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Inverter Interfaced Distributed Generation (DG) in the Smart Grid has the potential to contribute to the reactive power support of the overall power system. However, because of the large numbers and distributed nature of the DG units, using a fully centralized communication structure to achieve coordination can be prohibitive. A decentralized coordination approach is a good candidate solution to address this problem. In the literature, asymptotic consensus based algorithms have been proposed, in order to coordinate a set of DG units so that they collectively provide a certain reference reactive power. This paper presents an alternative decentralized coordination methodology that achieves the same objective in finite time. The protocol is based on linear iterative updates and known observability results from graph structured linear systems. In this paper, the methodology is customized to solve the reactive support coordination problem from distribution-connected inverters and it is modified to reduce the number of operations per step, ensuring applicability to a large distribution network. The IEEE 37-node test feeder is used as a test system, with added inverter interfaced generation in each node. For this sample system, the proposed approach is shown to coordinate the nodes faster than the asymptotic consensus approach.

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Finite Time Multi-Agent Coordination of Distributed Generation for Grid Reactive Support

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Abstract—Inverter Interfaced Distributed Generation (DG) in the Smart Grid has the potential to contribute to the reactive power support of the overall power system. However, because of the large numbers and distributed nature of the DG units, using a fully centralized communication structure to achieve coordination can be prohibitive. A decentralized coordination approach is a good candidate solution to address this problem. In the literature, asymptotic consensus based algorithms have been proposed, in order to coordinate a set of DG units so that they collectively provide a certain reference reactive power. This paper presents an alternative decentralized coordination methodology that achieves the same objective in finite time. The protocol is based on linear iterative updates and known observability results from graph structured linear systems. In this paper, the methodology is customized to solve the reactive support coordination problem from distribution-connected inverters and it is modified to reduce the number of operations per step, ensuring applicability to a large distribution network. The IEEE 37-node test feeder is used as a test system, with added inverter interfaced generation in each node. For this sample system, the proposed approach is shown to coordinate the nodes faster than the asymptotic consensus approach.

Index Terms—Distributed algorithm, reactive power control, smart-grid control.

I. INTRODUCTION

Moving towards the Smart Grid, it has been recognized ([1], [2]) that distributed resources have the potential to provide a wide range of valuable grid ancillary services, such as voltage support in the distribution system and/or active power reserves [3]. The future grid is expected to be characterized by increased participation of distributed agents in the control procedure [4]. In this context, it becomes increasingly important to develop viable procedures to monitor and control the numerous devices that are projected to be involved in the control flow.

The potential of grid-connected Distributed Generation (DG) units to provide grid support has been well documented [1],[7]. The question arise as to how a very large number of DG units will be controlled in order to collectively provide a desired level of grid support. In the literature, Multi-Agent Systems (MAS), consensus-based control theory has been extensively studied for the control of distributed systems, e.g. [23], [6]. Such approaches have been utilized in the literature for Smart-Grid applications: [8] discusses a class of consensus based linear iterative algorithms to be used for Distributed Energy Resources (DER) coordination and [9] presents an extension of that work for asynchronous communication between the DER's. [10] presents a hierarchical approach for controlling DER's using MAS theory, en route to achieving the Virtual Power Plant (VPP) concept. According to this paradigm, clusters of DER's will be able to collectively provide the same control functions or services actual power plants provide today. The feasibility of using distributed inverters for consensus-based reactive power support was

first proposed in [11], while a voltage control scheme is analyzed in [12].

The motivation behind this work is as follows. Inverter-interfaced DG's in the distribution grid have a margin for reactive power support of the grid [14]. At the same time the increasing penetration of DG's in the system cause significant voltage-related problems, like voltage rise [15]. Hence, controlling the reactive power injection of DG's in a coordinated fashion should be expected to improve the voltage profile in the grid. Centralized voltage control has been proposed [16], but it requires considerable communication overhead and all the calculations are performed centrally. Fully decentralized voltage control using a voltage-reactive power droop function have also been studied ([17],[18]). These approaches are sensitive to the topology of the distribution grid and its operating point. Voltage-control based on MAS methodologies was proposed in [19], where a two-level voltage control scheme is presented: all inverters participate in local control and request additional support when they reach their reactive power limits using a multi-agent consensus protocol. The outlined protocol converges asymptotically, and the convergence rate might increase considerably for large networks.

The MAS-based reactive power coordination methodologies already discussed in the literature rely on well-known *asymptotically* converging consensus algorithms [6]. This paper proposes the use of *finite time* converging distributed protocols, such as the protocols described in [13], to share a reactive power request among a set of DG units. In fact, the methodology in [13] is modified in order to address the following reactive power sharing problem: given a target amount of reactive power request from a set of inverter-interfaced DG units, how can each unit specify in finite time the reactive power it must contribute so that the sum of reactive powers is equal to the requested and each unit does not violate its rated power constraint.

The rest of the paper is organized as follows. Section II formally introduces the problem that is being addressed. Section III discusses key results regarding finite-time coordination between agents based on linear weighted updates and observability of graph-structured systems, and introduces an improved protocol for finite-time calculation of the sum of the initial values of the nodes in a connected graph. Section IV combines the previous results to provide the proposed reactive power coordination methodology. Section V presents numerical results from the application of the methodology to solve the problem for a test case based on the IEEE 27-nodes test feeder, and compare it with an asymptotically converging consensus protocol. Section VI offers conclusions and remarks regarding the direction for future research.

II. DECENTRALIZED REACTIVE POWER SHARING

Assume a group of N distributed generating units connected to nodes in a distribution network. Let each be represented by a node and let each node be uniquely represented by an integer. Without loss of generality, let the index set be $\mathcal{V} = \{1, 2, \dots, N\}$. It is assumed that each of the nodes is able to receive messages with a set of neighbors $\mathcal{N}_j^- \subseteq \mathcal{V}$. Define the in-degree D_j^- of node j as the number of nodes (including node itself) from which node j receives messages: $D_j^- = |\mathcal{N}_j^-|$. Thus, the availability of communications channels can be represented by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{E} = \{(j, i) \in \mathcal{V} \times \mathcal{V} : i \in \mathcal{N}_j^-\}$. The main standing assumptions for the sequel are that: i) Every node can receive messages from itself, i.e. $(i, i) \in \mathcal{E}, \forall i \in \mathcal{V}$ ii) The graph \mathcal{G} is connected, i.e. there exists a (directed) path between every pair of nodes in \mathcal{V} and iii) The graph is time-invariant, i.e. both the vertex set \mathcal{V} and the edge set \mathcal{E} remain constant with respect to time.

The problem that is addressed in this work is the following. At step $k = 0$, an initial set of nodes $\mathcal{I}_0 \subset \mathcal{V}$ receives information regarding the aggregate requested amount of reactive power support Q_r from the set of all DG units. Also, each DG's maximum and minimum reactive power is constrained by the total rated power S_r of the inverter, given the active power injection P of that DG at that instant of time:

$$-\sqrt{S_{r,j}^2 - P_j^2} \leq Q_j \leq \sqrt{S_{r,j}^2 - P_j^2} \quad (1)$$

Hence, at step $k = 0$, all nodes have information regarding their own maximum and minimum reactive power capacities:

$$\bar{q}_j = \sqrt{S_{r,j}^2 - P_j^2} \quad (2a)$$

$$\underline{q}_j = -\sqrt{S_{r,j}^2 - P_j^2} \quad (2b)$$

The purpose of this paper is to develop an iterative distributed protocol, based on message exchanges between neighbors, so that, after the protocol is executed, each node will have converged to its reactive power injection so that (1) is not violated and the sum of all injections is equal to Q_r . However, each node must only operate on local information and on information exchanges with neighbors. In the following sections, we will define how each node will initialize the protocol based on its local information and what information needs to be exchanged with neighbors so that each node converges to its own required reactive power injection in finite time.

III. FINITE - TIME ALGORITHMS FOR COOPERATION OF DER'S AND RL'S

In this section we derive the method outlined in section II to solve the coordination problem in a finite time. The proposed method is based on a protocol derived in [13], regarding the calculation of the initial state of a network of nodes using weighted linear updates. The main results from [13] regarding this protocol are summarized in section III-A.

A. Finite Time Algorithm for Initial State Calculation

Consider a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ of N nodes, that is connected, time-invariant and all nodes have self-loops, i.e. $\forall i \in \mathcal{V} : (i, i) \in \mathcal{E}$. Suppose that each node's information state is $s_j \in \mathbb{R}$, while $z \in \mathbb{R}^N$ is the state vector of the entire network. Suppose that for each node z_j is initialized at $z_j[0]$ and subsequently, at each step k all nodes perform weighted linear updates with their neighbors

$$z_j[k+1] = p_{jj}z_j[k] + \sum_{j \in \mathcal{N}_j^-} p_{ji}z_i[k] \quad (3)$$

where p_{ij} denotes the element in the i -th row and j -th column of the square matrix of update weights $P \in M_N$. Furthermore, at each step k , each node observes its neighbors' state and that of itself, forming an observation vector $y \in \mathbb{R}^{D_j^-}$:

$$y_j[k] = C_j z[k] \quad (4)$$

where $C_j \in M_{D_j^-, N}$ is an observation matrix.

Both P and $C_j, \forall j \in \mathcal{V}$ are randomly chosen structured matrices. Element p_{ij} of P is zero iff $(i, j) \notin \mathcal{E}$ and nonzero otherwise. C_j has a single nonzero element in each of its D_j^- rows, at a column corresponding to one of the in-neighbors of node j . The nonzero elements of P and C_j are chosen randomly according to a continuous distribution, as suggested in [13]. Let $Y_{j,k} = (y_j[0] \ y_j[1] \ \dots \ y_j[k])^T$ denote the consolidated vector of observations up to step k for node j . If the k -step observability matrix is defined as:

$$O_{j,k} = \begin{bmatrix} C_j \\ C_j P \\ \vdots \\ C_j P^k \end{bmatrix} \quad (5)$$

then the following linear expression holds:

$$Y_{j,k} = O_{j,k} z[0] \quad (6)$$

The expression (6) implies that, given the consolidated observations of node j up to step k , the initial state vector z_0 lies in an affine subspace \mathcal{A} that can be written as:

$$\mathcal{A} = \bar{z}_0 + Null(O_{j,k}) \quad (7)$$

where $\bar{z}_0 \in \mathbb{R}^N$ is one solution of (6). In the case that $O_{j,k}$ is full column rank, that subspace reduces to a single point in \mathbb{R}^N and node j can uniquely determine the initial state of the entire network. The following result from [13] addresses the issue of the rank of the observability matrix:

Theorem 1: ([13]) Let \mathcal{G} be a time-invariant connected graph with self-loops in all nodes, and suppose the nodes perform linear iterations as in (3-4). Then, for almost any choice of update matrix $P \in M_N$ and observation matrices $C_j \in M_{D_j^-, N}$ (both matrices are structured - subject to the restrictions imposed by the graph \mathcal{G}), the observability matrix of each node eventually becomes full column-rank, given enough iterations are performed. Furthermore, an upper bound for the number of steps required exists, i.e:

$$\forall j \in \mathcal{V}, \exists s_j \leq N - D_j^- ; rank(O_{j,s_j}) = N \quad (8)$$

Theorem 1 implies that all nodes will be able to calculate a unique solution to (6) after $s_{max} = \max_{j \in \mathcal{V}} (N - D_j^-)$ steps of linear iterations of the form (3-4), thus determining the initial state vector of the entire network.

With Theorem 1 at hand, one can determine a distributed protocol for initial state calculation at each node. The protocol, given in [13] consists of a distributed initialization phase, performed only once and an initial state calculation phase, performed continuously. In the initialization phase the nodes randomly choose their weight matrices (e.g. according to a uniform distribution) and engage in a distributed protocol allowing each node to obtain and store s_j, O_{j,s_j}, s_{max} . The matrix O_{j,s_j} is obtained columnwise as follows: The N nodes perform N separate linear update iterations, with each node executing (3-4) for $N - 2$ steps each. In each of the N iterations, the nodes

are initialized differently. In the i -th iteration the initial state vector is chosen as:

$$z_j^{(i)}[0] = \begin{cases} 1 & , \text{ if } i = j \\ 0 & , \text{ otherwise} \end{cases} \quad (9)$$

Hence, the consolidated observation vector for node j is, after $N-2$ steps:

$$Y_{j,N-2}^{(i)} = \begin{bmatrix} y_j^{(i)}[0] \\ \vdots \\ y_j^{(i)}[N-2] \end{bmatrix} = O_{j,N-2} \mathbf{e}_i \quad (10)$$

As suggested by (10), in the i -th iteration, the observations of node j are exactly the i -th column of $O_{j,N-2}$. The choice for $N-2$ steps is made because Theorem 1 states that at most $N - D_j^-$ steps are needed for full observability, and thus a valid upper bound for the number of steps is $N-2$, because every node's in-degree is greater than 1 in a connected graph. After the N protocols are completed, each node will have obtained the full observability matrix as:

$$O_{j,N-2} = \begin{bmatrix} Y_{j,N-2}^{(1)} & Y_{j,N-2}^{(2)} & \cdots & Y_{j,N-2}^{(N)} \end{bmatrix} \quad (11)$$

After obtaining $O_{j,N-2}$, each node can obtain s_j by checking the rank of its principal submatrices $O_{j,k}$, $k \leq N-2$, as:

$$s_j = \max\{k \in \mathbb{Z} : \text{rank}(O_{j,k}) = N\} \quad (12)$$

Once all nodes have calculated s_j they can converge to the number of steps needed to guarantee full observability for the entire network by performing the following iteration for $N-1$ steps, $\forall j \in \mathcal{V}$:

$$\begin{aligned} \hat{s}_{\max,j}[0] &= s_j \\ \hat{s}_{\max,j}[k+1] &= \max_{i \in \mathcal{N}_j^-} (\hat{s}_{\max,i}[k]) \end{aligned} \quad (13)$$

This iteration guarantees that all nodes will converge to s_{\max} .

Once the initialization protocol outlined by (9)-(13) has been completed, all nodes have obtained s_j , O_{j,s_j} , s_{\max} and are ready to engage in the initial state calculation phase.

In the initial state calculation phase, each node performs linear iterations with neighbors and observes their states as in (3)-(4). After s_j steps, node j calculates the initial value of all nodes by obtaining the unique solution of (6). The linear updates terminate after s_{\max} steps, at which point all nodes have uniquely determined the initial value $z[0]$. In the interest of space, the detailed protocol for each phase is not reproduced here, but the interested reader is referred to [13]. An revised version of both the initialization phase and the second stage, designed to address the specific problem of section II is developed in later sections.

B. Result 1: Improved Protocol for Initial State Calculation

The protocol mentioned above can be used to solve the DER/RL coordination problem of section II. However, certain modifications are made to the protocols in [13] to distribute the calculations evenly across steps. To motivate these modifications, consider the following: as outlined in section III-A, after s_j steps, node j must solve the linear system:

$$O_{j,s_j} z[0] = Y_{j,s_j} \quad (14)$$

The linear system in (14) is guaranteed to have a unique solution, as per Theorem 1, since O_{j,s_j} is guaranteed to have rank N . However, solution of the linear system (14) is known to cost $\mathcal{O}(N^2 m_j)$ operations with simple gaussian elimination. Since this matrix is pre-calculated and stored locally at each node and remains constant for

all subsequent operations of the network it is possible to factorize O_{j,s_j} in the initialize stage for each node, thus reducing the number of operations performed in node j at step s_j to $\mathcal{O}(N^2)$ by back substitution. Even after this modification, however, the number of required computations in the final step depends on the size of the network N , and thus it will scale badly if the protocol is applied to networks with a large number of nodes. This section, as well as the next, is dedicated to deriving a protocol that will solve the DER coordination problem and will require less computations per step.

Let $s_j D_j^- = m_j$ be the number of rows of O_{j,s_j} . From Theorem 1 O_{j,s_j} is rank N , $m_j \geq N$. Thus, the Singular Value Decomposition (SVD) of O_{j,s_j} can be written as [20]:

$$O_{j,s_j} = V \Sigma W^T \quad (15)$$

where $V \in M_{m_j, m_j}$ and $W \in M_{N, N}$ are orthogonal and $\Sigma = \begin{bmatrix} \Sigma_N \\ 0 \end{bmatrix}$, where $\Sigma_N = \text{diag}\{(\sigma_1 \ \sigma_2 \ \dots \ \sigma_N)^T\}$.

Let $V = [V_L \ V_R]$ where $V_L \in M_{m_j, N}$ and $V_R \in M_{m_j, m_j - N}$. Thus, the solution of (14) can be written as:

$$z[0] = W \Sigma_N^{-1} V_L^T Y_{j,s_j} \quad (16)$$

If we define the pseudo-inverse of O_{j,s_j} as $O_{j,s_j}^+ = W \Sigma_N^{-1} V_L^T$, then (16) is written as:

$$z[0] = O_{j,s_j}^+ Y_{j,s_j} \quad (17)$$

We can write O_{j,s_j}^+ as:

$$O_{j,s_j}^+ = \begin{bmatrix} O_{j(0)}^+ & O_{j(1)}^+ & \cdots & O_{j(s_j)}^+ \end{bmatrix} \quad (18)$$

where $O_{j(k)}^+ \in M_{N, D_{j,-}}$, $k \in \{0, 1, \dots, s_j\}$. Using this partition of O_{j,s_j}^+ and (17)

$$z[0] = \sum_{k=0}^{s_j} O_{j(k)}^+ y_j[k] \quad (19)$$

As suggested by (19), once node j obtains the observation vector $y_j[k]$ in step k it can calculate the k -th term of the sum in the right-hand side of (19). Thus, the initial state vector can be obtained by storing an intermediate estimate $\hat{z}_0[k]$ in each step, and updating it as:

$$\hat{z}_{0,j}[k] = \hat{z}_0[k-1] + O_{j(k)}^+ y_j[k] \quad (20a)$$

$$\hat{z}_{0,j}[-1] = 0 \quad (20b)$$

Note that (19) guarantees that $\hat{z}_{0,j}[s_j] = z[0]$. This means that, since the pseudo-inverse in (18) has been pre-calculated (15) and partitioned (18) in the initialization phase, node j will need to perform $\mathcal{O}(N \cdot D_j^-)$ operations at each step of the execution phase, instead of $\mathcal{O}(N^2)$ in a single (final) step, to obtain the initial state vector for all nodes. Hence, the calculation of the initial state is now fully distributed to s_j steps.

C. Result 2: Finite Time Consensus on Sum of Initial Values

In certain cases, calculation of the entire initial state vector of the network in each node is more than what is needed. For most applications, a consensus on the average of the initial values of the nodes is required and has been extensively studied in the literature [5], [6]. The observability-based protocol of section III-B suggests a modification of the algorithm that would allow each node to calculate the sum of the initial node values.

Let $\mathbf{1} = [1 \ 1 \ \dots \ 1]^T$ be an N -dimensional vector. Suppose that each node is required to calculate:

$$\mathbf{1}^T z[0] = \sum_{j=1}^N z_j[0] \quad (21)$$

Of course, each node could perform this calculation by using the protocol of section III-B to calculate the entire state $z[0]$ and subsequently use (21) to obtain the required sum. However, that would require unnecessary calculations per node in each step, namely $\mathcal{O}(N \cdot D_j^-)$, which can be reduced. Indeed, by multiplying each side of (19) by $\mathbf{1}^T$, we obtain:

$$\mathbf{1}^T z_0 = \sum_{k=0}^{s_j} \mathbf{1}^T O_{j(k)}^+ y_j[k] \quad (22)$$

By defining the D_j^- -dimensional vector:

$$a_j[k] = \left(\mathbf{1}^T O_{j(k)}^+ \right)^T \quad (23)$$

then (22) can be rewritten as:

$$\sum_{j=1}^N z[0] = \sum_{k=0}^{s_j} a_j^T[k] y_j[k] \quad (24)$$

Thus, each node can obtain the exact sum of the initial node states in at most s_j steps, by performing the following operations in each step $0 \leq k \leq s_j$:

$$z_j[k+1] = p_{jj} z_j[k] + \sum_{i \in \mathcal{N}_j^-} p_{ji} z_i[k] \quad (25a)$$

$$y_j[k] = C_j z[k] \quad (25b)$$

$$\hat{z}_j[k] = \hat{z}_j[k-1] + a_j^T[k] y_j[k] \quad (25c)$$

All three operations (25a)-(25c) cost $\mathcal{O}(D_j^-)$ computations. Hence, if all that is needed is the sum of the node initial values, the finite time algorithm requires, per step and per node, a number of computations that is linear with respect to the number of neighbors of the node, and does not depend on the size of the network, which is the desired feature we set out to achieve. The added advantage of this approach is that an upper bound (namely $N-2$) on the number of steps needed for exact convergence is known, while the convergence time of asymptotic average consensus algorithms depends on the needed accuracy and the second largest eigenvalue of the graph Laplacian [5]. The disadvantage of this protocol, however, is the need for an initialization phase and the need to store the vectors $a_j[k]$, $k \leq s_j$ locally at each node j .

IV. A FINITE TIME PROTOCOL FOR REACTIVE POWE SHARING AMONG DR'S

Based on the previous section we can now solve the reactive power sharing problem of Section II. Our proposed methodology consists of two phases. In phase one, shown in Fig 1. the nodes engage in a decentralized initialization protocol, so that the vectors $a_j[k]$, and the values s_j and s_{max} are calculated at each node j . These quantities have to be known at each node, in order for the initial state calculation protocol to work properly.

The second phase of the protocol is executed once a certain amount of reactive power Q_r is requested from the nodes in \mathcal{I}_0 . As shown in Fig. 2, all nodes initialize their information states $z_{j,1}$ and $z_{j,2}$ based on local information (only nodes in \mathcal{I}_0 know Q_r). Each node then performs operations (25a) - (25c) on each state separately. As shown

Each node j chooses random weights $p_{j,i} \forall i \in \mathcal{N}_j^-$
Each node j chooses random observation matrix C_j
for $\tilde{j} = 1$ **to** N
 $z_k[0] = \begin{cases} 1 & , \text{if } k = \tilde{j} \\ 0 & , \text{otherwise} \end{cases} , \forall k \in \mathcal{V}$
for $k = 0$ **to** $N-2$
for all $j \in \mathcal{V}$ **do**
 $z_j[k+1] = p_{jj} z_j[k] + \sum_{i \in \mathcal{N}_j^-} p_{ji} z_i[k]$
 $y_j[k] = C_j z[k]$
 $Y_{N-2}^{(\tilde{j})} \leftarrow \begin{bmatrix} Y_{N-2}^{(\tilde{j})} \\ y_j[k] \end{bmatrix}$
end for
end for
end for
for all $j \in \mathcal{V}$ **do**
 $O_{j,N-2} \leftarrow \begin{bmatrix} Y_{j,N-2}^{(1)} & Y_{j,N-2}^{(2)} & \dots & Y_{j,N-2}^{(N)} \end{bmatrix}$
Calculate and store $s_{j,s_{max}}$ (12)-(13)
Calculate the SVD $O_{j,s_j} = V \Sigma W^T$
 $O_{j,s_j}^+ = W \Sigma_N^{-1} V_L^T$
 $a_j[k] \leftarrow \left(\mathbf{1}^T O_{j(k)}^+ \right)^T$
end for

Fig. 1. Distributed protocol for the initialization phase

for all $j \in \mathcal{V}$ **do**
 $z_{j,1}[0] = \bar{q}_j - q_j$
 $z_{j,2}[0] = \begin{cases} \frac{Q_r}{|\mathcal{I}_0|} - q_j & , \text{if } j \in \mathcal{I}_0 \\ -q_j & , \text{else} \end{cases}$
 $\hat{z}_{j,l}[-1] = 0, \forall l \in \{1, 2\}$
end for
for $k = 0$ **to** s_{max}
for all $j \in \mathcal{V}$ **do**
 $y_{j,1}[k] = C_j z_{j,1}[k]$
 $y_{j,2}[k] = C_j z_{j,2}[k]$
if $k \leq s_j$ **then**
 $\hat{z}_{j,1}[k] = \hat{z}_{j,1}[k-1] + a_{j,1}^T[k] y_{j,1}[k]$
 $\hat{z}_{j,2}[k] = \hat{z}_{j,2}[k-1] + a_{j,2}^T[k] y_{j,2}[k]$
else
 $\hat{z}_{j,1}[k] = \hat{z}_{j,1}[k-1]$
 $\hat{z}_{j,2}[k] = \hat{z}_{j,2}[k-1]$
end if
 $z_{j,1}[k+1] = p_{jj} z_{j,1}[k] + \sum_{i \in \mathcal{N}_j^-} p_{ji} z_{i,1}[k]$
 $z_{j,2}[k+1] = p_{jj} z_{j,2}[k] + \sum_{i \in \mathcal{N}_j^-} p_{ji} z_{i,2}[k]$
end for
end for
for all $j \in \mathcal{V}$ **do**
 $\hat{q}_j[s_j] = \hat{q}_j = \underline{q}_j + \hat{z}_{j,2}[s_j] \frac{\bar{q}_j - \underline{q}_j}{\hat{z}_{j,1}[s_j]}$
end for

Fig. 2. Reactive Power Sharing Protocol for a graph of DG's

in Section III-C $\hat{z}_{j,1}[k]$ and $\hat{z}_{j,2}[k]$ will converge to the sum of the initial values of those states over all the network nodes, hence:

$$\hat{z}_{j,1}[s_{max}] = Q_r \quad (26a)$$

$$\hat{z}_{j,2}[s_{max}] = \sum_{j \in \mathcal{V}} \bar{q}_j - \underline{q}_j \quad (26b)$$

Hence:

$$\hat{q}_j[s_{max}] = \underline{q}_j + (Q_r - \sum_{j \in \mathcal{V}} \underline{q}_j) \frac{\bar{q}_j - \underline{q}_j}{\sum_{j \in \mathcal{V}} \bar{q}_j - \underline{q}_j} \quad (27)$$

The value of (27) is an acceptable solution to the reactive power sharing problem.

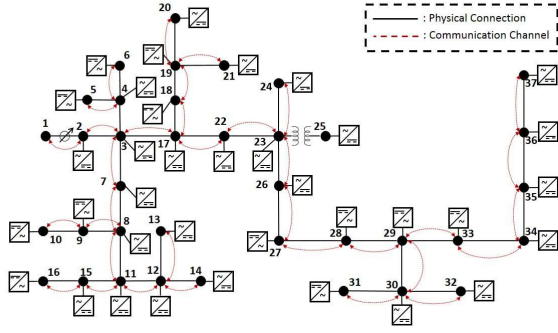


Fig. 3. The IEEE 37 node distribution feeder. Inverter interfaced generation is connected to each distribution node. Inter-node communication lines coincide with physical lines

TABLE I
LOCATION, RATED POWER AND ACTIVE INJECTION OF DG UNITS

Node	S_R (pu)	P (pu)	Node	S_R (pu)	P (pu)
1	0.00	0.00	20	0.88	0.75
2	1.00	0.80	21	0.97	0.80
3	1.00	0.85	22	0.98	0.80
4	1.00	0.85	23	0.91	0.75
5	1.00	0.80	24	0.89	0.70
6	1.00	0.90	25	0.87	0.65
7	1.00	0.85	26	0.88	0.69
8	0.95	0.80	27	0.89	0.72
9	0.85	0.75	28	0.86	0.70
10	0.95	0.80	29	0.85	0.70
11	1.00	0.85	30	0.90	0.75
12	1.05	0.90	31	0.90	0.70
13	1.00	0.85	32	0.87	0.70
14	1.00	0.80	33	0.88	0.70
15	1.00	0.80	34	0.90	0.65
16	0.90	0.75	35	0.90	0.70
17	1.00	0.80	36	0.90	0.70
18	1.05	0.80	37	0.90	0.70
19	1.05	0.85			

V. SIMULATION RESULTS

The proposed protocol can be used to share a total reactive power command among a group of DG units, proportionally to their maximum reactive power capacity. In this way, all DG units can participate in the voltage support of the transmission grid by adjusting their reactive power injection. To verify the approach, a network of 37 distribution nodes, shown in Fig. 3 was chosen as a test system. The physical topology of the network is based on the IEEE-37 node test distribution feeder. The nodes can exchange information with neighbors in an iterative fashion. The available communication channels for this example coincide with the physical distribution lines, as shown in Fig. 3. Inverter interfaced distributed generation units are connected to each node. The rated power S_R and the active power P of the DG unit connected to each bus are given in Table I. It is assumed that each node has full information regarding the state and capacity of the DG unit that is connected to it. The aim of this Section is to demonstrate that the proposed methodology achieves to coordinate each DG unit's reactive power injection, so that their total sum of all injections is equal to a desired amount that is initially known only to node 1, which is the interface with the substation ($\mathcal{I}_0 = \{1\}$). All inverters have been initialized using the protocol of Fig. 1.

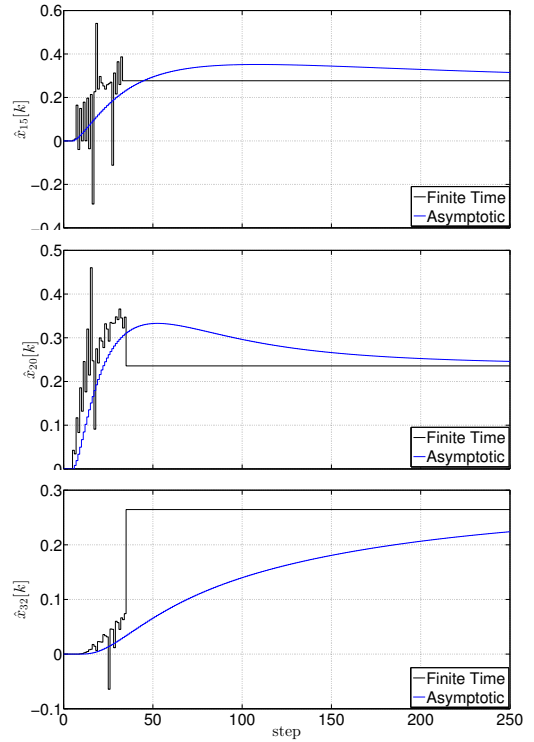


Fig. 4. Step-by-step execution of the inverter reactive power coordination algorithm: (a) Node 15 (b) Node 20 (c) Node 32

The evolution of the finite time reactive power coordination protocol has been simulated for a reactive power request of $Q_r = 10pu$ from the transmission grid (for a base power of $100KVA$). Fig. 4 shows, for various nodes, the step-by-step evolution of the proposed coordination algorithm (Fig. 2), i.e. $\hat{x}_j[k]$ is plotted. For comparison purposes, the result is plotted against the asymptotically converging consensus based algorithm of [8], that converges to the same value per node. Note that all nodes converge to an allocation of Q_r that is proportional to their maximum capacity. Also, it is worth noting that the proposed algorithm converges in a finite number of steps. It can be observed that the proposed protocol does not converge to the steady state solution as smoothly as the asymptotically converging consensus protocol. This however should not be an issue in practice, because the inverters will update their reactive power contribution only after the protocol converges - and each of them knows an upper bound s_{max} of the number of steps needed for that to happen. In other words, the transient behavior of the protocol in Fig. 4 is disregarded by the inverters.

The capability of the proposed protocol to allocate the reactive power command Q_r among the inverters is shown in Fig. 5. This figure illustrates the step-by-step evolution of the sum of the intermediate estimates of the feasible solution $\hat{x}_j[k]$, saturated by the upper and lower capacity limits of each inverter:

$$\hat{Q}[k] = \sum_{j=1}^N \left\{ \max \left[\underline{Q}_j, \min \left(\bar{Q}_j, \hat{x}_j[k] \right) \right] \right\} \quad (28)$$

The quantity $\hat{Q}[k]$ is compared to the same quantity when the asymptotic consensus algorithm in [8] is used. The decentralized reactive power coordination protocol is considered complete when \hat{Q} converges to Q_r . As a result, the step-by-step evolution of \hat{Q} is suitable for the comparison of the performance of the two

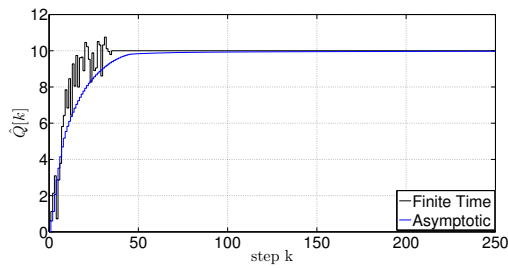


Fig. 5. Step-by-step evolution of the quantity $\hat{Q}[k]$: finite time versus asymptotic protocol

protocols. As shown in Fig. 5, both protocols successfully allocate the target reactive power among the DG units. However, the finite-time algorithm is characterized by faster convergence than the asymptotic consensus approach.

VI. CONCLUSIONS AND FUTURE WORK

This paper presented a new methodology for distributed coordination of the reactive power output of grid-connected DG units. Following a request for a total amount of reactive power to the unit connected closest to the substation, each DG unit exchanges messages with neighbors, eventually converging to the amount of reactive power it must inject so that the total reactive power injection is the requested amount. The methodology ensures that each unit will not violate its rated power constraint, contributing reactive power proportionally to its capacity, and that the reactive power request will be shared among all units in a finite number of steps (message-exchanges). The methodology is applicable to time-invariant communication networks and requires a number of operations per unit that increases linearly with that unit's number of neighbors. The proposed protocol is comprised of an initialization phase, so that the protocol parameters are identified for that specific communications topology, and a reactive power sharing phase, whereby the reactive power request is shared among the units. Simulations on a 37-node distribution feeder showed the applicability of the approach, as a reactive power request is properly shared among all available DG units. For this test system, the proposed methodology achieves faster power sharing than the asymptotic consensus algorithms discussed in the literature. Future work will be focused on expanding this distributed coordination methodology in other Smart Grid applications, such as active power sharing.

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