Extremum Seeking-Based Indirect Adaptive Control for Nonlinear Systems with Time-varying Uncertainties

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I. INTRODUCTION

Extremum Seeking (ES) method has often been used as a real-time optimization tool [1]. As opposed to the classical model-based control, extremum seeking does not need detailed modeling of the process. Since ES controllers do not depend on exact plant models and also can easily deal with multi-input systems, they have been used in many control applications, such as automotive brakes [2], electromagnetic actuators [3], [4] and stirred-tank bioreactors [5]. On the other hand, model-based Input-output feedback linearization has been proven to be powerful in the control design for trajectory tracking and stabilization of nonlinear systems [6]. One shortcoming of the feedback linearization approach is that it requires precise system modelling [6]. When there exist model uncertainties, a robust input-output linearization approach needs to be developed. For instance, high-gain observers [7] and linear robust controllers [8] have been proposed in combination with the feedback linearization techniques. Another approach to deal with model uncertainties is using adaptive control methods. Of particular interest to us is the modular approach to adaptive nonlinear control, e.g. [9]. In this approach, first the controller is designed by assuming all the parameters are known and then an identifier is used to guarantee certain boundedness of the estimation error. The identifier is independent of the designed controller and thus the approach is called 'modular'. A modular approach has been proposed in [10] for adaptive neural control of pure-feedback nonlinear systems, where the input-to-state stability (ISS) modularity of the controller-estimator is achieved and the closed-loop stability is guaranteed by the small-gain theorem.

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In this paper, we study a class of nonlinear systems which are input-output linearizable through static state feedback [11]. We assume that the uncertainties in the linearized model are additive as guaranteed by the 'matching condition' [12]. The control objective is to achieve asymptotic tracking of a desired trajectory. The robust controller for the uncertain nonlinear system can be designed according to the following guidelines. In the first step, we design a controller for the nominal model (i.e. when the uncertainties are assumed to be zero) so that the tracking error dynamics is asymptotically stable. In the second step, we use a Lyapunov reconstruction method [11] to show that the error dynamics are input-to-state stable (ISS) [13] where the estimation error in the parameters is the input to the system and the tracking error represents the system state. Finally, we use ES to guarantee that the error in the estimation of parameters are bounded and decreasing so that the the tracking error will be bounded and decreasing, as guaranteed by the ISS property. Learning-based adaptive control for nonlinear systems has been studied in [3], [4]. In these two papers, the problem of adaptive robust control of electromagnetic actuators was studied, where ES was used to tune the feedback gains of the nonlinear controller in [3] and ES was used to estimate the unknown model parameters in [4]. An extension to the general case of nonlinear systems was proposed in [15], [16]. We relax here the strong assumption, used in [15], [16], about the existence of an ISS feedback controller, and propose a constructive proof to design such an ISS feedback for the particular case of nonlinear systems affine in the control.

The rest of the paper is organized as follows. In Section II, we present notations, together with some fundamental definitions and results. In Section III, we provide our problem formulation. The nominal controller design is presented in Section IV. In Section V, three cases are considered for the uncertain system based on the structure of uncertainties term in the model and a robust controller is designed for each case such that ISS is guaranteed from the estimation errors input to the tracking errors state. The stability of the multi-parametric extremum seeking (MES) algorithm that is used for parameters estimation is provided in Section VI. Note that the proofs of our Theorems have been omitted due to space constraints, however, they will be reported in a future journal version of this work. Section VII is dedicated to an application example and the paper conclusion is given in Section VIII.

II. PRELIMINARIES

Throughout the paper, we use $\|\cdot\|$ to denote the Euclidean norm; i.e. for a vector $x \in \mathbb{R}^n$, we have $\|x\| \triangleq \|x\|_2 = \sqrt{x^T x}$, where x^T denotes the transpose of the vector x. The 1-norm of $x \in \mathbb{R}^n$ is denoted by $\|x\|_1$. In this paper, when we refer to a matrix norm, we mean the Frobenius norm, which for a matrix $A \in \mathbb{R}^{m\times n}$, with elements a_{ij}, is defined as $\|A\| \triangleq \sqrt{\sum_{i=1}^n \sum_{j=1}^n |a_{ij}|^2}$. We use the following norm properties for the need of our proof: 1) for any $x \in \mathbb{R}^n$, $\|x\| \leq \|x\|_1$, 2) for any $x,\ y \in \mathbb{R}^n$, $\|x\| - \|y\| \leq \|x - y\|$, 3) for any $x,\ y \in \mathbb{R}^n$, $x^T y \leq \|x\|\|y\|$. Given $x \in \mathbb{R}^m$, the signum function is defined as $\mathrm{sign}(x) \triangleq [\mathrm{sign}(x_1),\ \mathrm{sign}(x_2),\ \cdots,\ \mathrm{sign}(x_m)]^T$, where x_i denotes the i-th ($1 \leq i \leq m$) element of x. We have $x^T \mathrm{sign}(x) = \|x\|_1$. For an $n \times n$ matrix P, we denote by $P > 0$ if it is positive definite. Similarly, we denote by $P < 0$ if it is negative definite. We use $\mathrm{diag}\{A_1,\ A_2,\ \cdots,\ A_n\}$ to denote a diagonal block matrix with n blocks. For a matrix B, we denote $B(i,j)$ as the element that locates at the i-th row and j-th column of matrix B. We denote I_n as the identity matrix or simply I if the dimension is clear from the context. We use $\dot{f}$ to denote the time derivative of f and $f^{(r)}(t)$ for the r-th derivative of $f(t)$, i.e. $f^{(r)} \triangleq \frac{d^r f}{dt}$. We denote by $\mathbb{C}^k$ functions that are k times differentiable and by $\mathbb{C}^\infty$ a smooth function. A continuous function $\alpha : [0,a) \to [0,\infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$ [11, pp. 144]. A continuous function $\beta : [0,a) \times [0,\infty) \to [0,\infty)$ is said to belong to class $\mathcal{KL}$ if, for a fixed s, the mapping $\beta(r,s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r,s)$ is decreasing with respect to s and $\beta(r,s) \to 0$ as $s \to \infty$ [11, pp. 144].

Consider the system

$$\dot{x} \ = \ f(t,x,u) \tag{1}$$

where $f : [0,\infty) \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in x and u, uniformly in t. The input $u(t)$ is piecewise continuous, bounded function of t for all $t \geq 0$.

Definition 1 ([11]): The system (1) is said to be *input-to-sate stable* (ISS) if there exist a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ such that for any initial state $x(t_0)$ and any bounded input $u(t)$, the solution $x(t)$ exists for all $t \geq t_0$ and satisfies

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0) + \gamma(\sup_{t_0 \leq \tau \leq t} \|u(\tau)\|).$$

Theorem 1 ([11]): Let $V : [0,\infty) \times \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function such that

$$\begin{aligned} \alpha_1(\|x\|) \leq & V(t,x) \leq \alpha_2(\|x\|) \\ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t,x,u) \leq & -W(x), \quad \forall \|x\| \geq \rho(\|u\|) > 0 \end{aligned} \tag{2}$$

for all $(t,x,u) \in [0,\infty) \times \mathbb{R}^n \times \mathbb{R}^m$, where α_1, α_2 are class $\mathcal{K}_\infty$ functions, ρ is a class $\mathcal{K}$ function, and $W(x)$ is a continuous positive definite function on $\mathbb{R}^n$. Then, the system (1) is input-to-state stable (ISS).

III. PROBLEM FORMULATION

A. Nonlinear system model

We consider here affine uncertain nonlinear systems of the form:

$$\begin{aligned} \dot{x} \ &= \ f(x) + \Delta f(t,x) + g(x)u \\ y \ &= \ h(x), \end{aligned} \tag{3}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, $y \in \mathbb{R}^m$ ($p \geq m$), represent respectively the state, the input and the controlled output vectors, $\Delta f(t,x)$ is a vector field representing additive model uncertainties. The vector fields f, Δf, columns of g and function h satisfy the following assumptions.

Assumption 1: The function $f : \mathbb{R}^n \to \mathbb{R}^n$ and the columns of $g : \mathbb{R}^n \to \mathbb{R}^p$ are $\mathbb{C}^\infty$ vector fields on a bounded set X of $\mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}^m$ is a $\mathbb{C}^\infty$ vector on X. The vector field Δf is $\mathbb{C}^1$ on X.

Assumption 2: System (3) has a well-defined (vector) relative degree $\{r_1,\ r_2,\ \cdots,\ r_m\}$ at each point $x^0 \in X$, and the system is linearizable, i.e. $\sum_{i=1}^m r_i = n$.

Assumption 3: The desired output trajectories y_{id} ($1 \leq i \leq m$) are smooth functions of time, relating desired initial points $y_{id}(0)$ at $t = 0$ to desired final points $y_{id}(t_f)$ at $t = t_f$.

B. Control objectives

Our objective is to design a state feedback controller $u(x)$ so that for the uncertain nonlinear model (3) the tracking error is uniformly bounded. We stress here that the goal of parameters tuning is not for stabilization but for performance optimization. To achieve the control objective, we proceed as follows. First, we design a robust controller which can guarantee the input-to-state stability (ISS) of the tracking error dynamics w.r.t the estimation errors input. Then, we combine this controller with a model-free extremum-seeking algorithm to iteratively tune the uncertain parameters, to optimize online a desired performance cost function.

IV. NOMINAL CONTROLLER DESIGN

Under Assumption 2 and nominal conditions, i.e. when $\Delta f(t,x) = 0$, system (3) can be written as, e.g. [6]

$$y^{(r)}(t) \ = \ b(\xi(t)) + A(\xi(t))u(t), \tag{4}$$

where

$$\begin{aligned} y^{(r)}(t) \ &= \ [y_1^{(r_1)}(t),\ y_2^{(r_2)}(t),\ \cdots,\ y_m^{(r_m)}(t)]^T \\ \xi(t) \ &= \ [\xi^1(t),\ \cdots,\ \xi^m(t)]^T \\ \xi^i(t) \ &= \ [y_i(t),\ \cdots,\ y_i^{(r_i-1)}(t)], \quad 1 \leq i \leq m \end{aligned} \tag{5}$$

The functions $b(\xi)$, $A(\xi)$ can be written as functions of f, g and h, and $A(\xi)$ is non-singular in $\tilde{X}$, where $\tilde{X}$ is the image of the set of X by the diffeomorphism $x \to \xi$ between the states of system (3) and the linearized model (4).

At this point, we introduce one more assumption on system (3).

Assumption 4: The additive uncertainties $\Delta f(t,x)$ in (3) appear as additive uncertainties in the input-output linearized model (4)-(5) as follows (see also [12])

$$y^{(r)}(t) \ = \ b(\xi(t)) + A(\xi(t))u(t) + \Delta b(t,\xi(t)), \tag{6}$$

where $\Delta b(t,\xi(t))$ is $\mathbb{C}^1$ on $\tilde{X}$.

If we consider the nominal model (4) first, then we can define a virtual input vector $v(t)$ as

$$v(t) \;=\; b(\xi(t)) + A(\xi(t))u(t). \tag{7}$$

Combining (4) and (7), we can obtain the following input-output mapping

$$y^{(r)}(t) \;=\; v(t). \tag{8}$$

Based on the linear system (8), it is straightforward to apply a stabilizing controller for the nominal system (4) as

$$u_n = A^{-1}(\xi)\left[v_s(t,\xi) - b(\xi)\right], \tag{9}$$

where v_s is a $m \times 1$ vector and the i-th ($1 \le i \le m$) element v_{si} is given by

$$v_{si} = y_{id}^{(r_i)} - K_{r_i}^i(y_i^{(r_i-1)} - y_{id}^{(r_i-1)}) - \cdots - K_1^i(y_i - y_{id}). \tag{10}$$

Denote the tracking error as $e_i(t) \triangleq y_i(t) - y_{id}(t)$, we obtain the following tracking error dynamics

$$e_i^{(r_i)}(t) + K_{r_i}^i e^{(r_i-1)}(t) + \cdots + K_1^i e_i(t) = 0, \tag{11}$$

where $i \in \{1,\ 2,\ \cdots,\ m\}$. By selecting the gains K_j^i where $i \in \{1,\ 2,\ \cdots,\ m\}$ and $j \in \{1,\ 2,\ \cdots,\ r_i\}$, we can obtain global asymptotic stability of the tracking errors $e_i(t)$. To formalize this condition, we make the following assumption.

Assumption 5: There exists a non-empty set $\mathcal{A}$ where $K_j^i \in \mathcal{A}$ such that the polynomials in (11) are Hurwitz, where $i \in \{1,\ 2,\ \cdots,\ m\}$ and $j \in \{1,\ 2,\ \cdots,\ r_i\}$.

To this end, we define $z = [z^1,\ z^2,\ \cdots,\ z^m]^T$, where $z^i = [e_i,\ \dot{e}_i,\ \cdots,\ e_i^{(r_i-1)}]$ and $i \in \{1,\ 2, \cdots,\ m\}$. Then, from (11), we can obtain

$$\dot{z} \;=\; \tilde{A}z,$$

where $\tilde{A} \in \mathbb{R}^{n\times n}$ is a diagonal block matrix given by

$$\tilde{A} \;=\; \mathrm{diag}\{\tilde{A}_1,\ \tilde{A}_2,\ \cdots,\ \tilde{A}_m\}, \tag{12}$$

and $\tilde{A}_i$ ($1 \le i \le m$) is a $r_i \times r_i$ matrix given by

$$\tilde{A}_i \;=\; \begin{bmatrix} 0 & 1 & & & \\ 0 & & 1 & & \\ 0 & & & \ddots & \\ \vdots & & & & 1 \\ -K_1^i & -K_2^i & \cdots & \cdots & -K_{r_i}^i \end{bmatrix}.$$

As discussed above, the gains K_j^i can be chosen so that the matrix $\tilde{A}$ is Hurwitz. Thus, there exists a positive definite matrix $P > 0$ such that (see e.g. [11])

$$\tilde{A}^T P + P\tilde{A} \;=\; -I. \tag{13}$$

V. Robust Controller Design

A. Preliminary Analysis

We now consider the uncertain model (3), i.e. when $\Delta f(t,x) \neq 0$. The corresponding linearized model is given by (6) where $\Delta b(t,\xi(t)) \neq 0$. The global asymptotic stability of the error dynamics (11) cannot be guaranteed anymore due to the additive uncertainty $\Delta b(t,\xi(t))$. We use Lyapunov reconstruction techniques to design a new controller so that the tracking error is guaranteed to be bounded. The new controller for the uncertain model (6) is defined as

$$u_f \;=\; u_n + u_r, \tag{14}$$

where the nominal controller u_n is given by (9) and the robust controller u_r will be given later on based on particular forms of the uncertainty $\Delta b(t,\xi(t))$. By using the controller (14), from (6) we obtain

$$\begin{aligned} y^{(r)}(t) &= b(\xi(t)) + A(\xi(t))u_f + \Delta b(t,\xi(t)) \\ &= b(\xi(t)) + A(\xi(t))u_n + A(\xi(t))u_r + \Delta b(t,\xi(t)) \\ &= v_s(t,\xi) + A(\xi(t))u_r + \Delta b(t,\xi(t)), \end{aligned} \tag{15}$$

Further, the dynamics for z is given by

$$\dot{z} \;=\; \tilde{A}z + \tilde{B}\delta, \tag{16}$$

where $\tilde{A}$ is defined in (12), δ is a $m \times 1$ vector given by

$$\delta \;=\; A(\xi(t))u_r + \Delta b(t,\xi(t)), \tag{17}$$

and the matrix $\tilde{B} \in \mathbb{R}^{n\times m}$ is given by

$$\tilde{B} \;=\; \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \\ \vdots \\ \tilde{B}_m \end{bmatrix}, \tag{18}$$

with $\tilde{B}_i$ ($1 \le i \le m$) given by a $r_i \times m$ matrix such that

$$\tilde{B}_i(l,q) \;=\; \begin{cases} 1 & \text{if } l = r_i \text{ and } q = i \\ 0 & \text{otherwise} \end{cases}$$

If we apply $V(z) = z^T P z$ as a Lyapunov function for the dynamics (16), where P is the solution of the Lyapunov equation (13), then we obtain

$$\begin{aligned} \dot{V}(t) &= \frac{\partial V}{\partial z}\dot{z} \\ &= z^T(\tilde{A}^T P + P\tilde{A})z + 2z^T P\tilde{B}\delta \\ &= -\|z\|^2 + 2z^T P\tilde{B}\delta, \end{aligned} \tag{19}$$

where δ given by (17) depends on the robust controller u_r.

Next, we will design the controller u_r based on the particular forms of the uncertainties that appear in (6), i.e. $\Delta b(t,\xi(t))$. For notational convenience, the unknown parameter vector/matrix (which may be time-varying) is denoted by $\Delta(t)$ and the estimate for the unknowns is denoted by $\widehat{\Delta}(t)$. Further, the estimation error vector/matrix is given by $e_\Delta(t) = \Delta(t) - \widehat{\Delta}(t)$. The dimensions of Δ (and in turn, $\widehat{\Delta}$ and e_Δ) will be clear from the context.

B. Case 1: State-Independent Uncertainties

We consider the case when $\Delta b(t,\xi(t))$ is simply $\Delta(t)$, where $\Delta(t) = [\Delta_1(t),\ldots,\Delta_m(t)]^T$. Assume that we can obtain the estimate (e.g. by ES) of the unknown parameters $\Delta_i(t)$, which may be time-varying and is denoted by $\widehat{\Delta}_i(t)$, for $i = 1,2,\ldots,m$. Let $\widehat{\Delta}(t) = [\widehat{\Delta}_1(t),\ldots,\widehat{\Delta}_m(t)]^T$. We use the following robust controller

$$u_r = -A^{-1}(\xi)(\tilde{B}^T P z + \widehat{\Delta}(t)). \tag{20}$$

The closed-loop error dynamics can be written as

$$\dot{z} \;=\; f(z, e_\Delta), \tag{21}$$

where $e_\Delta(t)$ is the input to the system, $z(t)$ represents the system state and f is given by

$$f(z,e_\Delta) \;=\; (\tilde{A} - \tilde{B}\tilde{B}^T P)z + \tilde{B}e_\Delta.$$

Theorem 2: Consider the system (3), under Assumptions 1-5 and the assumption that $\Delta b(t,\xi(t)) = [\Delta_1(t),\ldots,\Delta_m(t)]^T$, with the feedback controller (14), where u_n is given by (9) and u_r is given by (20). Then, the closed-loop system (21) is ISS from the estimation errors input $e_\Delta(t) \in \mathbb{R}^m$ to the tracking errors state $z(t) \in \mathbb{R}^n$.

C. Case2: State-Dependent Uncertainties

We consider the second case when $\|\Delta b(t,\xi(t))\|$ is upper bounded by a function of the state $\xi(t)$, i.e.

$$\|\Delta b(t,\xi(t))\| \;\le\; \|\Delta(t)\|\|L(\xi)\|, \tag{22}$$

where $\Delta(t) \in \mathbb{R}^{m\times m}$ and $L(\xi)$ is a known bounded function. Assume that we can obtain the estimate (e.g. by ES) for $\Delta(i,j)$, which may be time-varying and is denoted by $\widehat{\Delta}(i,j)$, for $i,j = 1,2,\ldots,m$. Let $\widehat{\Delta}(t)$ be the matrix with the element $\widehat{\Delta}(i,j)$. We use the following robust controller

$$\begin{aligned} u_r = & -A^{-1}(\xi)\tilde{B}^T P z\|L(\xi)\|^2 \\ & - A^{-1}(\xi)\|\widehat{\Delta}(t)\|\|L(\xi)\|\mathrm{sign}(\tilde{B}^T P z). \end{aligned} \tag{23}$$

Similar to the previous case, the closed-loop error dynamics can be written in the form of

$$\dot{z} \;=\; f(t,z,e_\Delta), \tag{24}$$

where $e_\Delta(t)$ is the system input and $z(t)$ is the system state.

Theorem 3: Consider the system (3), under Assumptions 1-5 and the assumption that $\Delta b(t,\xi(t))$ satisfies (22), with the feedback controller (14), where u_n is given by (9) and u_r is given by (23). Then, the closed-loop system (24) is ISS from the estimation errors input $e_\Delta(t) \in \mathbb{R}^{m\times m}$ to the tracking errors state $z(t) \in \mathbb{R}^n$.

D. Case 3: Sum of a State-dependent Term and a Time-dependent Term

We consider the third case when $\Delta b(t,\xi(t))$ is composed of a state-dependent term and a time-dependent term, i.e.

$$\Delta b(t,\xi(t)) \;=\; \Delta(t)(Q(\xi) + \eta(t)), \tag{25}$$

where $\Delta(t) \in \mathbb{R}^{m\times m}$, $Q(\xi)$ is a known bounded function, the vector $\eta(t)$ is unknown but the upper bound for $\|\eta(t)\|$ is known to be C_1, i.e. $\|\eta(t)\| \le C_1$. Assume that we can obtain the estimate (e.g. by ES) for $\Delta(i,j)$, which may be time-varying and is denoted by $\widehat{\Delta}(i,j)$, for $i,j = 1,2,\ldots,m$. Let $\widehat{\Delta}(t)$ be the matrix with the element $\widehat{\Delta}(i,j)$ that locates at the i-th row and j-th column. We use the following robust controller

$$\begin{aligned} u_r \;=\; & -A^{-1}(\xi)[\tilde{B}^T P z\|Q(\xi)\|^2 + \widehat{\Delta}(t)\times Q(\xi) \\ & + \|\widehat{\Delta}(t)\|C_1\mathrm{sign}(\tilde{B}^T P z) + \tilde{B}^T P z C_1^2]. \end{aligned} \tag{26}$$

Similar to the previous two cases, the closed-loop error dynamics can be written in the following form

$$\dot{z} \;=\; F(t,z,e_\Delta), \tag{27}$$

where $e_\Delta(t)$ is the system input and $z(t)$ is the system state.

Theorem 4: Consider the system (3), under Assumptions 1-5 and the assumption that $\Delta b(t,\xi(t))$ satisfies (25), with the feedback controller (14), where u_n is given by (9) and u_r is given by (26). Then, the closed-loop system (27) is ISS from the estimation errors input $e_\Delta(t) \in \mathbb{R}^{m\times m}$ to the tracking errors state $z(t) \in \mathbb{R}^n$.

VI. Multi-parametric ES-based Adaptation

Let us now define the following cost function

$$J(\widehat{\Delta},t) \;=\; F(z(\widehat{\Delta}),t) \tag{28}$$

where $F : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^+$, $F(0,t) = 0$, $F(z,t) > 0$ for $z \neq 0$. We need the following assumptions on J.

Assumption 6: The cost function J has a local minimum at $\widehat{\Delta}^*(t) = \Delta(t)$.

Assumption 7: $|\frac{\partial J(\widehat{\Delta},t)}{\partial t}| < \rho_J$, for any $t \in \mathbb{R}^+$ and any $\widehat{\Delta} \in \mathbb{R}^p$.

Remark 1: Assumption 6 simply states that the cost function J has at least a local minimum at the true values of the uncertain parameters.

We can now present the following result for Case 1, i.e. the case that is studied in Section V-B.

Lemma 1: Consider the system (16) with the cost function (28), under Assumptions 6-7 and the assumption that $\Delta(t) = [\Delta_1(t),\ldots,\Delta_m(t)]^T$, with the feedback controller (14), where u_n is given by (9) and u_r is given by (20), and $\widehat{\Delta}(t)$ is estimated through the MES algorithm

$$\dot{\widehat{\Delta}}_i \;=\; a_i\sqrt{\omega_i}\cos(\omega_i t) - k_i\sqrt{\omega_i}\sin(\omega_i t)J(\widehat{\Delta},t), \tag{29}$$

where $i \in \{1,2,\ldots,p\}$, $a_i > 0$, $k_i > 0$, $\omega_i \neq \omega_j$, and $\omega_i > \omega^*$, with ω^* large enough, ensures that the norm of the tracking error admits the following bound

$$\|z(t)\| \le \beta(\|z(t_0)\|,t) + \gamma(\sup_{0\le\tau\le t}\|e_\Delta(\tau)\|)$$

where $\beta \in \mathcal{KL}$, $\gamma \in \mathcal{K}$ and $\|e_\Delta\|$ satisfies:

1) $(\frac{1}{\omega},d)$-Uniform Stability: For every $c_2 \in (d,\infty)$, there exists $c_1 \in (0,\infty)$ and $\hat{\omega} > 0$ such that for all $t_0 \in \mathbb{R}$ and for all $e_\Delta(0) \in \mathbb{R}^m$ with $\|e_\Delta(0)\| < c_1$ and for all $\omega > \hat{\omega}$,

$$\|e_\Delta(t,e_\Delta(0))\| < c_2, \quad \forall t \in [t_0,\infty)$$

TABLE I

SYSTEM PARAMETERS FOR THE MANIPULATOR EXAMPLE.

Parameter	Value
I_2	$\frac{5}{12}$ $[kg \cdot m^2]$
m_1	10 $[kg]$
m_2	5 $[kg]$
ℓ_1	1 $[m]$
ℓ_2	1 $[m]$
ℓ_{c_1}	0.5 $[m]$
ℓ_{c_2}	0.5 $[m]$
I_1	$\frac{10}{12}$ $[kg \cdot m^2]$
g	9.8 $[m/s^2]$

2) $(\frac{1}{\omega}, d)$-Uniform Ultimate Boundedness: For every $c_1 \in (0, \infty)$, there exists $c_2 \in (d, \infty)$ and $\hat{\omega} > 0$ such that for all $t_0 \in \mathbb{R}$ and for all $e_\Delta(0) \in \mathbb{R}^m$ with $\|e_\Delta(0)\| < c_1$ and for all $\omega > \hat{\omega}$,

$$\|e_\Delta(t, e_\Delta(0))\| < c_2, \quad \forall t \in [t_0, \infty)$$

3) $(\frac{1}{\omega}, d)$-Global Uniform Attractivity: For all $c_1, \ c_2 \in (d, \infty)$ there exists $T \in [0, \infty)$ and $\hat{\omega} > 0$ such that for all $t_0 \in \mathbb{R}$ and for all $e_\Delta(0) \in \mathbb{R}^m$ with $\|e_\Delta(0)\| < c_1$ and for all $\omega > \hat{\omega}$,

$$\|e_\Delta(t, e_\Delta(0))\| < c_2, \quad \forall t \in [t_0 + T, \infty)$$

Similar bounds can be derived for the two remaining cases but are omitted here because of space constraints.

VII. Mechatronic Example

A. Two-link Manipulator

We consider here a two-link robot manipulator. The dynamics for the manipulator is given by (see e.g. [14])

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) \ = \ \tau, \tag{30}$$

where $q \triangleq [q_1, q_2]^T$ denotes the two joint angles and $\tau \triangleq [\tau_1, \tau_2]^T$ denotes the two joint torques. The matrix H is assumed to be non-singular and is given by

$$H \ \triangleq \ \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix},$$

where

$$\begin{aligned} H_{11} &= m_1\ell_{c_1}^2 + I_1 + m_2[\ell_1^2 + \ell_{c_2}^2 + 2\ell_1\ell_{c_2}\cos(q_2)] + I_2 \\ H_{12} &= m_2\ell_1\ell_{c_2}\cos(q_2) + m_2\ell_{c_2}^2 + I_2 \\ H_{21} &= H_{12} \\ H_{22} &= m_2\ell_{c_2}^2 + I_2. \end{aligned} \tag{31}$$

The matrix $C(q, \dot{q})$ is given by

$$C(q, \dot{q}) \ \triangleq \ \begin{bmatrix} -h\dot{q}_2 & -h\dot{q}_1 - h\dot{q}_2 \\ h\dot{q}_1 & 0 \end{bmatrix},$$

where $h = m_2\ell_1\ell_{c_2}\sin(q_2)$. The vector $G = [G_1, G_2]^T$ is given by

$$\begin{aligned} G_1 &= m_1\ell_{c_1}g\cos(q_1) + m_2 g[\ell_2\cos(q_1 + q_2) + \ell_1\cos(q_1)] \\ G_2 &= m_2\ell_{c_2}g\cos(q_1 + q_2). \end{aligned} \tag{32}$$

In our simulations, we assume that the parameters take values according to [14], and summarized in Table I. The system dynamics (30) can be rewritten as

$$\ddot{q} \ = \ H^{-1}(q)\tau - H^{-1}(q)\left[C(q, \dot{q})\dot{q} + G(q)\right]. \tag{33}$$

Thus, the nominal controller is given by

$$\begin{aligned} u_n \ &= \ [C(q, \dot{q})\dot{q} + G(q)] \\ &+ \ H(q)\left[\ddot{q}_d - K_2(\dot{q} - \dot{q}_d) - K_1(q - q_d)\right], \end{aligned} \tag{34}$$

where $q_d = [q_{1d}, q_{2d}]^T$, denotes the desired trajectory and the feedback gains $K_1 > 0$, $K_2 > 0$, are chosen such that the tracking error go to zero asymptotically. For simplicity, we use the feedback gains $K_j^i = 1$ in (10) for $i = 1, 2$ and $j = 1, 2$ in our simulations. The reference trajectory is given by the following function from the initial time $t_0 = 0$ to the final time t_f, where

$$q_{id}(t) \ = \ \frac{1}{1 + \exp(-t)}, \quad i \ = \ 1, 2$$

Now we assume that the nonlinear model (30) is uncertain. In particular, we assume that there exist additive uncertainties in the model (33), i.e.

$$\ddot{q} = \ H^{-1}(q)\tau - H^{-1}(q)\left[C(q, \dot{q})\dot{q} + G(q)\right] + \Delta b(q, t). \tag{35}$$

We will illustrate our approach for the uncertain model (35). In the following, we consider the cost function

$$J \ = \ Q_1 \int_0^{t_f} \|q - q_d\|^2 \mathrm{d}t + Q_2 \int_0^{t_f} \|\dot{q} - \dot{q}_d\|^2 \mathrm{d}t, \tag{36}$$

where $Q_1 > 0$ and $Q_2 > 0$ denote the weighting parameters.

B. MES Based Adaptation

Due to space limitation, we report hereafter only the case presented in Section V-B, when $\Delta b(q, t)$ is simply a time-varying vector $\Delta(t)$, where $\Delta(t) = [\Delta_1(t), \Delta_2(t)]^T$. However, we underline that we have successfully tested the remaining cases and that all the results will be reported in a longer journal version of this work.

Here the controller is designed according to Theorem 2 and the two unknowns $\Delta_1(t)$ and $\Delta_2(t)$ are identified by the MES (29) such that the cost function J in (36) is minimized.

We simulate the system with

$$\begin{aligned} \Delta_1(t) \ &= \ 1 - 0.14\sin(0.01t) \\ \Delta_2(t) \ &= \ 1 - 0.12\cos(0.01t). \end{aligned} \tag{37}$$

The estimate of these two parameters $\widehat{\Delta}_i$ $(i = 1, 2)$ are computed using a discrete version of (29), given by

$$\begin{aligned} \Delta_i(k+1) \ &= \Delta_i(k) + t_f(\alpha_i\sqrt{\omega_i}\cos(\omega_i t_f k) \\ &- \kappa_i\sqrt{\omega_i}\sin(\omega_i t_f k)J), \quad i = 1, 2 \end{aligned} \tag{38}$$

where $k = 0, 1, 2, \cdots$ denotes the iteration index and $\widehat{\Delta}_i$ $(i = 1, 2)$ start from zero initial conditions. The parameters used in the cost function (36) and the MES (38) are summarized in Table II. For more details about the tuning of the parameters in the MES, we refer the reader to [1]. However, we underline here that the frequencies ω_1 and ω_2 have been selected high enough to ensure efficient

TABLE II

PARAMETERS USED FOR CASE 1.

Q_1	Q_2	α_1	α_2	ω_1	ω_2	t_f	κ_1	κ_2
0.325	0.325	0.01	0.01	9.9	9.8	4	0.01	0.01

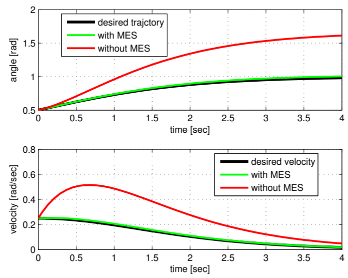

Fig. 1. Case 1: Obtained trajectories vs. reference trajectories for q_1 (in the top plot) and $\dot{q}_1$ (in the bottom plot).

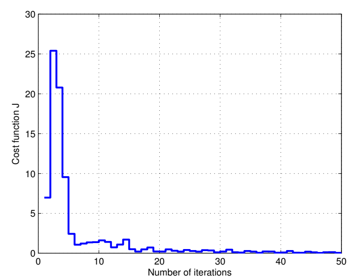

Fig. 2. Case 1: The zoom-in cost function vs. the number of iterations.

exploration on the search space and to ensure convergence and that the amplitudes α_i and κ_i $(i = 1, 2)$ of the dither signals have been selected such that the search is fast enough. The uncertain parameter vector $\widehat{\Delta}$ is updated for each cycle, i.e. at the end of each cycle at $t = t_f$, the cost function J is updated, and the new estimate of the parameters is computed for the next cycle. The purpose of using MES along with the ISS-based controller is to improve the performance of the controller by better estimating the unknown parameters over many cycles, hence decreasing the estimation errors over time to provide better trajectory tracking. As shown in Fig. 1, the ISS-based controller combined with ES greatly improves the tracking performance. Fig. 2 show that the cost function starts at an initial value around 7 and decreases below 1 within 20 iterations. Moreover, the estimate of the unknown parameters converge to a neighborhood of the true parameter trajectories within 100 iterations, as shown in Fig. 3.

VIII. CONCLUSION

In this paper, we have studied the problem of extremum seeking-based indirect adaptive control for nonlinear systems affine in the control with time-varying parametric uncertainties. We have proposed a robust controller which renders the feedback dynamics ISS w.r.t the parameter estimation errors. Then we have combined the ISS feedback controller with a model-free ES algorithm to obtain a learning-based

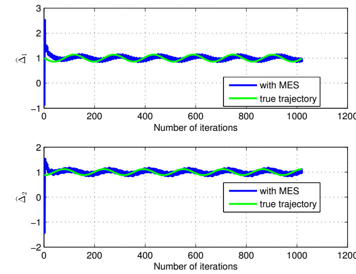

Fig. 3. Case 1: Estimate of parameter Δ_1 (in the top plot) and parameter Δ_2 (in the bottom plot).

adaptive controller. We have presented the stability proof of this controller and have shown a detailed application of this approach on a two-link robot manipulator example. Future works will deal with considering controllers under input constraints, using different ES algorithms with less restrictive tuning conditions, and comparing the obtained controllers to some available classical nonlinear adaptive controllers.

REFERENCES

[1] K. B. Ariyur and M. Krstic, *Real Time Optimization by Extremum Seeking Control*. New York, NY, USA: John Wiley & Sons, Inc., 2003.

[2] S. Drakunov, U. Ozguner, P. Dix, and B. Ashrafi, "Abs control using optimum search via sliding modes," *Control Systems Technology, IEEE Transactions on*, vol. 3, no. 1, pp. 79–85, 1995.

[3] M. Benosman and G. Atinc, "Multi-parametric extremum seeking-based learning control for electromagnetic actuators," in *American Control Conference (ACC), 2013*, June 2013, pp. 1914–1919.

[4] G. Atinc and M. Benosman, "Nonlinear backstepping learning-based adaptive control of electromagnetic actuators with proof of stability," in *Decision and Control (CDC), 2013 IEEE 52nd Annual Conference on*, Dec 2013, pp. 1277–1282.

[5] T. Zhang, M. Guay, and D. Dochain, "Adaptive extremum seeking control of continuous stirred-tank bioreactors," *AIChE Journal*, vol. 49, no. 1, pp. 113–123, 2003.

[6] A. Isidori, *Nonlinear Control Systems (Communications and Control Engineering)*. Springer London, UK, 1995.

[7] L. Freidovich and H. Khalil, "Performance recovery of feedback-linearization-based designs," *Automatic Control, IEEE Transactions on*, vol. 53, no. 10, pp. 2324–2334, Nov 2008.

[8] A. L. D. Franco, H. Bourles, E. De Pieri, and H. Guillard, "Robust nonlinear control associating robust feedback linearization and h infinity control," *Automatic Control, IEEE Transactions on*, vol. 51, no. 7, pp. 1200–1207, July 2006.

[9] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and Adpative Control Design*. New York: Wiley, 1995.

[10] C. Wang, D. J. Hill, S. S. Ge, and G. Chen, "An iss-modular approach for adaptive neural control of pure-feedback systems," *Automatica*, vol. 42, no. 5, pp. 723–731, 2006.

[11] H. Khalil, *Nonlinear Systems*, 3rd ed. Prentice Hall, 2002.

[12] M. Benosman, F. Liao, K.-Y. Lum, and J. L. Wang, "Nonlinear control allocation for non-minimum phase systems," *Control Systems Technology, IEEE Transactions on*, vol. 17, no. 2, pp. 394–404, March 2009.

[13] E. D. Sontag and Y. Wang, "On characterizations of the input-to-state stability property," *Systems and Control Letters*, vol. 24, no. 5, pp. 351 – 359, 1995.

[14] M. W. Spong, "On the robust control of robot manipulators," *Automatic Control, IEEE Transactions on*, vol. 37, no. 11, pp. 1782–2786, 1992.

[15] M. Benosman, "Extremum-seeking based adaptive control for nonlinear systems," in *IFAC World Congress, 2014*, to appear.

[16] ——, "Learning-based adaptive control for nonlinear systems," in *Control Conference (ECC), 2014 European*, June 2014, pp. 920–925.