligned} \mathcal{C} = \{ & x \in \mathcal{X} : \; \exists u \in \mathcal{U}, (A_i x + B_i u + B_w w) \in \mathcal{C}, \\ & \forall i = 1, \ldots, \ell, \forall w \in \mathcal{W}\}. \end{aligned} \quad (9)$$

III. Constrained Control for Soft Landing

Next we reformulate Problem 1 as the control of a constrained system and we show that trajectories that satisfy the constraints achieve soft landing. To this end we define the soft-landing constraints

$$v(t) \leq \gamma_{\max}(\varepsilon_{\max} - d(t)) \quad (10a)$$

$$v(t) \geq \gamma_{\min}(\varepsilon_{\min} - d(t)) \quad (10b)$$

where $\gamma_{\min}, \gamma_{\max} \in \mathbb{R}_+$, $\gamma_{\min} < \gamma_{\max}$, are spatial deceleration coefficients. See Figure 1 for a representation of constraints (10).

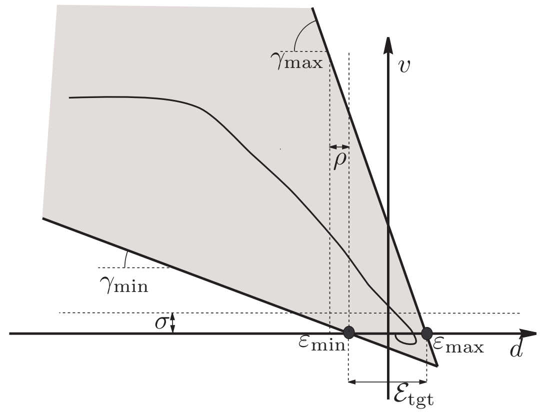

Fig. 1: Schematics of the soft landing constraints

Definition 6: Given system $\dot{x} = f(x)$, $y = h(x)$ subject to the constraints $x \in \mathcal{X}$ and $y \in \mathcal{Y}$, a trajectory $x(\cdot) : \mathbb{R}_{0+} \to \mathbb{R}^{n_x}$ is admissible for $t \in [0, \infty)$, (or simply, *admissible*, for shortness) if $x(t) \in \mathcal{X}$, $y(t) \in \mathcal{Y}$ for all $t \in [0, \infty)$.

The following result holds.

Theorem 1: Consider the constrained system obtained from (2) and (10),

$$\dot{x}(t) = f(x(t), u(t), \vartheta(t)) \quad (11a)$$

$$y(t) = \begin{bmatrix} d(t) \\ v(t) \end{bmatrix} = h(x(t)) \quad (11b)$$

$$[y(t)]_2 \leq \gamma_{\max}(\varepsilon_{\max} - [y(t)]_1) \quad (11c)$$

$$[y(t)]_2 \geq \gamma_{\min}(\varepsilon_{\min} - [y(t)]_1) \quad (11d)$$

$$x(t) \in \mathcal{X}, \; u(t) \in \mathcal{U} \quad (11e)$$

Any admissible trajectory of (11) is a solution of Problem 1.

Proof (sketch): For the considered case $d(0) < \varepsilon_{\min} < 0$, (10a) enforces $v(t) \leq v_{\max}(|\xi(t)|)$. An admissible trajectory of (11) enforces that for all $t \in \mathbb{R}_{0+}$, $x(t) \in \mathcal{X}$, $u(t) \in \mathcal{U}$, and (10a) is satisfied. Finally, we show that any admissible trajectory must (eventually) reach a state x such that $[h(x)]_1 = d \in \mathcal{E}_{\text{tgt}}$, $[h(x)]_2 = v = 0$. Due to (10b), for every x such that $[h(x)]_1 < \varepsilon_{\min}$, $v = [h(x)]_2 > 0$. On the other hand for every x such that $[h(x)]_1 > \varepsilon_{\max}$, $v = [h(x)]_2 < 0$. By the continuity of the admissible trajectories of (11) and since $\dot{d}(t) = v(t)$, we have that any admissible trajectory must intersect or accumulate at a point of the set $\{x \in \mathbb{R}^{n_x} : [h(x)]_1 \in \mathcal{E}_{\text{tgt}}, [h(x)]_2 = 0\}$. ■

Remark 1: By allowing $\mathcal{E}_{\text{tgt}}$ to be a set, we obtain a more general form of the problem of soft landing to a specific point. In general, $\mathcal{E}_{\text{tgt}}$ is small and it can also be a single point. However, choosing $\mathcal{E}_{tgt}$ as a single point as opposed to a set forces the desired state to the border of the feasible set of (10). This may cause robustness problems and also make the problem infeasible [7], especially in the presence of disturbances.

The following result also holds.

Corollary 1: Given any $\rho > 0$, $d(0) < \varepsilon_{\min} - \rho < 0$ (see Figure 1), for any admissible trajectory of (11) there exists $\hat{t} \in \mathbb{R}_+$ such that $d(\hat{t}) = [h(x(\hat{t})]_1 = \varepsilon_{\min} - \rho$ and

$$\hat{t} \in \left[\frac{1}{\gamma_{\max}} \log\left(\frac{\varepsilon_{\max} - d(0)}{\Delta\varepsilon + \rho} \right), \frac{1}{\gamma_{\min}} \log\left(\frac{\varepsilon_{\min} - d(0)}{\rho} \right) \right] \quad (12)$$

where $\Delta\varepsilon = \varepsilon_{\max} - \varepsilon_{\min} > 0$.

The proof is omitted due to limited space and it is based on computing two extremal trajectories "riding" the constraints. Given any $\sigma > 0$, from any $d(0) < \varepsilon_{\min} - \sigma/\gamma_{\min}$ time bounds for reaching velocity σ (see Figure 1) can also be obtained.

By Theorem 1, solving Problem 1 amounts to generating an admissible trajectory for $t \in [0, +\infty)$ for system (11). In order to generate such an admissible trajectory, several constrained control techniques can be applied. In this paper we consider numerical algorithms that restricts the system to be linear and discrete time, hence relaxing the admissibility of the continuous time trajectory to the admissibility of the discrete time trajectory. Thus, it is assumed that the sampling period T_s is small enough not to cause significant constraints violations in the intersamping.

Next, we propose a strategy based on MPC and control invariant sets that overcomes some of the limitations of the current approaches, such as the limits in manipulating only a reference for the reference governor and the guaranteed feasibility for standard MPC, and that is capable of handling uncertainties in the model parameters.

A. Soft landing control by control invariant sets

Due to Theorem 1, Problem 1 is solved by generating admissible trajectories for (11). A method to achieve that is to compute a control invariant set for the constrained system and to maintain the state in such a set, which is always possible if the set is control invariant.

While the approach described next applies to general nonlinear systems, here we consider (2) to be a discrete-time linear system,

$$\begin{aligned} x(k+1) &= A(p)x(k) + B(p)u(k) + B_w w(k) && \text{(13a)} \\ y(k) &= Cx(k) && \text{(13b)} \end{aligned}$$

where $x \in \mathcal{X} \subseteq \mathbb{R}^{n_x}$, $u \in \mathcal{U} \subseteq \mathbb{R}^{n_u}$, $y = [d\ v]' \in \mathcal{Y} \subseteq \mathbb{R}^2$, $w \in \mathcal{W} \subseteq \mathbb{R}^{n_w}$ is the additive disturbance, and $p \in \mathcal{P} \subseteq \mathbb{R}^{n_p}$ is the parameter vector which represents uncertainty in the plant[1]. It is assumed that $\mathcal{X}$, $\mathcal{U}$, $\mathcal{W}$ are polytopes. First, we assume that there are no disturbances, i.e., $\mathcal{W} \equiv \{0\}$, and no uncertainties, i.e., p is known and does not change, so that for simplicity $A = A(p)$, $B = B(p)$.

Let $\bar{\mathcal{X}} = \{x \in \mathcal{X} :\ \text{(11) holds for } y = Cx\}$, $\mathcal{C}$ be a control invariant set of (13), subject to $x \in \bar{\mathcal{X}}$, $u \in \mathcal{U}$, and $\mathcal{C}_u(x) \subseteq \mathcal{U}$ be the corresponding RAI set

$$\begin{aligned} \mathcal{C} &= \{x \in \bar{\mathcal{X}} :\ \exists u \in \mathcal{U},\ Ax + Bu \in \mathcal{C}\} && \text{(14)} \\ \mathcal{C}_u(x) &= \{u \in \mathcal{U} :\ Ax + Bu \in \mathcal{C}\}. && \text{(15)} \end{aligned}$$

Obviously, $\mathcal{C} \subseteq \bar{\mathcal{X}}$. Thus, if $x(0) \in \mathcal{C}$ and for all $k \in \mathbb{Z}_{0+}$, $u(k) \in \mathcal{C}_u(x(k))$, $x(k) \in \bar{\mathcal{X}}$ for all $k \in \mathbb{Z}_{0+}$.

For system (13) with no uncertainties, by Algorithm 1 the *maximal* CI (mCI) set, $\mathcal{C}^\infty$, can be computed [9].

Given $\mathcal{C}^\infty \equiv \{x \in \mathbb{R}^{n_x} : H^\infty x \leq K^\infty\}$ we obtain also the RAI set $\mathcal{C}_u^\infty(x) = \{u \in \mathcal{U} : Ax + Bu \in \mathcal{C}^\infty\} = \{u \in \mathbb{R}^{n_u} :\ H_u^\infty x + J_u^\infty u \leq K_u^\infty\}$. Given the state $x \in \mathcal{C}^\infty$, consider the finite time optimal control problem

$$\begin{aligned} \min_U \quad & F(x_N) + \sum_{i=0}^{N-1} L(x_i, u_i) && \text{(16a)} \\ \text{s.t.} \quad & x_{i+1} = Ax_i + Bu_i && \text{(16b)} \\ & H^\infty(Ax_i + Bu_i) \leq K^\infty && \text{(16c)} \\ & u_i \in \mathcal{U} && \text{(16d)} \\ & x_0 = x && \text{(16e)} \end{aligned}$$

where $N \in \mathbb{Z}_+$ is the prediction horizon, $U = [u_0, \ldots, u_{N-1}]$ is the control sequence, and F, L, are the final and stage cost, respectively. The following result holds.

Theorem 2: Consider system (13) where $\mathcal{W} = \{0\}$ and p is known, and let $x(0) \in \mathcal{C}^\infty$. Let the control input be chosen so that at any $k \in \mathbb{Z}_{0+}$, $u(k) = \tilde{u}_0$, where $\tilde{U}$ is any feasible (yet not necessarily optimal) solution of (16), for $x = x(k)$. Then, the obtained trajectory solves Problem 1. ■

The proof is omitted due to limited space, and follows directly from using the CI set constraint (16c).

Thus, the MPC strategy based on (16) generates trajectories that solve Problem 1. Next, we consider the case where the physical system model is not exactly known or its linear model is an approximation of nonlinear dynamics.

IV. Robust Soft-landing Control

Consider the case where system (2) is subject to uncertainty. Let f_d be the discrete time formulation of the dynamics (2) and let $\ell, \eta \in \mathbb{Z}_+$, a set of matrices $\{(A_i, B_i)\}_{i=1}^{\ell}$, a matrix $B_w \in \mathbb{R}^{n_x \times n_w}$, and a polytope $\mathcal{W} = \text{co}\{w_1, \ldots, w_\eta\} \subset \mathbb{R}^{n_w}$ exist, such that for all $x \in \mathcal{X}$, $u \in \mathcal{U}$, $\vartheta \in \mathcal{O}$

$$f_d(x, u, \vartheta) \in \text{co}(\{A_i x + B_i u\}_{i=1}^{\ell}) \oplus \text{co}(\{B_w w_i\}_{i=1}^{\eta}),$$

In addition, for all $x \in \mathcal{X}$, $u \in \mathcal{U}$, $\vartheta \in \mathcal{O}$, let $y = Cx$ for some matrix $C \in \mathbb{R}^{2 \times n_x}$. Then, we reformulate controlling (11) as controlling the constrained polytopic linear difference inclusion .

$$\begin{aligned} x(k+1) &\in \text{co}(\{A_i x(k) + B_i u(k)\}_{i=1}^{\ell}) \\ &\qquad \oplus \text{co}(\{B_w w_i\}_{i=1}^{\eta}) && \text{(17a)} \\ y(k) &= Cx(k) && \text{(17b)} \\ [y(k)]_2 &\leq \gamma_{\max}(\varepsilon_{\max} - [y(k)]_1) && \text{(17c)} \\ [y(k)]_2 &\geq \gamma_{\min}(\varepsilon_{\min} - [y(k)]_1) && \text{(17d)} \\ & x(k) \in \mathcal{X},\ u(k) \in \mathcal{U} && \text{(17e)} \end{aligned}$$

We can repeat the approach of Section III, whereas now we compute a RCI set for (17). For such computation we implement the Pre-set operator as defined in Algorithm 2.

Algorithm 2 $\text{Pre}(\Omega_h, \mathcal{O})$ computation for (17)

1) $\Omega_h \equiv \{x \in \mathbb{R}^{n_x} : H^{(h)} x \leq K^{(h)}\}$, $K^{(h)} \in \mathbb{R}^{n_q^{(h)}}$
   $\mathcal{U} = \{u \in \mathbb{R}^{n_u} :\ H_u u \leq K_u\}$,
2) $[\tilde{K}^{(h)}]_i = [K^{(h)}]_i - \max_{w \in \mathcal{W}} [H^{(h)} B_w w]_i$,
   $i = 1, \ldots, n_q^{(h)}$
3) $\bar{\Omega}_{h+1} = \{x \in \mathbb{R}^{n_x}, u \in \mathbb{R}^{n_u} :\ H_u u \leq K_u$
   $H^{(h)}(A_i x + B_i u) \leq \tilde{K}^{(h)}, i = 1, \ldots, \ell\}$
4) $\Omega_{h+1} = \text{proj}_x(\bar{\Omega}_{h+1}) \cap \Omega_h$

When Algorithm 1 with the Pre-set operator implemented as in Algorithm 2 is applied to (17) terminates, we obtain the RCI set $\mathcal{C}^\infty \equiv \{x \in \mathbb{R}^{n_x} : H^\infty x \leq K^\infty\}$ and, from that, the RAI set $\mathcal{C}_u^\infty(x) = \{u \in \mathcal{U} : A_i x + B_i u + B_w w \in \mathcal{C}^\infty, \forall i = 1, \ldots, \ell, \forall w \in \mathcal{W}\} = \{u \in \mathbb{R}^{n_u} : H_u^\infty x + J_u^\infty u \leq K_u^\infty\}$.

Remark 2: The set $\mathcal{C}_u^\infty = \{(x, u) \in \mathbb{R}^{n_x + n_u} : u \in \mathcal{C}_u^\infty(x)\}$ is equal to $\bar{\Omega}_{\bar{h}+1}$ where $\bar{h} \in \mathbb{Z}_{0+}$ is the smallest index such that $\Omega_{\bar{h}+1} = \Omega_{\bar{h}}$ in Algorithm 1

Given $x \in \mathcal{C}^\infty$, consider the finite time optimal control

[1] Relating (13) to (2), we have $[w'\ p']' = \vartheta$.

problem

$$\min_{U} \quad F(x_N) + \sum_{i=0}^{N-1} L(x_i, u_i) \tag{18a}$$

$$\text{s.t.} \quad x_{i+1} = \bar{A}x_i + \bar{B}u_i + B_w\bar{w} \tag{18b}$$

$$H_u^\infty x_i + J_u^\infty u_i \leq K_u^\infty \tag{18c}$$

$$u_i \in \mathcal{U} \tag{18d}$$

$$x_0 = x \tag{18e}$$

where $N \in \mathbb{Z}_+$ is the prediction horizon, $U = [u_0, \ldots, u_{N-1}]$ is the control sequence, F, L, are the final and stage cost, respectively, $(\bar{A}, \bar{B}) \in \mathrm{co}(\{(A_i, B_i)\}_{i=1}^{\ell}$, and $\bar{w} \in \mathcal{W}$. Then, the following result holds.

Theorem 3: Consider (17) and let $x(0) \in \mathcal{C}^\infty$. Let the control input be chosen so that at any $k \in \mathbb{Z}_{0+}$, $u(k) = \tilde{u}_0$, where $\tilde{U}$ is any feasible (yet not necessarily optimal) solution of (18) for $x = x(k)$. Then, we obtain a trajectory that solves Problem 1. ■

The proof is omitted due to limited space, and follows directly from using the RCI set constraint (18c).

Remark 3: The prediction model can be any model in the difference inclusion. Convergence is independent of the chosen model, although the closed-loop performance is not.

Remark 4: Constraint (18c) can be enforced at the first step of the prediction horizon only. Since $\mathcal{C}_\infty$ is invariant, at the subsequent control step a feasible input will still exist.

V. Case study in Transportation Systems

We consider a large automated transportation vehicle moving on a straight line that has to stop in $\mathcal{E}_{\mathrm{tgt}} \equiv [-0.5, 0.5]$m. The dynamics of the vehicle are

$$\dot{d}(t) = v(t), \tag{19a}$$

$$\dot{v}(t) = \frac{k_a}{rm}\chi(t) - \frac{c_0\mu g}{m} - \frac{c_1}{m}v(t), \tag{19b}$$

$$\dot{\chi}(t) = -\frac{1}{\tau_a}\chi(t) + \frac{1}{\tau_a}u(t), \tag{19c}$$

where d[m], v[m/s], and χ are position, velocity, and traction actuator state, respectively. Equations (19a) and (19b) describe the system's longitudinal dynamics, where m[kg] is the vehicle mass, r[m] is the wheel radius, and k_a[Nm] is the traction actuator gain. In (19b) the resisting forces to the vehicle motion are the rolling resistance and the bearing friction, modeled through the coefficients c_0, c_1, respectively. For the conditions considered in this application the airdrag is small, and hence it is ignored (or linearized and included in c_1). Equation (19c) models the traction actuator dynamics. We consider also the case where (19) is affected by uncertainty in the bearing friction, mass, and actuator time constant, that is $m \in [(1-\delta_m)\,\bar{m}, (1+\delta_m)\,\bar{m}]$, $c_1 \in [(1-\delta_{c1})\,\bar{c}_1, (1+\delta_{c1})\,\bar{c}_1]$, $\tau_a \in [(1-\delta_{\tau a})\ \bar{\tau}_a, (1+\delta_{\tau a})\,\bar{\tau}_a]$, where δ_m, δ_{c1}, $\delta_{\tau a}$ define the relative uncertainty.

We obtain the model for control design from (19), that formulated in discrete-time with $T_s = 1$s, and augmented with a constant state ζ, representing the effect of the rolling resistance as a measured disturbance, i.e., $\zeta(k) = c_0\mu g/m$, for all $k \in \mathbb{Z}_{0+}$ and another state representing a 1-step delayed velocity. For the case of no uncertainty $\delta_j = 0$, for $j \in \{m, c1, \tau a\}$, the resulting discrete state model is the linear system (13) with known p where $x \in \mathbb{R}^5$, $x(k) = [d(k)\ v(k)\ \chi(k)\ \zeta(k)\ v(k-1)]'$ and $y \in \mathbb{R}^2$, $y = [d\ v]'$, and $w \in [-\delta_w(c_0\mu g/m), +\delta_w(c_0\mu g/m)]$, w[m/s^2] is an additive bounded disturbance that models also uncertainty in the rolling resistance (especially due to the friction coefficient μ) and δ_w expresses the relative uncertainty.

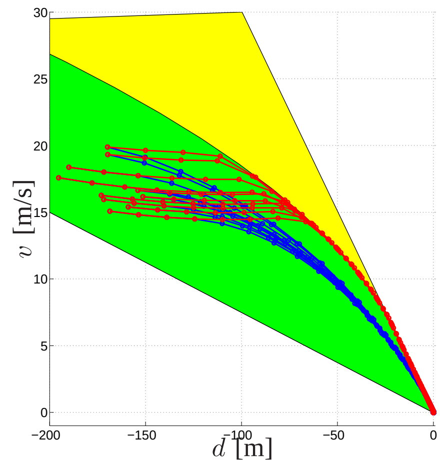

Fig. 2: Control invariant set for the nominal vehicle model

We consider upper and lower bounds on the states, $d \in [-200, 1]$, $v \in [-5, 30]$, $\chi \in [-1, 1]$, and on the input $u \in [-1, 1]$, and the *quasi-monotonicity* velocity constraint,

$$v(k) \leq v(k-1) + \gamma_v(d(k) - d_{\min}) \tag{20}$$

where $d_{\min} = -200$ and $\gamma_v > 0$ is the monotonicity relaxation gain. Constraint (20) ensures that when $d = d_{\min} = -200$ the vehicle can only decelerate, and progressively relax this requirement γ_v as the vehicle gets closer to the stopping point to counteract disturbances.

We formulate (16) with (16a) as

$$J = \sum_{i=0}^{N-1} [y_i]_1 Q [y_i]_1 + u_i R u_i \tag{21}$$

where $Q, R \in \mathbb{R}_{0+}$ are weighting matrices, $N = 5$, and we compute the mCI set $\mathcal{C}^\infty$ using Algorithm 1. In Figure 2 we show $\mathrm{proj}_y(\mathcal{C}^\infty)$, superimposed to $\mathrm{proj}_y(\bar{\mathcal{X}}) \supseteq \mathrm{proj}_y(\mathcal{C}^\infty)$.

In Figure 2 we also show the closed-loop trajectories of position and velocity obtained from 10 random initial conditions where from each we generated trajectories for 2 different calibrations of the cost function (21), $Q = 1$, $R = 1$ (red) and $Q = 0$, $R = 1$ (blue). The trajectories obtained for $Q = 1$ evolve close to the highest velocity allowed by the control invariant set. The time-histories for position, velocity, and input for all the simulations are reported in Figure 3.

Next, we consider the case when there is uncertainty in the parameters, namely $\delta_m = \delta_{\tau a} = 0.25$, $\delta_{c1} = 0.20$, $\delta_w = 0.15$. We model the system as a pLDI (17) which

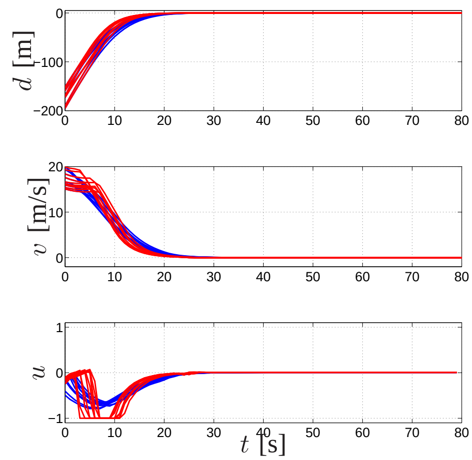

Fig. 3: Position, velocity, and input time history for the simulations of the nominal vehicle

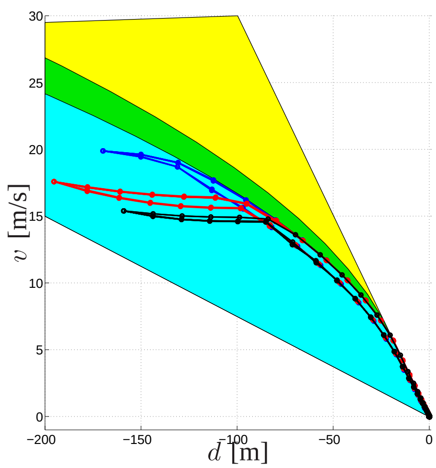

Fig. 4: Robust control invariant set for the uncertain model of the vehicle

requires $\ell = 8$, $\eta = 2$, and we compute the robust control invariant set $\tilde{\mathcal{C}}^{\infty}$ using Algorithm 1 and Algorithm 2. In Figure 4 we show the $\mathrm{proj}_y(\tilde{\mathcal{C}}^{\infty})$ for the case of a system subject to uncertainty, superimposed to $\mathrm{proj}_y(\mathcal{C}^{\infty})$, and to $\mathrm{proj}_y(\bar{\mathcal{X}})$. Indeed, the RCI set is smaller than the nominal CI set. In Figure 4 we also show the trajectories of position and velocity obtained from 3 initial conditions, each with 10 values of the parameters randomly chosen at their maximum or minimum. The control input is generated as described in Section IV with cost function (21) for $Q = 1$, $R = 1$, with a fixed prediction model being always the same vertex of the pLDI. The trajectories tend to group into two bundles, which correspond to the minimum and maximum value of the mass, which is the parameter with highest sensitivity. The position, velocity, and input time-histories are presented in Figure 5.

VI. Conclusions

We have presented a design for general soft-landing controllers. The approach reduces the problem to the generation

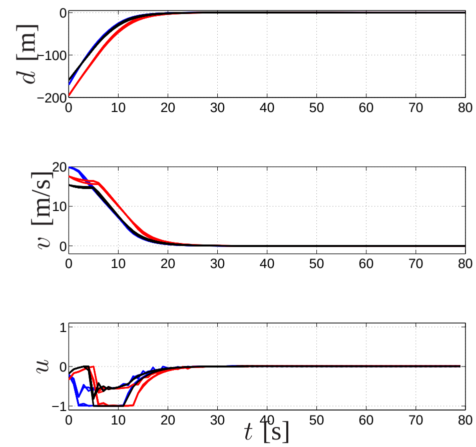

Fig. 5: Position, velocity, and input time history for the simulations of the uncertain model of the vehicle

of admissible time-unbounded trajectories of a constrained system, compute a control invariant set for the constrained system and implement an MPC to maintain the state in the set. By using polytopic linear difference inclusions, for which robust control invariant sets can be computed, the approach is applicable also to uncertain systems. We have demonstrated our approach through a case study in transportation systems.

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