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Daniel J. Burns, Walter K. Weiss, and Martin Guay

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I. INTRODUCTION

Vapor compression systems (VCS), such as heat pumps, refrigeration and air-conditioning systems, are widely used in industrial and residential applications (Fig. 1A). The introduction of variable speed compressors, electronically-positioned valves, and variable speed fans to the vapor compression cycle has greatly improved the flexibility of the operation of such systems Whereas fixed speed machines meet the heating load required to regulate the room temperature by cycling the compressor in an on-off fashion, variable speed machines allow the cooling capacity of the system to be directly matched to the load. Further, the selection of the actuator values required to meet the load is not unique, and these sets of actuator values consume different amounts of electrical power [1]. Therefore, an energy optimal approach to vapor compression system control selects combinations of actuator values that both meet the load requirement and minimize electrical power consumption.

In typical feedback control architectures for these systems (Fig. 1B), the increased degrees-of-freedom provided by variable actuators imply that multiple simultaneous signals can be regulated. In addition to the zone temperature, internal process signals can be regulated in order to improve efficiency. Previous work has shown that the energy efficiency of these systems is strongly dependent on these setpoints [2], however, determining appropriate setpoints is not always straightforward.

In a feedback controller for a typical air conditioning system (Fig. 1B), setpoints may include (1) zone temperatures selected by the user and (2) internal machine signals, the regulation of which is required for delivering the required cooling capacity in the presence of given thermodynamic boundary conditions such as heat load and outdoor air temperature. Assuming there exist flexibility with actual zone temperatures, the optimization of setpoints of type (1) have been extensively investigated, especially in the context of a model predictive controller where disturbances such as ambient temperature and occupancy may be predicted over some horizon. The interested reader may refer to [3], [4], [5] for more information on problems of this type. However, in this paper, we consider the optimization of setpoints of type (2); that is, internal machine process variables whose steady state values determine the energy consumption of the vapor compression machine.

Often, setpoints of type (2) are simply given as a constant evaporator superheat temperature. In this case, it is assumed that the superheat temperature is a good surrogate for overall cycle efficiency, and by regulating the cycle such that all the refrigerant passing through the evaporator becomes saturated vapor upon exiting, it is assumed that the overall process is performed efficiently. However, strict measurement of superheat requires at least one temperature and one pressure measurement (and perhaps more sensors depending on the assumptions made regarding pressure losses in the evaporator), and these sensors are often too expensive to be included in commercial systems. Additionally, for systems with multiple evaporators, requiring independent regulation of both superheat temperature and zone temperature may not even be possible with the typical set of actuators, because the number of regulated variables may exceed the number

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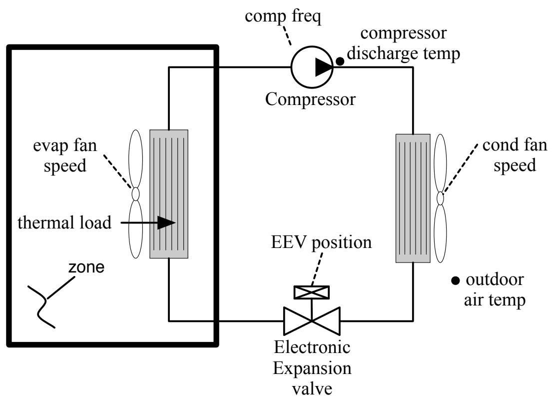

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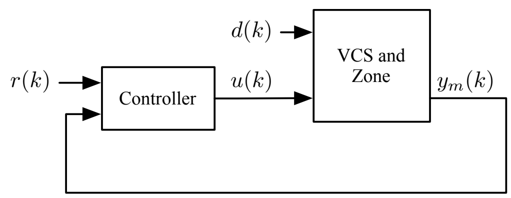


B.

Fig. 1. A. The vapor compression system under study consists of a variable speed compressor, condensing heat exchanger, electronically controlled expansion valve, and evaporating heating exchanger. The inputs to the VCS that are manipulated by the control system include (i) the compressor frequency, (ii) the condenser fan speed, (iii) the EEV position, and (iv) the evaporator fan speed. B. A feedback controller is nominally configured to use measurements $y(k)$ to drive regulated variables of a vapor compression system and zone to their setpoints $r(k)$ in the presence of disturbances $d(k)$ such as changes in outdoor air temperature and heat load.

of controls. Therefore, alternatives to superheat setpoints for regulating cycle capacity and efficiency are desired.

In this paper, we select the compressor discharge temperature as a signal to be regulated by feedback controller. The discharge temperature is often measured for equipment protection making it a commonly available signal, and because the refrigerant state at this location in the cycle is always superheated, this signal is a one-to-one function of the disturbances over the full range of expected operating points. (Contrast this with evaporator superheat temperature, which is not defined for values less than zero and produces no change in sensible temperature when two-phase refrigerant exits the evaporator. One of the main challenges of superheat regulation is that low superheat temperature, which is good for efficiency, is easily perturbed to zero in the presence of disturbances, causing the loss of signal information and therefore of feedback control.) Because discharge temperature changes with heat loads and outdoor air temperatures, its setpoint cannot be regulated to a constant, but instead must vary with these conditions. It is the aim of this paper to automate the generation of such setpoints in order to maximize energy efficiency.

Recently, model-free methods that operate in realtime and aim to optimize a cost have received increased attention and have demonstrated improvements in the optimization of vapor compression systems and other HVAC applications [2], [6], [7], [8] To date, the dominant extremum seeking algorithm that appears in the HVAC research literature is the traditional perturbation-based algorithm first developed in the 1920s [9] and re-popularized in the late 1990s by an elegant proof of convergence for a general class of nonlinear systems [10].

While most extremum seeking techniques optimize a performance metric by estimating its gradient and driving inputs such that the metric is optimized, the way in which the gradient is estimated has a strong influence on its convergence properties. In the traditional perturbation-based method, a sinusoidal term is added to the input at a slower frequency than the natural plant dynamics, inducing a sinusoidal response in the performance metric [11]. The extremum seeking controller then filters and averages this signal to obtain an estimate of the gradient. Averaging the perturbation introduces yet another (and slower) time scale in the optimization process. For thermal systems such as vapor compression machines where the dynamics are already on the order of tens of minutes, the slow convergence properties of perturbation-based extremum seeking become impediments to wide-scale deployment.

However, new extremum seeking approaches have been developed that estimate the gradient of the performance metric in a way that does not introduce two time scales. Time-varying extremum seeking uses adaptive filtering techniques to estimate the parameters of the gradient function from measured data, eliminating averaging in the controller [12]. In this paper, we apply time-varying extremum seeking to the problem of obtaining setpoints that optimize energy efficiency in a vapor compression system.

The rest of the paper is organized as follows: a description of time-varying extremum seeking control (TV-ESC) is provided in Section II. In Section III, a comparison of TV-ESC and perturbation-based ESC is provided in simulation using a simple example. In Section IV, experimental results for the application of the TV-ESC on an air conditioning system are presented. Lastly, concluding remarks are offered in section V.

II. Extremum Seeking Controller

The ESC provides a regulating feedback controller with setpoints for discharge temperature r that will minimize power consumption z. We follow the discrete-time ESC update law outlined in [13].

The equilibrium cost $z = \ell(r^*)$ satisfies the following optimality conditions:

$$\frac{\partial \ell(r^*)}{\partial r} = 0 \tag{1}$$

$$\frac{\partial^2 \ell(r^*)}{\partial r \partial r^T} > \beta I \quad \forall r \in R \tag{2}$$

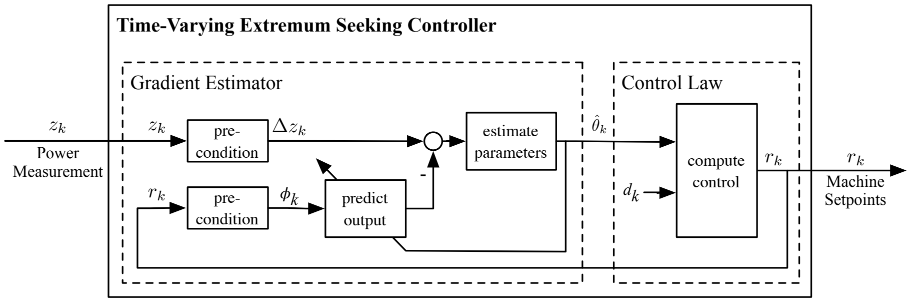


Fig. 2. Overview of the TV-ESC algorithm.

where β is a strictly positive constant.

Let $\phi_k = \Delta r_k$. The dynamics of the cost function can be parametrized as:

$$\Delta z_k = \theta_k^T \Delta r_k = \phi_k^T \theta_k \tag{3}$$

Let the estimator for (3) be

$$\Delta \hat{z}_k = \hat{\theta}_k^T \Delta r_k = \phi_k^T \hat{\theta}_k \tag{4}$$

where $\hat{\theta}_k$ is the vector of parameter estimates. The output prediction error is defined as $e_k = \Delta z_k - \Delta \hat{z}_k$.

The dynamical system operates at the faster time-scale with sampling time $\varepsilon \Delta t$ while the steady-state optimization operates at the slow time scale with sampling time Δt, where ε is a time-scale separation parameter. The parameter estimate update approach is as follows:

$$\begin{aligned}
\Sigma_{k+1}^{-1} =& \Sigma_k^{-1} + \varepsilon \left(\frac{1}{\alpha} - 1 \right) \Sigma_k^{-1} - && (5)\\
& \frac{\varepsilon}{\alpha^2} \Sigma_k^{-1} \phi_k (1 + \frac{1}{\alpha} \phi_k^T \Sigma_k^{-1} \phi_k)^{-1} \phi_k^T \Sigma_k^{-1} && (6)\\
\bar{\hat{\theta}}_{k+1} =& Proj \left[\hat{\theta}_k + \frac{\varepsilon}{\alpha} \Sigma_k^{-1} \phi_k (1 + \frac{1}{\alpha} \phi_k^T \Sigma_k^{-1} \phi_k)^{-1} (e_k), \Theta_0 \right] && (7)
\end{aligned}$$

Where $\Sigma \in \mathbb{R}^{n_\theta \times n_\theta}$ is the covariance matrix and $Proj$ is an orthogonal projection operator. For a more detailed discussion on this operator see [13] and [14].

The gradient descent controller is given by:

$$r_{k+1} = r_k - \varepsilon k_g \hat{\theta}_k + \varepsilon d_k \tag{8}$$

where d_k is a bounded dither signal and k_g is the optimization gain.

Together, the iterative extremum seeking routine is given by:

$$\begin{aligned}
r_{k+1} &= r_k - \varepsilon k_g \hat{\theta}_k + \varepsilon d_k && (9a)\\
\phi_k &= \Delta r_k = r_{k+1} - r_k && (9b)\\
\Delta \hat{z}_k &= \phi_k^T \hat{\theta}_k && (9c)\\
\Sigma_{k+1}^{-1} &= \Sigma_k^{-1} + \varepsilon (\frac{1}{\alpha} - 1) \Sigma_k^{-1} - && (9d)\\
& \frac{\varepsilon}{\alpha^2} \Sigma_k^{-1} \phi_k (1 + \frac{1}{\alpha} \phi_k^T \Sigma_k^{-1} \phi_k)^{-1} \phi_k^T \Sigma_k^{-1} && \\
\bar{\hat{\theta}}_{k+1} &= Proj \left[\hat{\theta}_k + \frac{\varepsilon}{\alpha} \Sigma_k^{-1} \phi_k (1 + \frac{1}{\alpha} \phi_k^T \Sigma_k^{-1} \phi_k)^{-1} (e_k), \Theta_0 \right] && (9e)
\end{aligned}$$

As shown in Figure 2, at the k^{th} iteration step, the ESC algorithm uses the difference between current r_k and next input r_{k+1}, and the difference between measured Δz_k and predicted $\Delta \hat{z}_k$ change in power consumption for the gradient estimation. The estimated gradient will be used to parameterize the unknown but measured cost function describing power consumption. The gradient is estimated by employing a recursive least squares filter with forgetting factor α. Further, the estimated gradient is used to compute the gradient descent controller which will reduce power consumption. The new setpoint is provided to the feedback regulator, and the ESC algorithm is repeated.

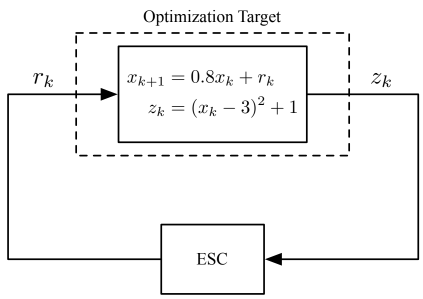
Note that the time-varying extremum seeking controller does not require averaging the effect of the perturbation as in the case of the traditional perturbation-based extremum seeking controller. For this reason, time-varying extremum seeking converges substantially faster, as demonstrated in an example in the following section.

III. Comparison of time-varying and perturbation extremum seeking control

To illustrate the differences in convergence rate between perturbation-based ESC and TV-ESC, these two methods are used to optimize a Hammerstein system consisting of first-order linear difference equation and a static output nonlinearity (see Figure 3A). The equations for this system are given by

$$x_{k+1} = 0.8 x_k + r_k \tag{10}$$

$$z_k = (x_k - 3)^2 + 1 \tag{11}$$



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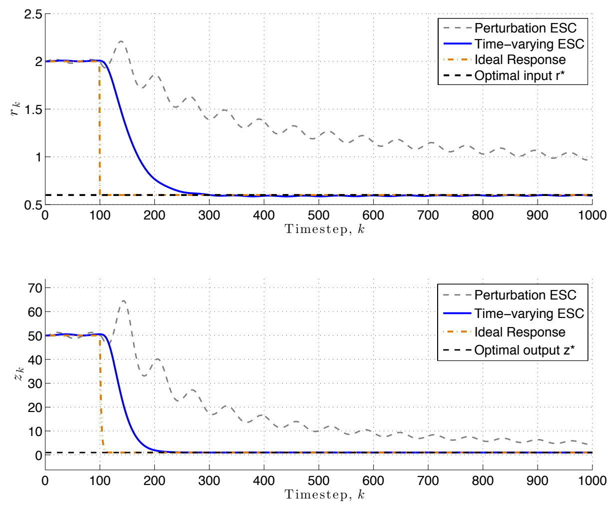

B.

Fig. 3. Comparing TV-ESC with perturbation ESC. For this application, TV-ESC converges considerably faster to the optimum.

which has a single optimum point at

$$r^* = 0.6 \tag{12}$$

$$z^* = 1. \tag{13}$$

Note that the pole location in the difference equation component establishes a dominant timescale and therefore sets a fundamental limit for the convergence rate.

In order to illustrate the difference in convergence rates, a discrete-time perturbation-based extremum seeking controller (perturb-ESC) and a time-varying extremum seeking controller (TV-ESC) is applied to the problem of finding the input r that minimizes the output z, without a model of the process or any explicit knowledge of the nature of the optimum. Reasonable effort is made to obtain algorithm parameters for both ESC methods that achieve the best possible convergence rates. The parameters for the perturbation based ESC used for simulation are

$$d_k = 0.2\sin(0.1k) \tag{14}$$

$$\omega_{LP} = 0.03 \tag{15}$$

$$K = -0.005 \tag{16}$$

Where d_k is the sinusoidal perturbation, ω_{LP} is the cutoff frequency for a first-order low-pass averaging filter, and K is the adaptation gain. Note that the high-pass washout filter was not used as convergence rate was improved without it. For details of a discrete-time perturb-ESC formulation, see [15].

The parameters used for the TV-ESC are

$$d_k = 0.001\sin(0.1k) \tag{17}$$

$$k_g = 0.001 \tag{18}$$

$$\alpha = 0.1 \tag{19}$$

$$\varepsilon = 0.4 \tag{20}$$

Where k_g is the adaptation gain, α is the forgetting factor, and ε is the timescale separation factor. No projection algorithm was needed for this example.

Simulations are performed starting from an initial input value of $r = 2$ and the ESC methods are turned on after 100 steps. The resulting simulations are shown in Figure 3B. The perturb-ESC method converges to a neighborhood around the optimum in about 4000 steps (not shown in the figure), while the TV-ESC method converges in about 250 steps.

The fast convergence characteristic of TV-ESC is well suited to the optimization of thermal systems with their associated long time constants. In the next section, we apply the TV-ESC algorithm to the problem of selecting setpoints for the discharge temperature of a vapor compression machine and present experimental results.

IV. Experimental Results

A. Laboratory Description

In order to validate the proposed scheme for realtime optimization of compressor discharge temperature setpoints, an experiment using TV-ESC on a production vapor compression system is performed. A residential split-ductless style room air conditioner is configured as the device-under-test. The indoor unit is installed in a 64 ft^3 adiabatic test chamber and the outdoor unit is installed in a 128 ft^3 adiabatic test chamber. The device-under-test is a variable refrigerant flow (VRF) air conditioner with variable speed compressor, indoor and outdoor fan speeds and computer-controlled electronic expansion valve. The system has a nominal rated cooling capacity of 2.8 kW.

A balance-of-plant system is constructed to regulate thermodynamic test conditions during the experiment. In particular, the indoor unit test chamber includes variable capacity electric heaters capable of supplying up to 5 kW of power representing the thermal load, and the outdoor unit includes electric heaters and a variable capacity chilled water fan coil system capable of matching the heat rejected by the air conditioner outdoor unit so that the outdoor air temperature can be held constant. External control loops on the balance-of-plant system are designed to regulate the indoor heat load and the outdoor air temperature. With the indoor air temperature regulated by the device-under-test, a realistic test of air conditioner performance under normal operating conditions is obtained.

The actuators for both the device-under-test and the balance-of-plant are controlled by an external data acquisition and control system (National Instruments), which is also used to record various experimental measurements, and prototype custom control algorithms. The device-under-test is configured to regulate the room temperature using the compressor speed and the discharge temperature is regulated using the electronic expansion valve. Setpoints to this discharge temperature control loop are determined with the time-varying extremum seeking controller. Previous experiments indicate that the dominant time constant of the room air conditioner closed loop system is approximately 7 *min* and is associated with the indoor room air temperature dynamics. This time constant sets the fundamental limit of convergence rate of an optimization algorithm that operates on the steady state manifold, $\ell(r^*)$.

B. Experimental Conditions and Results

A 2000 W heat load is applied to the indoor unit test chamber, with the outdoor air temperature regulated to 35°C. The device-under-test is set to regulate the indoor test chamber to 25°C. The TV-ESC parameters used in this experiment are

$$d_k = \sin(0.001k) \tag{21}$$

$$k_g = 5 \tag{22}$$

$$\alpha = 0.001 \tag{23}$$

$$\varepsilon = 0.004 \tag{24}$$

wherein the TV-ESC algorithm is executed once every second. Initially, the discharge temperature setpoint is set to 72°C, which is 10°C higher than the optimal discharge temperature determined *a priori* for these operating conditions. Physically, discharge temperatures higher than optimal indicate an evaporator that has an excessive amount of superheated refrigerant, that is refrigerant in the vapor state. It is expected that as the discharge temperature setpoint is decreased, the cooling capacity of the evaporator will be increased as a larger fraction of the heat exchanger performs useful cooling, leading to a lower room temperature. In turn, the inner room temperature feedback loop will lower the compressor speed, decreasing overall power consumption. At the initial operating condition of this experiment, the device-under-test consumes about 525 W, yielding an observed coefficient-of-performance (COP) of 2000 $W/525\ W = 3.8$.

At $t = 5$ *min*, the TV-ESC algorithm is switched on and begins estimating the gradient of the mapping from discharge temperature setpoint r_k to electrical power consumption z_k. Figure 4 shows the evolution of the discharge temperature setpoint output from the TV-ESC controller in the top plot, and the measured power consumption of the device-under-test provided as a measurement in the middle plot. The bottom plot shows the constant thermodynamic conditions during the test (indoor and outdoor air temperature and heat load).

From about $t = 5$ *min* to $t = 20$ *min* the discharge temperature setpoint is decreased, causing little change in the power consumption and the identified gradient estimate in

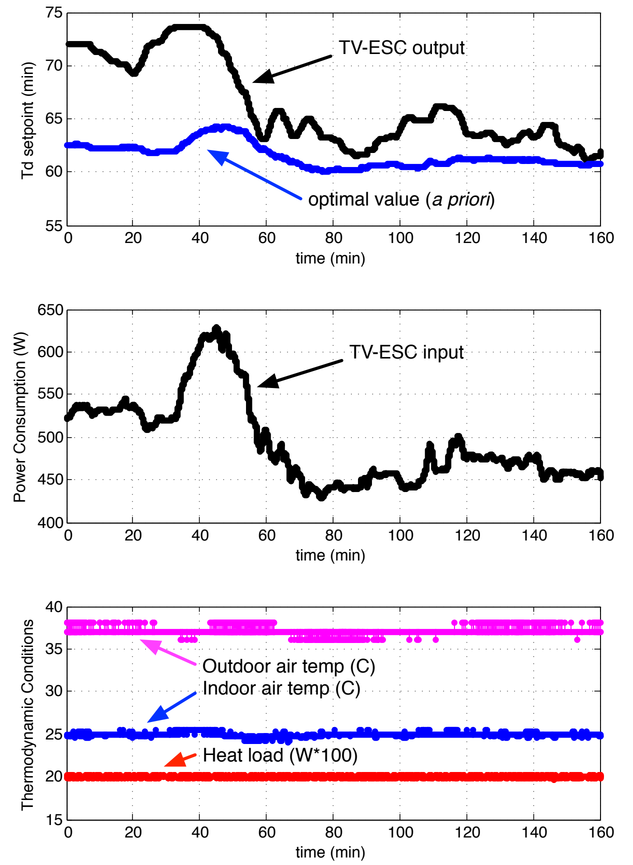

Fig. 4. Top: The discharge temperature setpoint (black) is driven to the optimal setpoint (blue) determined from previous experiments in about 160 *min*. Middle: The power consumption for this test is driven from an initial value of about 525 W to about 450 W while the thermodynamic conditions are held constant (bottom).

this range is near zero. However, from about $t = 20$ *min* to $t = 40$ *min*, the discharge temperature is increased, causing a large increase in power consumption as cooling capacity is dramatically reduced and compressor speeds must be increased to maintain room temperature. As a result, the gradient is estimated as a large positive value, and the TV-ESC controller drives the discharge temperature from 74°C to 63°C (very near to the optimal value of 62°C) from $t = 40$ *min* to $t = 60$ *min*.

For the remainder of the experiment, the average value of the discharge temperature approaches the optimal value. The change in power consumption is relatively slow after $t = 80$ *min* (expect for a sharp increase at around $t = 110$ *min*), indicating insensitivity in the power consumption to discharge temperatures as the optimal value is approached (*i.e.*, the region near the minimum of the convex mapping is nearly "flat."). The power consumption is driven to a final value of 450 W, yielded an observed COP of 2000 $W/450\ W = 4.4$, which is an improvement of 14%.

The optimization is shown in the plant input-output space in Figure 5. Starting at 72°C and 525 W, the discharge temperature is steered toward 62°C and 450 W. However,

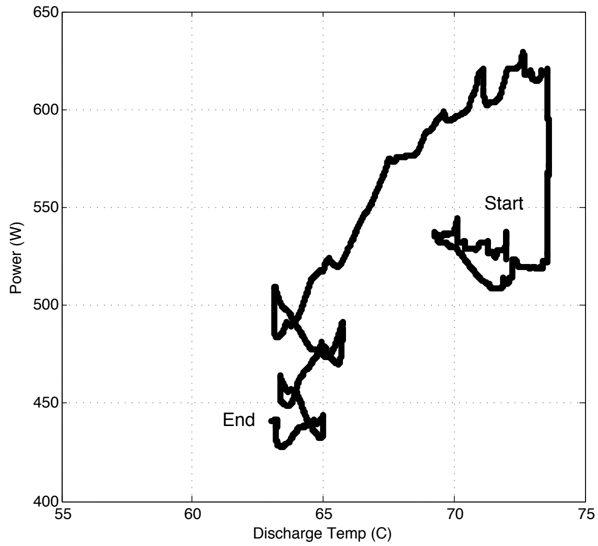

Fig. 5. Power consumption optimization for the room air conditioner.

it should be noted that the convergence timescale is on the order as the dominant plant time scale, and therefore some dynamics are active during optimization. As a result, this figure should not be interpreted as the steady state performance map, but one that is distorted due to influence of transients. However, this highlights the main advantage of TV-ESC: because the gradient can be estimated without averaging over multiple perturbations, convergence can proceed much faster—at about one time scale slower than the dominant plant dynamics rather than two.

V. Conclusion

In this paper, we proposed an alternative signal for the regulation of the vapor compression cycle in place of the often difficult to measure evaporator superheat, namely the compressor discharge temperature. We apply time-varying extremum seeking to the problem of determining setpoints for an inner loop feedback controller that uses the electronic expasion valve to regulate discharge temperature.

TV-ESC is shown in experiments to provide a discharge temperature setpoint that drives power consumption from 525 W to 450 W for a particular set of thermodynamic conditions. Moreover, this optimzation occurs over reasonable timescales (less than three hours in the case of a vapor compression system a slow mode with a 7 *min* time constant), whereas the time required for optimization using the traditional pertubation-based ESC method would have been impractical.

References

[1] M. Guay and D. Burns, "A comparison of extremum seeking algorithms applied to vapor compression system optimization," in *Proceedings of the 2014 American Controls Conference*, 2014.

[2] D. Burns and C. Laughman, "Extremum seeking control for energy optimization of vapor compression systems," in *International Refrigeration and Air Conditioning Conference*, 2012.

[3] F. Oldewurtel, A. Parisio, C. Jones, D. Gyalistras, M. Gwerder, V. Stauch, B. Lehmann, and M. Morari, "Use of Model Predictive Control and Weather Forecasts for Energy Efficient Building Climate Control," *Energy and Buildings*, vol. 45, pp. 15–27, Feb. 2012. [Online]. Available: http://control.ee.ethz.ch/index.cgi?page=publications;action=details;id=3863

[4] X. Zhang, G. Schildbach, D. Sturzenegger, and M. Morari, "Scenario-Based MPC for Energy-Efficient Building Climate Control under Weather and Occupancy Uncertainty," in *European Control Conference*, Zurich, Switzerland, Jul. 2013, pp. 1029–1034. [Online]. Available: http://control.ee.ethz.ch/index.cgi?page=publications;action=details;id=4464

[5] Y. Ma, F. Borrelli, B. Hencey, B. Coffey, S. Bengea, and P. Haves, "Model Predictive Control for the Operation of Building Cooling Systems," *IEEE TRANSACTIONS ON CONTROL SYSTEMS TECHNOLOGY*, vol. 20, no. 3, pp. 796–803, MAY 2012.

[6] H. Sane, C. Haugstetter, and S. Bortoff, "Building hvac control systems - role of controls and optimization," in *Proceedings of the 2006 American Controls Conference*, 2006.

[7] P. Li, Y. Li, and J. E. Seem, "Efficient Operation of Air-Side Economizer Using Extremum Seeking Control," *Journal of Dynamic Systems, Measurement, and Control*, vol. 132, no. 3, May 2010.

[8] V. Tyagi, H. Sane, and S. Darbha, "An extremum seeking algorithm for determining the set point temperature for condensed water in a cooling tower," in *American Control Conference, 2006*, June 2006, pp. 5 pp.–.

[9] M. Leblanc, "Sur l'*é*lectrification des chemins de fer au moyen de courants alternatifs de fr*é*quence *é*lev*é*e," Revue G*é*n*é*rale de l'Electricit*é*, 1922.

[10] M. Krstic, "Performance Improvement and Limitations in Extremum Seeking Control," *Systems & Control Letters*, vol. 39, no. 5, pp. 313–326, April 2000.

[11] Y. Tan, W. Moase, C. Manzie, D. Nesic and, and I. Mareels, "Extremum seeking from 1922 to 2010," in *29th Chinese Control Conference (CCC)*, july 2010, pp. 14 –26.

[12] M. Guay, D. Dochain, and S. Dhaliwal, "A time-varying extremum seeking control technique," in *Proc. 2013 ACC, Washington, D.C.*, 2013.

[13] M. Guay, "A time-varying extremum-seeking control approach for discrete-time systems," *Journal of Process Control*, vol. 24, no. 3, pp. 98 – 112, 2014. [Online]. Available: http://www.sciencedirect.com/science/article/pii/S0959152413002527

[14] G. Goodwin and K. Sin, *Adaptive Filtering Prediction and Control*, ser. Dover Books on Electrical Engineering Series. Dover Publications, Incorporated, 2013. [Online]. Available: http://books.google.ca/books?id=0_m9j_YM91EC

[15] N. J. Killingsworth and M. Krstic, "PID tuning using extremum seeking: online, model-free performance optimization," *IEEE Control Systems Magazine*, vol. 26, no. 1, pp. 70–79, Feb. 2006. [Online]. Available: http://ieeexplore.ieee.org.proxy.queensu.ca/articleDetails.jsp?arnumber=1580155

[16] Alleyne, A., *et. al.*, "THERMOSYS 4 Toolbox," University of Illinois at Urbana-Champaign. `http://arg.mechse.illinois.edu/thermosys`, 2012.