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Economical Manufacturing from Optimal Control Perspective: Simplification, Methods and Analysis

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Abstract: This paper presents a preliminary study of economical manufacturing from optimal control perspective. We begin with modeling two manufacturing systems as linear and nonlinear constrained continues-time dynamics, respectively. Economical manufacturing, characterized by minimal economical cost, is formulated as optimal control problems with variations on the cost function and system dynamics. The optimal control problems are interesting and challenging to solve due to constraints, nonlinearity, battery components, and consideration of energy price profiles. Rigorous analysis establishes the characteristics of optimal solutions and/or properties of optimal control problems. Methods and techniques to solve these optimal control problems are discussed for further investigation.

Keywords: Optimal control, manufacturing, batteries, energy management, demand response

1. INTRODUCTION

Environmentally sustainable manufacturing practices, e.g. green manufacturing, attract increasing attentions from stakeholders to research communities for regulatory requirements, product stewardship, public image and potential competitive advantages (Rusinko, 2007). Strategies to realize green manufacturing focus on making products with less material and energy, more use of renewable resources, eliminating wastes, etc. While all resources consumed and wastes produced by manufacturing affect the environment to certain extents, energy efficiency, or efficient and effective utilization of energy resources, has been identified as one of the key drivers or metrics for sustainability (Duflou et al., 2012; Salonitis and Ball, 2013).

Energy efficiency in manufacturing can be gained at various levels: machine, process, and system levels in Deif (2011) or device/process, line/cell/multi-machine system, facility, multi-factory system, and enterprise/global supply chain levels in Duflou et al. (2012). For instance, Diaz et al. (2010, 2011) consider machine tool design and operation strategies for energy efficiency; Diaz et al. (2009) redesigns a process by introducing a kinetic energy recovery system for energy recycling during deceleration of spindles; Salonitis (2012) reduces the energy consumption of a grinding process by optimizing process steps. Salonitis and Ball (2013) discuss energy efficiency at the machine tool and manufacturing system level, and identify several barriers to effective energy reduction.

This paper considers economical manufacturing aiming to minimize the economical cost affected by consumed energy as well as energy price. This is different from green manufacturing which focuses on energy efficiency. Ignoring the energy price information during design and operation, current green manufacturing practices do not necessarily lead to energy efficient and environmentally friendly operation. This is particularly true when the energy price reflects energy efficiency of energy suppliers and thus fluctuates. For simplicity, we only consider electricity as the energy resource of manufacturing processes, and pose economical manufacturing as the demand side management or demand response problem in Gellings (1985); Albadi and El-Saadany (2007); Samadi et al. (2010). Optimal control problems are consequently formulated and analyzed rigorously.

Manufacturing processes can be continuous or discrete over time, and the models can be complicated. Even at the device level, the energy consumption modeling is challenging and relies largely on measurements (Salonitis and Ball, 2013; Duflou et al., 2012). This work avoids getting into the tedious modeling of a manufacturing process and its energy consumption by considering two simplified continuous-time models, and concentrates on characterizing the properties of economical manufacturing subject to electricity price fluctuations. It is worth noting that this work is related to the recent research on power management of hybrid electric vehicle (Lin et al., 2003) and micro-grid (Katiraei et al., 2008).

This paper is organized as follows. Section 2 models manufacturing systems and formulates the economical manufacturing as optimal control problems. In Section 3, characteristics of optimal control problems and corresponding optimal solutions are analyzed. Conclusion is made in Section 4.

2. PRELIMINARIES

Manufacturing sectors, such as petroleum refining, metals processing, chemical production, semi-conductor manufacturing, and paper product manufacturing, are characterized by semi-continuous processes (Duflou et al., 2012). Accordingly, a manufacturing system has continuous and discrete states, and the dynamics could be both time and event-driven, i.e., in hybrid nature. Seeking optimal scheduling and control of manufacturing systems resorts to solve optimal control of hybrid systems, which is either difficult or computationally prohibitive. Interested readers are referred to Ramadge and Wonham (1987); Pepyne and Cassandras (2000); Shaikh and Caines (2007); Vasudevan et al. (2012) and references therein for details. This paper presents a preliminary study on scheduling the entire manufacturing process with consideration of energy price, and thus considers continuous dynamics for simplicity and ignore the discrete-event dynamics.

2.1 Modeling of a Conventional Manufacturing System

Dynamics A manufacturing system typically consists of machines that can perform a variety of tasks on a family of parts. Different types of machines may have distinctive dynamics, which pose significant difficulty in analyzing and solving the economical manufacturing problem due to the complexity in the system and energy consumption models. We assume the energy consumption of each machine is directly related to its productivity, which is practical in motion control systems (Wang et al., 2013). Consider the productivity and product produced as the state of a manufacturing process, and the electric power as its input. We have the following simplified manufacturing system dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \alpha(u_e) \end{aligned} \tag{1}$$

where $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$ denote the product and productivity, respectively, $\alpha(u_e) : \mathbb{R} \to \mathbb{R}$ is continuous, u_e the power taken from the power grid, and $\mathbb{R}$ is the set of real numbers. Denote the state vector by $x = (x_1, x_2)^T$. Note that including the productivity as a state implies that it cannot jump. Also x_1 and x_2 can be vectors so that their ith components represent, respectively, the product and productivity of the ith machine.

Remark 1. The dynamics (1) are essentially a black-box model. We assume $\alpha(u_e)$ can be identified by measuring the power consumption and produced products. Typically, in order to identify $\alpha(u_e)$, we assume that $\alpha(u_e)$ has linear parameterizations as follows

$$\alpha(u_e) = \sum_{k=0}^{p} \theta_k u_e^k$$

where $\theta_k, 0 \leq k \leq p$ is constant to be identified. For simplicity, we take $\alpha(u_e) = u_e$. A slight generalization to this special case is taking $\alpha(u_e)$ as a linear function of x. The methodology presented in the sequel may still be applicable provided that $\alpha(u_e)$ is convex.

Constraints In terms of state constraints, it is clear that $0 \leq x_1$, i.e., the product quantitiy cannot be negative. Similarly, productivity should satisfy the following constraint

$$0 \leq x_2 \leq v_{\max} \tag{2}$$

where $v_{\max} > 0$ is constant and represents the highest possible productivity of the process. Without loss of generality, we assume that the initial product quantity is 0. The constraint on x_1, i.e., $0 \leq x_1$, is implied by constraint (2), and thus can be omitted. The state constraint is therefore given by (2).

Control constraints literally reflect the limitations of hardware such as transformers, power electronics, and electric motors. We assume that specifications of each hardware can be cast into a set of constraints on u_e, a union of which constitutes the control constraints. To further simplify the presentation, we assume that the specifications of electric motors lead to the following constraint

$$a_1 \leq u_e \leq a_2 \tag{3}$$

where a_1 and a_2 are constant and represent the lower and upper bounds of u_e, respectively.

Objectives The target of a manufacturing process is to produce a fixed product quantity within a certain period $[0, T]$, and minimize the cost, typically the energy bill. The energy cost due to operating the manufacturing process over an interval $[0, T]$ may vary according to different hardware. For instance, if the process is not equipped with bi-directional inverters, the regenerative power due to slowing down the manufacturing process is wasted. Thus its corresponding cost function is written as follows

$$E_1(x, u_e) = \int_0^T \max(0, p(t)u_e(t))\mathrm{d}t \tag{4}$$

where $p(t)$ is the electricity price. For simplicity, we assume that $p(t)$ is known. Readers are referred to Mohsenian-Rad and Leon-Garcia (2010) and references therein for details on pricing models.

Remark 2. Control engineers typically act as process planners who optimize process parameters to reduce energy consumption. In this paper, the process parameter is the productivity. The energy saving achieved by optimizing the productivity however might be limited for certain manufacturing systems. For instance, the laser source and chiller are responsible for more than 80% of the total consumed energy for a laser cutting machine tool according to Duflou et al. (2010).

Alternatively, if bi-directional inverters are installed to enable the recycling of regenerated electricity, the energy cost is given by

$$E_2(x, u_e) = \int_0^T p(t)u_e(t)\mathrm{d}t. \tag{5}$$

2.2 Modeling of A New Manufacturing System

We next present a simplified model of a new manufacturing system including a battery component. The battery component is used as an additional power source to complement the power grid. Battery dynamics can be very complicated (Fuller et al., 1994). Here we take the following simple battery model (Wang et al., 2014)

$$\begin{aligned} \dot{x}_3 &= \eta I \\ V &= h(x_3) + RI \end{aligned} \tag{6}$$

where x_3 represents the state of charge (SOC) of the battery, I the charge (> 0) and discharge current (< 0), V the terminal voltage, η the charge efficiency constant, h is a smooth function and denotes the open circuit voltage (OCV), and $R > 0$ is the internal resistance. The power delivered by the battery component is therefore given by

$$u_b = VI = h(x_3)I + RI^2.$$

Note that $u_b > 0$ during charge and $u_b < 0$ during discharge.

With the battery as an additional power source to the manufacturing process, we modify (1) by including u_b as another input. We have the model of the new manufacturing system as follows

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= u_e - (h(x_3)I + RI^2) \\ \dot{x}_3 &= \eta I. \end{aligned} \tag{7}$$

Notation $x = (x_1, x_2, x_3)^T$ is abused to denote the state vector of system (7).

Constraints The battery SOC is subject to constraints, which is typically denoted by $0 \le x_3 \le 1$, with $x_3 = 0$ and $x_3 = 1$ representing the zero and full charge states, respectively. Combining (2), we have the state constraints of the new manufacturing system as follows

$$\begin{aligned} 0 \le x_2 \le v_{\max} \\ 0 \le x_3 \le 1. \end{aligned} \tag{8}$$

The battery component introduces the charge and discharge current I as an extra control input. In order to ensure a safe operation of the battery, I is subject to the constraint $|I| < I_{\max}$, where $I_{\max} > 0$ is constant. Including the battery component also changes the original control constraint related to hardware limitations. Since we assume that the control constraint results from limitations of electric motors, the original constraint (3) is changed accordingly to: $a_1 \le u_e - (h(x_3)I + RI^2) \le a_2$. We finally have control constraints as follows

$$\begin{aligned} &|I| < I_{\max} \\ &a_1 \le u_e - (h(x_3)I + RI^2) \le a_2. \end{aligned} \tag{9}$$

Objectives The target of the new manufacturing system is to produce a fixed product quantity within a certain period $[0, T]$, and minimize the energy cost. The objectives can be captured by the cost functions (4)-(5).

2.3 Problem Formulation

This work considers and evaluates the following optimal control problems for system (1).

Problem 3. Given plant (1), initial state $x(0) = x_0 = (0, 0)^T$, final state $x(T) = x_f = (r, 0)^T$, and final time T, find control u_e^* which minimizes cost function $E_1(x, u_e)$ in (4), subject to state and control constraints (2)-(3).

Problem 4. Given plant (1), initial state $x(0) = x_0 = (0, 0)^T$, final state $x(T) = x_f = (r, 0)^T$, and final time T, find control u_e^* which minimizes cost function $E_2(x, u_e)$ in (5), subject to state and control constraints (2)-(3).

This work considers and evaluates the following optimal control problems for system (7).

Problem 5. Given plant (7), initial state $x(0) = x_0 = (0, 0, 1)^T$, final state $x(T) = x_f = (r, 0, 1)^T$, and final time T, find control (u_e^*, I^*) which minimizes cost function $E_1(x, u_e)$ in (4), subject to state and control constraints (8)-(9).

Problem 6. Given plant (7), initial state $x(0) = x_0 = (0, 0, 1)^T$, final state $x(T) = x_f = (r, 0, 1)^T$, and final time T, find control (u_e^*, I^*) which minimizes the cost function $E_2(x, u_e)$ in (5), subject to state and control constraints (8)-(9).

In Problems 5-6, the initial and final SOCs of the battery are assumed the same and at the full charge state. This assumption is made to simplify the problem. From application point of view, it is reasonable to assume the initial and final SOCs of the battery are the same. However, assuming the battery starts at the full charge state may not be optimal because it eliminates the freedom to store energy at the beginning of production.

3. MAIN RESULTS

For simplicity, we assume that Problems 3 - 6 have feasible solutions. Existence of optimal solutions for Problems 3-6 can be established by verifying results in Cesari (1965), and thus its detailed discussion is omitted here.

3.1 Case 1: $p(t)$ is Constant

Without loss of generality, we take $p(t) = 1$.

Optimal Solutions of Problem 3: Assume that Problem 3 admits optimal controls which are piecewisely continuous over time. This allows us to reformulate the cost function and yields

$$\begin{aligned} E_1(x, u_e) &= \sum_{k=1}^{m} \int_{t_k}^{t_{k+1}} u_e(t) \mathrm{d}t \\ &= \sum_{k=1}^{m} [x_2(t_{k+1}) - x_2(t_k)] \end{aligned} \tag{10}$$

where $u_e(t) > 0, \forall t \in [t_k, t_{k+1}]$ and $t_1 = 0$, and $t_k, 1 \le k \le m$ is the time instant when control is discontinuous. Without loss of generality, we assume $x_2(t)$ achieves its maximum at t_m. From $u_e(t) > 0, \forall t \in [t_k, t_{k+1}]$, we know

$$x_2(t_k) \le x_2(t_{k+1}), \quad 1 \le k \le m - 1.$$

On the other hand, we have

$$x_2(t_k) \ge x_2(t_{k+1}), \quad 2 \le k \le m - 2.$$

With this fact, (10) can be further reduced to

$$\begin{aligned} E_1 &= x_2(t_1) + x_2(t_m) + \sum_{k=1}^{m-1} [x_2(t_k) - x_2(t_{k+1})] \\ &\ge x_2(t_m) = \max_{t \in [0,T]} x_2(t) \end{aligned}$$

which implies that the minimum of the cost function should not be less than the maximum of $x_2(t)$. Next we show the existence of a feasible solution giving $E_1 = \max_{t\in[0,T]} x_2(t)$, and thus Problem 3 is equivalent to $\min_{u_e} \max_{t\in[0,T]} x_2(t)$, which can be solved analytically.

To further analyze the properties of optimal solutions to Problem 3, we assume a feasible solution that has the maximum productivity $\bar{x}_2(t)$ at $t = \bar{t}$, and consider the following problem.

Problem 7. Given plant (1) and $\bar{t}$, initial state $x(0) = x_0 = (0, 0)^T$, final state $x(\bar{t}) = (\bar{r}, \bar{x}_2)^T$, with fixed $\bar{r}$ and $\bar{x}_2$, find control u_e^* which minimizes cost function $E_1(x, u_e)$ over $[0, \bar{t}]$, subject to state and control constraints (2)-(3).

We have the following result about Problem 7.

Proposition 8. A feasible solution to Problem 7 implies the existence of another feasible solution where control u_e is non-negative over $[0, \bar{t}]$.

Proof. Assume that Problem 7 has a feasible solution where control $u_e^1(t)$ is negative during a certain interval $[t_1, t_2]$ and the corresponding state at t_2 is denoted $x(t_2) = (x_1(t_2), x_2(t_2))^T$, i.e.,

$$x_2(t) = \int_0^t u_e^1(t)\mathrm{d}t, \quad x_1(t_2) = \int_0^{t_2} x_2(t)\mathrm{d}t.$$

It is clear that one can always find another $\hat{u}_e$ defined by

$$\hat{u}_e = \begin{cases} u_e^1(t) + \delta u_e(t), & t \in [0, t_1] \\ 0, & t \in [t_1, t_2] \end{cases}$$

such that

$$\hat{x}_2(t) = \int_0^t \hat{u}_e(t)\mathrm{d}t$$

with $\hat{x}_2(t_2) = x_2(t_2)$ and

$$\hat{x}_1(t_2) = \int_0^{t_2} \hat{x}_2(t)\mathrm{d}t = x_1(t_2).$$

Hence another feasible solution to Problem 7 can be constructed as

$$u_e^2(t) = \begin{cases} \hat{u}_e(t), & t \in [0, t_2] \\ u_e^1(t), & t \in [t_2, \bar{t}] \end{cases}$$

which is non-negative.

Remark 9. The proof of Proposition 8 is based on assumption that the feasible solution has only one time interval with negative control. This is without loss of generality.

Given Proposition 8, we have the following conclusion about the optimal solution of Problem 7.

Proposition 10. The optimal solution of Problem 7 is not unique and satisfies the following properties

(1) the optimal control $u_e^* \geq 0$ for all $t \in [0, \bar{t}]$
(2) the optimal value is $E_1^* = \bar{x}_2$.

Remark 11. Proposition 10 means the optimal solution of Problem 3 keeps accelerating until reaching the max productivity, and the peak productivity is equal to the cost value. This allows us to compute the optimal solution of Problem 3 analytically.

Optimal Solutions of Problem 4: For Problem 4, we have

$$E_2(x, u_e) = 0$$

for any feasible solution, and thus any feasible solution is optimal. This is because the cost function does not penalize losses during manufacturing, and all electric energy flowing into manufacturing system is recycled.

Optimal Solutions of Problem 6: We rearrange cost function (5) as follows

$$\begin{aligned} E_4 &= \int_0^T u_e(t)\mathrm{d}t \\ &= \underbrace{\int_0^T (u_e - (h(x_3)I + RI^2))\mathrm{d}t}_{E_{41}} \\ &\quad + \underbrace{\int_0^T (h(x_3)I + RI^2)\mathrm{d}t}_{E_{42}} \end{aligned} \tag{11}$$

where E_{41} is the energy cost due to manufacturing, and E_{42} the cost due to losses in the course of charging and discharging the battery. Notice that

$$\begin{aligned} E_{42} &= \int_0^T h(x_3)\frac{1}{\eta}\dot{x}_3(t)\mathrm{d}t + \int_0^T RI^2\mathrm{d}t \\ &= \frac{1}{\eta}\int_0^T h(x_3)\mathrm{d}x_3(t) + \int_0^T RI^2\mathrm{d}t \\ &= \int_0^T RI^2\mathrm{d}t \end{aligned}$$

If the hysteresis effect is considered in the battery operation, the term $\int_0^T h(x_3)\mathrm{d}x_3(t)$ is non-zero, and in fact, positive.

Since the definition of the cost function implies that it incurs loss/cost to use the battery, and is free to use the energy from the power grid, the optimal solution of Problem 5 should reduce to that of Problem 3.

Optimal Solutions of Problem 5: Similar to Problem 6, optimal solutions of Problem 5 can be established by solving Problem 3.

3.2 Case 2: $p(t)$ is Time-Varying

In this case, the energy price is assumed to be independent of the instantaneous power consumption of the manufacturing system itself. Instead, energy suppliers might set the price to ensure the safe operation of the power grid and influence the consumers' behavior. We also assume that $p(t)$ for $t \in [0, T]$ is priori before planning the manufacturing process, and leave the case where $p(t)$ is unknown for future study.

When $p(t)$ is a pre-determined non-constant positive mapping: $\mathbb{R}^+ \to \mathbb{R}^+$, optimal solutions of Problems 3-6 are difficult to characterize and solve in a closed-form. This is because the optimal solution is $p(t)-$dependent. Direct optimal control approach, transcription (by discretization) of continuous-time optimal control problems to numerical optimization problems, has been widely adopted to solve optimal or sub-optimal solutions.

We first show that Problems 3-4 are convex, and thus direct optimal control approach can be used to solve for global optimal solutions. Then, as an example of applying direct optimal control approach, Problem 3 is transcribed to a numerical optimal control problem by discretization over time. The transcription results in a convex optimization problem but with a non-smooth cost function, which is further reformulated as a smooth convex optimization problem.

Problems 3-4 have linear constraints. For the cost function (4), its convexity can be readily established by considering results in (Boyd and Vandenberghe, 2004, Sec. 3.2). For completeness, the verification procedure is sketched as follows: assuming two feasible control $u_e^1(t), u_e^2(t)$ defined over $[0, T]$, and denoting the corresponding costs

$$E(u_e^1) = \int_0^T p(t)\max(0, u_e^1(t))\mathrm{d}t$$
$$E(u_e^2) = \int_0^T p(t)\max(0, u_e^2(t))\mathrm{d}t$$

we have, for $0 \le \theta \le 1$,

$$\begin{aligned} &E(\theta u_e^1 + (1-\theta)u_e^2) \\ &\quad = \int_0^T p(t)\max(0, \theta u_e^1(t) + (1-\theta)u_e^2(t))\mathrm{d}t \\ &\quad \le \int_0^T p(t)[\max(0, \theta u_e^1(t)) + \max(0, (1-\theta)u_e^2(t))]\mathrm{d}t \\ &\quad = \theta E(u_e^1) + (1-\theta)E(u_e^2). \end{aligned}$$

Since both constraints and cost functions are convex in x and u, Problems 3-4 are convex. Optimal solutions however might not be unique, because both of the cost functions (4)-(5) are not strictly convex.

We discretize both cost function (4) and system dynamics (1) on a mesh in the time domain using the Euler integration rule. For simplicity, the time grid is uniform and denoted by $\{t_i\}_{i=0}^N \in [0, T]$, with the step size $\Delta = T/(N+1)$, $t_0 = 0$ and $t_N = T$. The state and control variables are discretized on the mesh $\{t_i\}_{i=0}^N$ as follows: $X_i = x(t_i)$ for $i = 0, \ldots, N$, and $U_i = u_e(t_i)$, for $i = 1, \ldots, N$. For notational convenience, let $X = [X_0, \ldots, X_N]^T$ and $U = [U_1, \ldots, U_N]^T$.

The cost function (4) is discretized as follows

$$E_1^d(U) = \sum_{i=1}^N \Delta p(t_i)\max(0, U_i) \tag{12}$$

The system dynamics (1) are similarly discretized, via the Euler integration rule, into the following linear equations

$$X_i - X_{i-1} - \Delta B U_i = 0, \quad 1 \le i \le N \tag{13}$$

where $B_e = [0, 1]^T$. The discretization of state and control constraints gives, for $1 \le i \le N$,

$$\begin{aligned} 0 \le X_{i,2} \le v_{\max} \\ u_{\min} \le U_i \le u_{\max} \end{aligned} \tag{14}$$

where $X_{i,2}$ represents the 2nd component of X_i. The initial and final conditions are also denoted by linear constraints as follows

$$X_0 = 0, \quad X_N = x_f. \tag{15}$$

Abbreviating constraints (13)-(15) as $G(X, U) \le 0$, we have the non-smooth convex optimization problem:

$$\begin{aligned} &\min_{X,U} E_1^d(U) \\ &\text{subject to} \quad G(X, U) \le 0. \end{aligned} \tag{16}$$

We have the following result about (16).

Proposition 12. Problem (16) is equivalent to the following smooth convex optimization problem

$$\begin{aligned} &\min_{X,U,\zeta} E_1^{d,\zeta}(\zeta) \\ &\text{subject to} \quad G(X, U) \le 0 \\ &\qquad\qquad\quad \zeta \ge 0 \\ &\qquad\qquad\quad \zeta \ge U \end{aligned} \tag{17}$$

where $\zeta = (\zeta_1, \cdots, \zeta_N)^T$, $E_1^{d,\zeta} = \sum_{i=1}^N \Delta p(t_i)\zeta_i$.

Proof. We know $\max(0, U_i)$ is equivalent to

$$\min \zeta_i \quad \text{subject to} \quad \zeta_i \ge 0, \quad \zeta_i \ge U_i.$$

In the feasible set $G(X, U) \le 0$, we have

$$\begin{aligned} &\min_{X,U} E_1^d(U) \\ &= \Delta \min_{X,U} \sum_{i=1}^N p(t_i)\max(0, U_i) \\ &= \Delta \min_{X,U} \sum_{i=1}^N p(t_i)\{\min_{\zeta_i} \zeta_i, \text{ subject to } \zeta_i \ge 0, \ \zeta_i \ge U_i\} \\ &= \Delta \min_{X,U} \sum_{i=1}^N \{\min_{\zeta_i} p(t_i)\zeta_i, \text{ subject to } \zeta_i \ge 0, \ \zeta_i \ge U_i\} \\ &= \Delta \min_{X,U,\zeta} \sum_{i=1}^N p(t_i)\zeta_i, \text{ subject to } \zeta_i \ge 0, \ \zeta_i \ge U_i \end{aligned}$$

where the last equality is obtained by considering that

$$\sum \min a_k \quad \text{subject to} \quad a_k \ge 0, \quad a_k \in \mathcal{A}_k$$

is equivalent to

$$\min \sum a_k \quad \text{subject to} \quad a_k \ge 0, \quad a_k \in \mathcal{A}_k.$$

This completes the proof.

Remark 13. Smooth optimization problem is preferred in practice for the abundance and efficiency of solvers. Proposition 12 facilitates the application of direct optimal control approach to Problem 3.

Different from Problems 3-4, because of non-convexity in the x_2-dynamics, transcription of Problems 5-6 leads to non-convex optimization problems.

3.3 Case 3: $p(t)$ is Determined by u_e

In this case, the energy price is uniquely determined by the power consumption of the manufacturing system itself, i.e., the energy suppliers set the price according to local power consumption. Depending on the price profile $p(u_e)$, Problems 3-6 have different characteristics.

For instance, if the price profile is given as follows

$$p(u_e) = k u_e$$

with $k > 0$, then both cost function $E_1 = \int_0^T k u_e^2(t)\mathrm{d}t$ and $E_2 = \int_0^T k\max(0, u_e^2(t))\mathrm{d}t$ are convex. Problems 3-4 can be readily solved by direct optimal control approach. Hamiltonian approach in Wang et al. (2013) might also be applicable to characterize and further solve the optimal solutions for better accuracy. Taking $k > 0$ means the regenerative energy flowing back to the power grid is also penalized, which is unrealistic. A more realistic price profile is

$$p(u_e) = \begin{cases} k u_e, & u_e \ge 0 \\ -k u_e, & \text{otherwise} \end{cases}$$

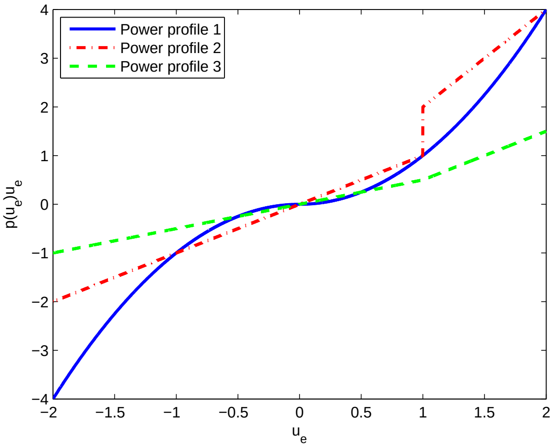

Fig. 1. Profiles of integrated functions

which yields an integrated function of (5) similar to the solid line in Fig. 1. The solid line shows that the integrated function is a non-convex function of u_e. Consistently, the discretization of system dynamics, cost functions, and constraints leads to an optimization problem, which has a non-convex cost function.

An alternative profile of the price might be taken in the form of piecewise constant, for instance

$$p(u_e) = \begin{cases} k_1, & u_e \leq \overline{u} \\ k_2, & \text{otherwise.} \end{cases} \tag{18}$$

Without loss of generality, we consider the cost function (5). The integrated function in (5), with the price profile given by (18) and $k_1 = 1, k_2 = 2$, is illustrated by the dash dot line in Fig. 1. We verify that the integrated function is not convex. Given u_e^1, u_e^2, and $u_e = \theta u_e^1 + (1 - \theta)u_e^2 < \overline{u}_e$, we have

$$p(u_e)u_e = \begin{cases} k_1 u_e, & \text{if } u_e < \overline{u}_e \\ k_2 u_e, & \text{otherwise.} \end{cases}$$

For $u_e^1 < \overline{u}_e, u_e^2 > \overline{u}_e$, we can always take a $\theta \in [0, 1]$ such that $u_e = \theta u_e^1 + (1 - \theta)u_e^2 > \overline{u}_e$, and have

$$p(u_e)u_e = k_2 u_e > \theta k_1 u_e^1 + (1 - \theta)k_2 u_e^2.$$

Meanwhile, we can always take another $\theta \in [0, 1]$ such that $u_e = \theta u_e^1 + (1 - \theta)u_e^2 < \overline{u}_e$, and have

$$p(u_e)u_e = k_1 u_e < \theta k_1 u_e^1 + (1 - \theta)k_2 u_e^2.$$

Since the integrated function is non-convex in u_e, cost function (5) and its disretization over time is also non-convex in u_e. Hence, the direct transcription of Problems 3-6 leads to non-convex optimization problems. Specifically, Problem 4 can be reduced to a mixed integer linear programming problem by introducing a binary variable $\mu(i)$ at each discretization time step i

$$\mu(i) = \begin{cases} 1, & u_e(i) < \overline{u}_e \\ 0, & \text{otherwise} \end{cases}$$

The binary variable $\mu(i)$ allows us to reparameterize the integrated function at time step i as follows

$$\mu(i)k_1 u_e(i) + (1 - \mu(i))k_2 u_e(i).$$

The mixed linear programming problem requires additional constraints

$$\mu(i) = \{0, 1\}.$$

Fig. 1 shows another power profile in the dash line, which corresponds to the following price profile

$$p(u_e(t)) = \begin{cases} k_1, & u_e < \overline{u}_e \\ k_2 - \dfrac{\overline{u}_e(k_2 - k_1)}{u_e}, & \text{otherwise} \end{cases} \tag{19}$$

with $k_2 > k_1$. The integrated function of cost functions (4)-(5) corresponding to the dash line is convex, Problems 3-4 are therefore convex and the global optimal solutions can be solved numerically.

We remark that the direct transcription of Problems 5-6 always renders non-convex optimization problems no matter what price profile is chosen. This is because such non-convexity comes from the dynamics.

4. CONCLUSION

This work conducts a preliminary study of economical manufacturing where the energy cost is optimized instead of the consumed energy. Economical manufacturing addresses the energy price fluctuation. With the productivity of the manufacturing process selected as the process parameter to capture the energy consumption and the system model, economical manufacturing is investigated in the optimal control framework. Future work includes investigation of economical manufacturing with a more realistic system and energy consumption models and pricing mechanisms.

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REFERENCES

Albadi, M.H. and El-Saadany, E.F. (2007). Demand response in electricity markets: an overview. In *the Proc. of the IEEE Power Engineering Society General Meeting*, 1–5.

Boyd, S.P. and Vandenberghe, L. (2004). *Convex Optimization*. Cambridge University Press.

Cesari, L. (1965). Existence theorems for optimal solutions in Pontryagin and Lagrange problems. *SIAM J. Control Optim.*, 3(3), 475–498.

Deif, A.M. (2011). A system model for green manufacturing. *Advances in Production Engineering & Management*, 6(1), 27–36.

Diaz, N., Choi, S., Helu, M., Chen, Y., Jayanathan, S., Yasui, Y., Kong, D., Pavanaskar, S., and Dornfeld, D. (2010). Machine tool design and operation strategies for green manufacturing. In *Proc. of the 4th CIRP Int. Conf. on High Performance Cutting.*

Diaz, N., Redelsheimer, E., and Dornfeld, D. (2011). Energy consumption characterization and reduction strategies for milling machine tool use. In *Proc. of the 18th CIRP Int. Conf. on Life Cycle Engineering*, 263–267.

Diaz, N., Helu, M., Jarvis, A., Tönissen, S., Dornfeld, D., and Schlosser, R. (2009). Strategies for minimum energy operation for precision machining. In *the Proc. of MTTRF 2009 Annual Meeting.*

Duflou, J.R., Kellens, K., Devoldere, T., Deprez, W., and Dewulf, W. (2010). Energy related environmental impact reduction opportunities in machine design: case study of a laser cutting machins. *International Journal of Sustainable Manufacturing*, 2, 80–98.

Duflou, J.R., Sutherland, J.W., Dornfeld, D., Herrmann, C., Jeswiet, J., Kara, S., Hauschild, M., and Kellens, K. (2012). Towards energy and resource efficient manufacturing: A processes and systems approach. *CIRP Annals-Manufacturing Technology*, 61, 587–609.

Fuller, T.F., Doyle, M., and Newman, J. (1994). Simulation and optimization of the dual lithium ion insertion cell. *J. Electrochem. Soc.*, 141(1), 1–10.

Gellings, C. (1985). The concept of demand-side management for electric utilities. *Proc. IEEE*, 73, 1468–1470.

Katiraei, F., Iravani, R., Hatziargyriou, N., and Dimeas, A. (2008). Microgrids management. *IEEE Power Energy Mag.*, 6, 54–65.

Lin, C.C., Peng, H., Grizzle, J.W., and Kang, J.M. (2003). Power management strategy for a parallel hybrid electric truck. *IEEE Trans. Contr. Syst. Technol.*, 11, 839–849.

Mohsenian-Rad, A.H. and Leon-Garcia, A. (2010). Optimal residential load control with price prediction in real-time electricity pricing environments. *IEEE Trans. Smart Grid*, 1, 120–133.

Pepyne, D.L. and Cassandras, C.G. (2000). Optimal control of hybrid systems in manufacturing. *Proc. IEEE*, 88, 1108–1123.

Ramadge, P.J. and Wonham, W.M. (1987). Supervisory control of a class of discrete event process. *SIAM J. Control Optim.*, 25(1), 206–230.

Rusinko, C.A. (2007). Green manufacturing: an evaluation of environmentally sustainable manufacturing practices and their impact on competitive outcomes. *IEEE Trans. Eng. Manag.*, 54(3), 445–454.

Salonitis, K. (2012). Efficient grinding processes: an energy efficiency point of view. In *the Proc. of the 10th Int. Conf. on Manufacturing Research*, 541–546.

Salonitis, K. and Ball, P. (2013). Energy efficient manufacturing from machine tools to manufacturing systems. In *the Proc. of the 46th CIRP Conf. on Manufacturing Systems*.

Samadi, P., Mohsenian-Rad, A.H., Schober, R., Wong, V.W., and Jatskevich, J. (2010). Optimal real-time pricing algorithm based on utility maximization for smart grid. In *the Proc. of the 1st IEEE Int. Conf. on Smart Grid Communications*, 415–420.

Shaikh, M.S. and Caines, P.E. (2007). On the hybrid optimal control problem theory and algorithms. *IEEE Trans. Automat. Control*, 52(9), 1587–1602.

Vasudevan, R., Gonzalez, H., Bajcsy, R., and Sastry, S.S. (2012). Consistent approximations for the optimal control of constrained switched systems. Http://arxiv.org/abs/1208.0062.

Wang, Y., Ueda, K., and Bortoff, S.A. (2013). A Hamiltonian approach to compute an energy efficient trajectory for a servomotor system. *Automatica*, 49(12), 3550–3561.

Wang, Y., Fang, H., Sahinoglu, Z., Wada, T., and Hara, S. (2014). Adaptive estimation of the state of charge for lithium-ion batteries: nonlinear geometric observer approach. *IEEE Trans. Contr. Syst. Technol.*, PP(99), 1.