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Abstract

Optimal power flow (OPF) is a well known challenging optimization problem for power systems engineering. There have been a myriad of works dedicated to solving OPF problems in a centralized way, i.e. using a centralized solver, which necessitates for a central control center to receive and transmit a large amount of data. Recently, few attempts have been done to solve OPF problems in a distributed way, which means that the computations are distributed over the whole network, reducing the need for communications (with a central control center) and increasing the robustness of the control to loss of communications and to unexpected faults in the network nodes. Furthermore, a decentralized solution to the OPF problem has the advantage to be flexible to the network topology and size, since the computations are distributed over the network. In this paper we propose some preliminary results on a distributed OPF algorithm, to solve the original OPF problem, i.e. without any convexification steps. We propose a consensus-based distributed OPF algorithm, which uses consensus algorithm to estimate the optimization variables between neighboring nodes in a given network. We test the performance of this algorithm on the IEEE 14-node test-case.

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Consensus-based Distributed Optimal Power Flow Algorithm

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***Abstract*—Optimal power flow (OPF) is a well known challenging optimization problem for power systems engineering. There have been a myriad of works dedicated to solving OPF problems in a centralized way, i.e. using a centralized solver, which necessitates for a central control center to receive and transmit a large amount of data. Recently, few attempts have been done to solve OPF problems in a distributed way, which means that the computations are distributed over the whole network, reducing the need for communications (with a central control center) and increasing the robustness of the control to loss of communications and to unexpected faults in the network nodes. Furthermore, a decentralized solution to the OPF problem has the advantage to be flexible to the network topology and size, since the computations are distributed over the network. In this paper we propose some preliminary results on a distributed OPF algorithm, to solve the original OPF problem, i.e. without any convexification steps. We propose a consensus-based distributed OPF algorithm, which uses consensus algorithm to estimate the optimization variables between neighboring nodes in a given network. We test the performance of this algorithm on the IEEE 14-node test-case.**

I. Introduction

Optimal power flow (OPF) problem is a classical problem for power system engineering. Under constraints of power generation limits, voltage limits , transmission thermal limits as well as other security constraints, the solution of OPF determines the active and reactive power generation and the voltages of buses to minimize the cost of generation. The OPF problem is crucial to the secure operation of the power grids and wholesale power market including the day-ahead and real-time market.

Due to its significance, the OPF problem has received considerable attentions since 1960's [3] and still remains a hot research topic until now. Despite all efforts, the OPF problem remains difficult to solve. The main reasons can be summarized into two aspects: 1) OPF is a non-convex, nonlinear optimization problem. The non-convexity, caused by quadratic relations between voltages of adjacent buses prevents a guarantee to a global optimal solution in general. 2) The OPF is a large scale optimization problem with a large number of decision variables and constraints in complex power generation, voltages in all buses, transmission thermal limits. The computational complexity needs to reduced in the design of algorithm for practical implementation.

In practice, such difficulties in solving OPF are avoided by approximation. The original complex AC power flow problem is approximated by a DC-power flow problem, which is a linear programming problem. This approximation is reported to have acceptable level of accuracy for transmission networks and thus, utilized by current ISOs [11]. However, the DC power flow solution is inaccurate for distribution networks [5]. This cannot satisfy the emerging demand of smart grid technology. Thus, there is another line of research that focuses on a global optimal solution to the AC OPF problem. Algorithms that can achieve the global optimal solution under certain assumptions, are proposed based on SDP relaxation such as [6], [9], [4].

Even though the non-convexity of the problem can be handled in several circumstances, the centralized algorithms still cannot meet the current requirement of modern power networks. Nowadays, various renewable energy sources such as distributed generation and power storage are involved as part of the smart grid technology. Due to the diverse, time-varying, volatile load or storage connected to the grid, the centralized OPF problem is desirable to be solved to reflect any significant change in the power network. To satisfy the requirement of the daily operation of the grid and wholesale power market, it is reported that the OPF problem is usually solved every five minutes by the power planning authority such as regional transmission operator (RTO) or independent system operator (ISO). As a result, the large scale OPF problem in the era of smart grids technology has so high requirement for the efficiency and accuracy that a centralized algorithm is unable to be effective any more.

To provide a scalable, fast solution to large scale optimization problems, distributed optimization algorithms are proposed. In a distributed framework, the original centralized problem is divided into a certain number of small scale subproblems. Each subproblem is solved by a single agent as a computation entity with agent-to-agent communication capabilities. A certain communication between adjacent agents is required during the computation process to exchange necessary data according to a certain protocol. Thus, all agents can solve the centralized problem collaboratively in a parallel fashion. Distributed optimization algorithms have been widely studied in the research community. The idea of distributed optimization was first introduced in the seminal work [13]. In [10], an unconstrained nonlinear optimization problem is solved by multiple agents cooperatively using sub-gradient method to minimize the individual objective function and also sharing information among agents in the neighborhood. For constrained optimization problem, distributed algorithms have been proposed based on penalty function method [7], alternative direction method of multiplier (ADMM) [1], [14].

Distributed optimization algorithms have also been adapted to solve the OPF problems. In [8] and [2], the original power network is divided into several groups based on geographical regions. Maximal cliques are formed from a modified network of the regional groups according to certain rules. Each clique is associated with a relaxed SDP problem and solved by in a distributed fashion. In [12], each bus is treated as an individual agent. Each agent maintains an estimate of voltages of the adjacent buses and formulates the reduced OPF problem by minimizing the local cost of power generation and the estimation error. The local equality constraint for each agent is the power balance equation of each bus with the voltage variables of other buses replaced by estimates. The estimates are reported to converge to true values when the algorithm reaches a feasible solution. Simulation results on IEEE test cases are provided in those works, which shows

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a great reduction of CPU times in the decentralized algorithms. Such an improvement becomes more significant as the number of buses grows.

In this paper, we proposed a novel consensus-based distributed OPF algorithm. The algorithm is run by individual agents which own a portion of the power grid and solve local optimization problem. The agent not only use the voltage and power variables that observed locally, but also estimates the voltage and power variables coupled with the local power balance constraints. The agents exchange their local data and estimates with the neighbors to form a correct estimation. To improve the estimation, we proposed to use a consensus-filter to process the data shared among the neighboring agents. The local objective function of each agent is an augmented cost function that consists of both power cost and the estimation errors. A penalty function-like method is used to minimize the local objective.

The rest of the report is organized as follows. We first introduce some notations and define the OPF problem in Section II. Then the proposed consensus-based approach for distributed optimization problem is presented in Section III. Numerical results are in Section IV. We conclude our report in Section V.

II. Notations and problem formulation

The power network is modelled as a graph $(\mathcal{N}, \mathcal{E})$, where the vertex set $\mathcal{N}$ is the set of all buses in the network , the link set $\mathcal{E}$ is the transmission line that connects two buses in the grid. Each bus $i \in \mathcal{N}$can be connected to generators or loads. The set of buses that connected to generators is denoted as $\mathcal{N}^{\mathcal{G}} \subset \mathcal{N}$. For convenience, the bus connected to a generator is called generator bus. The bus without any generator is called load bus. We use $i \sim j$ to denote that the bus i is connected to bus j.

The active and reactive generation power at the bus i are denoted as P_i^G and Q_i^G. The power demand of at the bus i is denoted as P_i^D, Q_i^D. The complex voltage of the bus i is denoted as $V_i = e_i + \mathbf{j} f_i$ with e_i being the real part of the voltage, f_i the imaginary part and $\mathbf{j} = \sqrt{-1}$. For simplicity, we refer to e_i and f_i collectively by V_i. Denote the admittance-to-ground of bus i as y_{ii} and denote the line admittance between two buses i and j as y_{ij}. The admittances are complex numbers given as $y_{ij} = g_{ij} + \mathbf{j} b_{ij}$. Clearly, if $i \not\sim j$, then $y_{ij} = 0$, finally, $\mathcal{N}(i)$ denotes the set of all neighbors of node i.

The OPF problem can be stated as

$$\min_{P_i^G, Q_i^G, e_i, f_i, i \in \mathcal{N}} \sum_{i \in \mathcal{N}^G} F_i(P_i^G) \tag{1}$$

$$\text{s.t.} P_i^G - P_i^D = \mathbf{Re}\{|V_i|^2 y_{ii}^* + \sum_{i \sim j} V_i y_{ij}^* (V_i - V_j)^*\}, i \in \mathcal{N} \tag{2}$$

$$Q_i^G - Q_i^D = \mathbf{Im}\{|V_i|^2 y_{ii}^* + \sum_{i \sim j} V_i y_{ij}^* (V_i - V_j)^*\}, i \in \mathcal{N} \tag{3}$$

$$\underline{V}_i \leq e_i^2 + f_i^2 \leq \bar{V}_i, i \in \mathcal{N} \tag{4}$$

$$\underline{P}_i^G \leq P_i^G \leq \bar{P}_i^G, \underline{Q}_i^G \leq Q_i^G \leq \bar{Q}_i^G, i \in \mathcal{N}. \tag{5}$$

where the cost function F_i is usually defined as quadratic $F_i(P_i^G) = c_{2,i}(P_i^G)^2 + c_{1,i} P_i^G$. Equation (2) and (3) are real and reactive power balance equations for the bus i. The inequality (4) is the upper and lower bound on the magnitude of the voltages. The inequality (5) is the bound on the real and reactive generation power.

The non-convexity of the OPF comes from the quadratic relationships between the voltages in power balance equations (2) and (3). This poses difficulty to solve the OPF problem. Moreover, the number of the buses is often too large to handle in the centralized OPF form. In the following section, we proposed our consensus-based distributed algorithm to solve the centralized version of OPF (1).

III. Consensus-based Distributed OPF Algorithm

In this section, we introduce the direct approach of our distributed algorithm. We assume that each bus in the network is attached to an agent. Each agent can be viewed as an entity with computation and communication capabilities. For the purpose of brevity, we do not distinguish the agent of the bus and the bus itself, unless there is a particular need.

The entire centralized OPF is formulated as a combination of sub-problems that is solved by each bus. For the ease of presentation, we assume that all buses are connected to a generator, *i.e.* $\mathcal{N}^G = \mathcal{N}$. For each bus i, we formulate a local sub-problem of OPF. The local decision variables at the bus i are P_i^G, Q_i^G, e_i, f_i. Note that there are coupling between the bus i and its neighbor of the voltage in the power balance equations, as shown in (2) and (3). Thus, in the process of solving the optimization problem, the bus i estimates the voltages of the neighboring bus j, which is denoted as $e_{j(i)}, f_{j(i)}$. The voltage $V_{j(i)} = e_{j(i)} + \mathbf{j} f_{j(i)}$ represents the complex voltage of bus $j \sim i$ estimated by the bus i. The estimates are used to replace the real voltages of bus j in the power balance equations of the bus i. Those estimates are also part of the decision variables for the bus i. The local OPF problem for bus i can be written as follows.

$$\min_{P_i^G, Q_i^G, e_i, f_i, e_{j(i)}, f_{j(i)}} c_{i,2}(P_i^G)^2 + c_{i,1} P_i^G, \tag{6}$$

$$\text{s.t.} P_i^G - P_i^D = \mathbf{Re}\{|V_i|^2 y_{ii}^* + \sum_{i \sim j} V_i y_{ij}^* (V_i - V_{j(i)})^*\}, i \in \mathcal{N}, \tag{7}$$

$$Q_i^G - Q_i^D = \mathbf{Im}\{|V_i|^2 y_{ii}^* + \sum_{i \sim j} V_i y_{ij}^* (V_i - V_{j(i)})^*\}, i \in \mathcal{N}, \tag{8}$$

$$\underline{V}_i \leq e_i^2 + f_i^2 \leq \bar{V}_i, \tag{9}$$

$$\underline{V}_{j(i)} \leq e_{j(i)}^2 + f_{j(i)}^2 \leq \bar{V}_{j(i)}, j \in \mathcal{N}(i), \tag{10}$$

$$\underline{P}_i^G \leq P_i^G \leq \bar{P}_i^G, \underline{Q}_i^G \leq Q_i^G \leq \bar{Q}_i^G, i \in \mathcal{N}. \tag{11}$$

$$V_j = V_{j(i)}, i \sim j, \ i \in \mathcal{N}, \tag{12}$$

where the constraint (12) ensures that the estimate of the voltage of bus j formed at the bus i is consistent with the true value V_j. To solve the local OPF problem (6), each bus i has to achieve two goals. 1) minimization of the generation cost. 2) the estimates $e_{j(i)}$, $f_{j(i)}$ follow the real voltages e_j, f_j, *i.e.* the constraint (12) is satisfied. Since the generation cost is already included in the local OPF problem (6), the goal 2) remains to be considered. Here, we adopt a consensus-like method to update the estimates at each iteration k by exchanging information among adjacent buses.

1) Consensus Algorithm of the Estimates: For each pair of voltage estimate $e_{j(i)}$, $f_{j(i)}$, the bus i maintains a pair of consensus variables $\hat{e}_{j(i)}$, $\hat{f}_{j(i)}$ and updates these variables using the real value of e_j, f_j obtained from the bus j according to the following rules.

$$\begin{aligned} \hat{e}_{j(i)}(k+1) &= \hat{e}_{j(i)}(k) + \gamma(e_j(k) - \hat{e}_{j(i)}(k)), \\ \hat{f}_{j(i)}(k+1) &= \hat{f}_{j(i)}(k) + \gamma(f_j(k) - \hat{f}_{j(i)}(k)), \end{aligned} \tag{13}$$

where $0 < \gamma < 1$ is the consensus gain by design. In our algorithm, the estimates such as $e_{j(i)}$, $f_{j(i)}$ are not replaced by true values like e_j, f_j from other buses directly. Instead, those values are passed through a consensus filter. The consensus variables can be viewed as intermediate variables in the filter and will be used to update the estimates in the local optimization which will be described later.

Following this algorithm, the bus i and j exchange the real voltages as well as their consensus variables at each iteration k. In other words, the bus i receives the true voltage $V_j(k)$ and the consensus variables $\hat{e}_{i(j)}, \hat{f}_{i(j)}$ and send its own voltage $V_i(k)$ and the consensus variables $\hat{e}_{j(i)}, \hat{f}_{j(i)}$ to its neighbors. Since the data exchange only occurs among the neighbors, there is not too much burden in the communication as compared to the centralized algorithm, which requires all data from every bus collected and uploaded to a computation center.

2) Local Optimization Problem: To minimize both the generation cost and estimation error, the local optimization problem (6) is reformulated . For bus $i \in \mathcal{N}$, the local optimization problem is written as

$$\min_{P_i^G, Q_i^G, e_i, f_i, e_{j(i)}, f_{j(i)}} F(P_i^G) + \rho_i \sum_{i\sim j} (\|e_{j(i)} - \hat{e}_{j(i)}\|^2 + \|f_{j(i)} - \hat{f}_{j(i)}\|^2 + \|e_i - \hat{e}_{i(j)}\|^2 + \|f_i - \hat{f}_{i(j)}\|^2) \quad (14)$$

$$\text{s.t.} P_i^G - P_i^D = \mathbf{Re}\{|V_i|^2 y_{ii}^* + \sum_{i\sim j} V_i y_{ij}^* (V_i - V_{j(i)})^*\}, i \in \mathcal{N} \quad (15)$$

$$Q_i^G - Q_i^D = \mathbf{Im}\{|V_i|^2 y_{ii}^* + \sum_{i\sim j} V_i y_{ij}^* (V_i - V_{j(i)})^*\}, i \in \mathcal{N} \quad (16)$$

$$\underline{V}_i \leq e_i^2 + f_i^2 \leq \bar{V}_i, \quad (17)$$

$$\underline{V}_{j(i)} \leq e_{j(i)}^2 + f_{j(i)}^2 \leq \bar{V}_{j(i)}, j \in \mathcal{N}/i \quad (18)$$

$$\underline{P}_i^G \leq P_i^G \leq \bar{P}_i^G, \underline{Q}_i^G \leq Q_i^G \leq \bar{Q}_i^G, i \in \mathcal{N}. \quad (19)$$

where $\rho_i > 0$ is a large positive number selected as a penalty factor in order to make sure that the estimation errors are minimized with a higher priority. This is necessary because without such a penalty, each bus is prone to minimize his own generation cost selfishly based on a set of biased estimation.

The decision variables of the problem (14) are only related to the bus i and its neighbors. So it is a local nonlinear optimization problem with a small scale. This problem can be solved efficiently using the state-of-art nonlinear programming solvers such as `IPOPT` or `fmincon`.

Next, we summarize the distributed algorithm for bus i as follows.

1) initialize the values of $P_i^G(0), Q_i^G(0), e_i(0), f_i(0), e_{j(i)}(0), f_{j(i)}(0), \hat{e}_{j(i)}(0), \hat{f}_{j(i)}(0)$
2) At iteration $k+1$, if the stop criterion is satisfied (e.g. the differences between consecutive estimates are small enough $\|e_{i(j)}(k) - e_{i(j)}(k+1)\| < \epsilon$), then stop. If not, each agent sends the variables $e_i(k), f_i(k), \hat{e}_{j(i)}(k), \hat{f}_{j(i)}(k)$ to neighbors $j \sim i$.
3) Update the consensus variables $\hat{e}_{j(i)}(k), \hat{f}_{j(i)}(k)$ according to (13).
4) Update the decision variables by solving the optimization problem (14) to (19), then go to step 2).

IV. Numerical tests

In this section, we present a numerical example to illustrate the application of the distributed algorithm proposed in Section III.

We consider the IEEE14 case which is a benchmark case for OPF problem and apply the proposed approach to solve the problem. The topology of the 14-bus system is illustrated in Figure 1.

Due to the high number of buses, it takes 200 iterations for the distributed algorithm to converge. The voltage and the estimates profile of some of the buses are illustrated on the Figures 2 to 11 (due to space limitation we did not include the voltage profile of all the buses). We can see clearly that the estimated voltages at

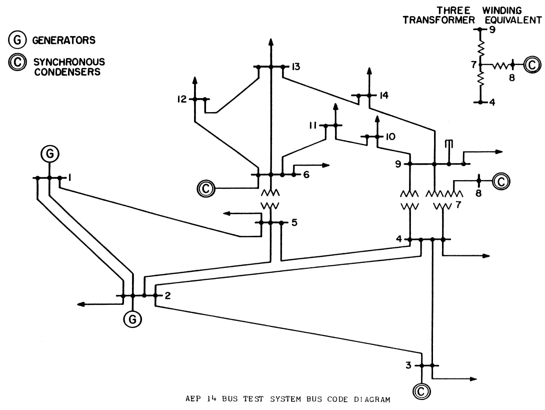

Fig. 1: IEEE14 system topology

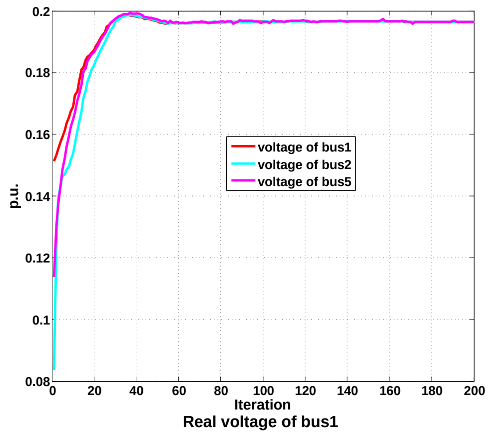

Fig. 2: The real voltage of bus 1 and the estimates in IEEE14 system

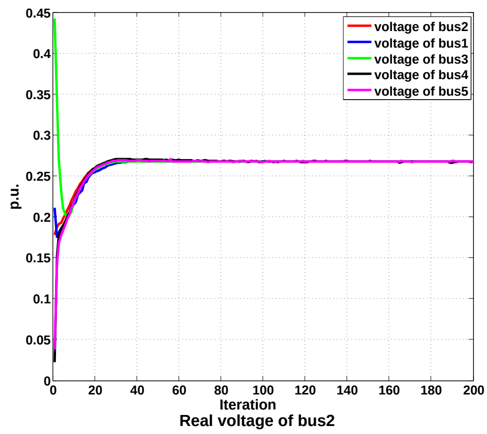

Fig. 3: The real voltage of bus 2 and the estimates in IEEE14 system

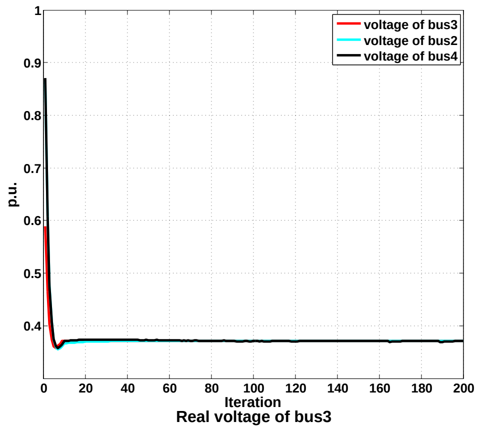

Fig. 4: The real voltage of bus 3 and the estimates in IEEE14 system

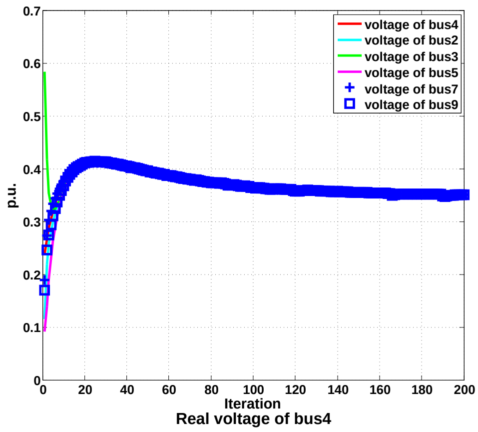

Fig. 5: The real voltage of bus 4 and the estimates in IEEE14 system

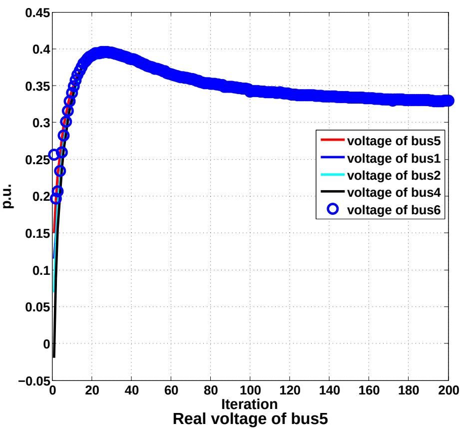

Fig. 6: The real voltage of bus 5 and the estimates in IEEE14 system

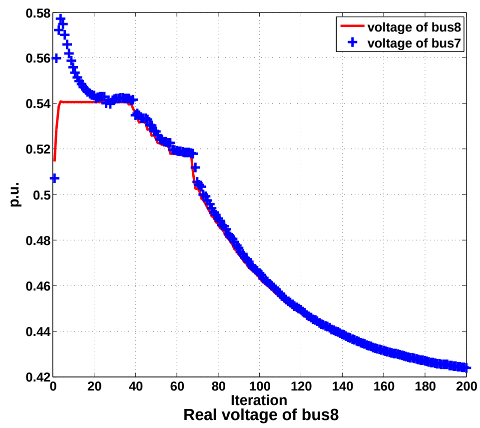

Fig. 7: The real voltage of bus 8 and the estimates in IEEE14 system

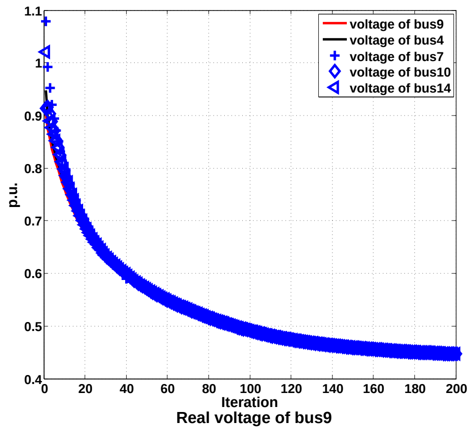

Fig. 8: The real voltage of bus 9 and the estimates in IEEE14 system

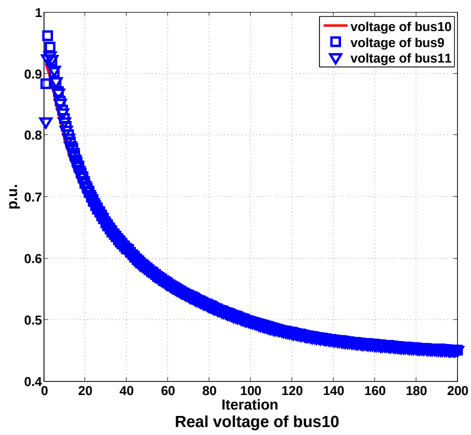

Fig. 9: The real voltage of bus 10 and the estimates in IEEE14 system

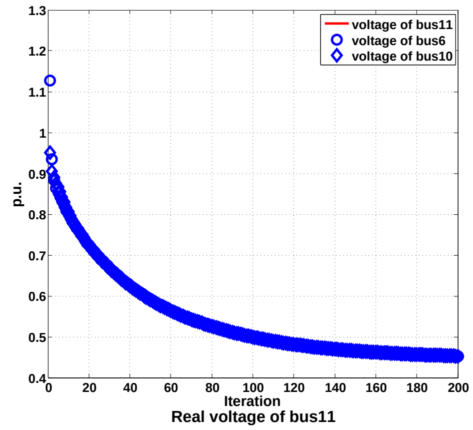

Fig. 10: The real voltage of bus 11 and the estimates in IEEE14 system

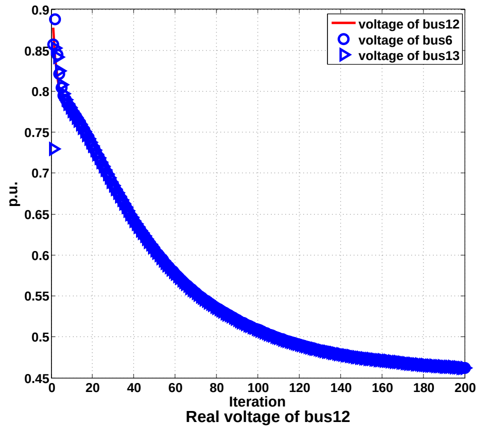

Fig. 11: The real voltage of bus 12 and the estimates in IEEE14 system

each node synchronize with the actual voltages of their neighboring nodes. The generation power generated by both the distributed and centralized algorithm are summarized in the Table I. Although, the two algorithms do not converge necessarily to the same optimal solutions, the distributed algorithm converges to a feasible solution and the cost function value at this solution is close to the cost function value at the centralized solution. Of course, these are preliminary results, that we wanted to share with the audience, but further analysis of the algorithm in terms of robustness and convergence are understudy and will be presented in our future reports.

TABLE I: Power generation results of 14 Bus OPF problem

Bus number	Distributed	Centralized
Bus 1	1.98	1.38
Bus 2	$0.58 + 0.38\mathbf{j}$	$1.31 + 0.22\mathbf{j}$
Bus 3	$0.21 + 0.17\mathbf{j}$	$0.3\mathbf{j}$
Bus 6	$0.24\mathbf{j}$	$0.1\mathbf{j}$
Bus 8	$0.07\mathbf{j}$	$0.08\mathbf{j}$

V. Conclusion

We have studied a consensus-based distributed optimization algorithm and its application to the optimal power flow problem (the original OPF problem, i.e. without any convexification steps). The algorithm is run by multiple agents, each agent solves its local OPF problem under the constraint of the power balance equation of the bus related to it. The agent estimates the voltages of its neighbors and exchange information to reach consensus. A consensus filtering method is utilized to ensure the convergence of the estimation. Numerical results are provided to demonstrate the effectiveness of this distributed optimization problem. The consensus are reached between all variables and their estimates. The algorithm provides a feasible solution which is obtained via local computations only. However, since the OPF problem is non-convex, there is no guarantee that a global optimal solution is attained (we remind the reader that this is not usually guaranteed even for centralized solvers, due to the non-convexity of the problem). Thus, it would be interesting to further evaluate the proposed algorithm in terms of its convergence robustness w.r.t. the network size/ initial conditions, and compare its convergence point to various centralized solvers to evaluate the gap between the obtained solution and the centralized, possibly global, optimal solutions.

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