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Extremum Seeking-Based Indirect Adaptive Control for Nonlinear Systems with State-Dependent Uncertainties

Mouhacine Benosman and Meng Xia *

Abstract

We study in this paper the problem of adaptive trajectory tracking for nonlinear systems affine in the control with bounded state-dependent uncertainties. We propose to use a modular approach, in the sense that we first design a robust nonlinear state feedback which renders the closed loop input to state stable (ISS) between an estimation error of the uncertain parameters and an output tracking error. Next, we complement this robust ISS controller with a model-free multiparametric extremum seeking (MES) algorithm to estimate the model uncertainties. The combination of the ISS feedback and the MES algorithm gives an indirect adaptive controller. We show the efficiency of this approach on a two-link robot manipulator example.

1 Introduction

Input-output feedback linearization has been proven to be a powerful control design for trajectory tracking and stabilization of nonlinear systems [1]. The basic idea is to first transform a nonlinear system into a simplified linear equivalent system and then use the linear design techniques to design controllers in order to satisfy stability and performance requirements. One shortcoming of the feedback linearization approach is that it requires precise system modelling [1]. When there exist model uncertainties, a robust input-output linearization approach needs to be developed. For instance, high-gain observers [2] and linear robust controllers [3] have been proposed in combination with the feedback linearization techniques. Another approach to deal with model uncertainties is using adaptive control methods. Of particular interest to us is the modular approach to adaptive nonlinear control, e.g. [4]. In this approach, first the controller is designed by assuming all the parameters are known and then an identifier is used to guarantee certain boundedness of the estimation error. The identifier is independent of the designed controller and thus this is called 'modular' approach.

On the other hand, extremum seeking (ES) method is a model-free control approach, e.g.[5], which has been applied to many industrial systems, such as electromagnetic actuators [6, 7], compressors [8], and stirred-tank bioreactors [9]. Many papers have dedicated to analyzing the ES algorithms convergence when applied to a static or a dynamic known maps, e.g.[10, 5, 11, 12], however, much fewer papers have been dealing with the use of ES in the case of static or dynamic uncertain maps. The case of ES applied to an uncertain static and dynamic mapping, was investigated in [13], where the authors considered systems with constant parameter uncertainties. However, in [13], the authors used ES to optimize a given performance (via optimizing a given performance function), and complemented the ES algorithm with classical model-based filters/estimators to estimate the states and the unknown constant parameters of the system, which is one of the main differences with the approach that we want to present here (originally introduced by the authors for a specific mechatronics application in [7, 14]), where the ES is not only used to optimize a given performance but is also used to estimate the uncertainties of the model, without the need for extra model-based filters/estimators.

In this work, we build upon the existing ES results to provide a framework which combines ES results and robust model-based nonlinear control to propose an ES-based indirect adaptive controller, where the ES algorithm is used to estimate, in closed-loop, the uncertainties of the model. Furthermore, we focus here on a particular class of nonlinear systems which are input-output linearizable through static state feedback [15]. We assume that the uncertainties in the linearized model are bounded additive as guaranteed by the 'matching condition' [16]. The control objective is to achieve asymptotic tracking of a desired trajectory. The proposed adaptive control is designed as follows. In the first step, we design a controller for the nominal model (i.e. when the uncertainties are assumed to be zero) so that the tracking error dynamics is asymptotically stable. In the second step, we use a Lyapunov reconstruc-

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tion method [17] to show that the error dynamics are input-to-state stable (ISS) [15, 18] where the estimation error in the parameters is the input to the system and the tracking error represents the system state. Finally, we use ES to estimate the uncertain model parameters so that the the tracking error will be bounded and decreasing, as guaranteed by the ISS property. To validate the results, we apply our results on a two-link robotic manipulators [19].
Similar ideas of ES-based adaptive control for nonlinear systems have been introduced in [6, 7]. In these two works, the problem of adaptive robust control of electromagnetic actuators was studied, where ES was used to tune the feedback gains of the nonlinear controller in [6] and ES was used to estimate the unknown parameters in [7]. An extension to the general case of nonlinear systems was proposed in [20, 21]. We relax here the strong assumption, used in [20, 21], about the existence of an ISS feedback controller, and propose a constructive proof to design such an ISS feedback for the particular case of nonlinear systems affine in the control.

The rest of the paper is organized as follows. In Section 2, we present notations, and some fundamental definitions and results that will be needed in the sequel. In Section 3, we provide our problem formulation. The nominal controller design are presented in Section 4. In Section 5, a robust controller is designed which guarantees ISS from the estimation errors input to the tracking errors state. In Section 6, the ISS controller is complemented with an MES algorithm to estimate the model uncertainties. Section 7 is dedicated to an application example and the paper conclusion is given in Section 8.

2 Preliminaries

Throughout the paper, we use $\|\cdot\|$ to denote the Euclidean norm; i.e. for a vector $x \in \mathbb{R}^n$, we have $\|x\| \triangleq \|x\|_2 = \sqrt{x^T x}$, where x^T denotes the transpose of the vector x. The 1-norm of $x \in \mathbb{R}^n$ is denoted by $\|x\|_1$. We use the following norm properties for the need of our proof:

1. for any $x \in \mathbb{R}^n$, $\|x\| \leq \|x\|_1$;
2. for any $x,\ y \in \mathbb{R}^n$, $\|x\| - \|y\| \ \leq\ \|x - y\|$;
3. for any $x,\ y \in \mathbb{R}^n$, $x^T y \ \leq\ \|x\|\|y\|$.

Given $x \in \mathbb{R}^m$, the signum function is defined as

$$\mathrm{sign}(x) \ \triangleq\ [\mathrm{sign}(x_1),\ \mathrm{sign}(x_2),\ \cdots,\ \mathrm{sign}(x_m)]^T,$$

where x_i denotes the i-th $(1 \leq i \leq m)$ element of x and

$$\mathrm{sign}(x_i) \ = \ \begin{cases} 1 & \text{if } x_i \ > \ 0 \\ 0 & \text{if } x_i \ = \ 0 \\ -1 & \text{if } x_i \ < \ 0 \end{cases}$$

We have $x^T \mathrm{sign}(x) \ = \ \|x\|_1$.

For an $n \times n$ matrix P, we denote by $P > 0$ if it is positive definite. Similarly, we denote by $P < 0$ if it is negative definite. We use $\mathrm{diag}\{A_1,\ A_2,\ \cdots,\ A_n\}$ to denote a diagonal block matrix with n blocks. For a matrix B, we denote $B(i,j)$ as the element that locates at the i-th row and j-th column of matrix B. We denote I_n as the identity matrix or simply I if the dimension is clear from the context.

We use $\dot{f}$ to denote the time derivative of f and $f^{(r)}(t)$ for the r-th derivative of $f(t)$, i.e. $f^{(r)} \ \triangleq\ \frac{d^r f}{dt}$. We denote by $\mathbb{C}^k$ functions that are k times differentiable and by $\mathbb{C}^\infty$ a smooth function. A continuous function $\alpha : [0, a) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\alpha(r) \to \infty$ as $r \to \infty$ [15]. A continuous function $\beta : [0, a) \times [0, \infty) \to [0, \infty)$ is said to belong to class $\mathcal{KL}$ if, for a fixed s, the mapping $\beta(r, s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$ [15].

Consider the system

$$\dot{x} \ = \ f(t, x, u) \tag{2.1}$$

where $f : [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is piecewise continuous in t and locally Lipschitz in x and u, uniformly in t. The input $u(t)$ is piecewise continuous, bounded function of t for all $t \geq 0$.

Definition 2.1. ([15, 22]) *The system (2.1) is said to be* input-to-sate stable *(ISS) if there exist a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ such that for any initial state $x(t_0)$ and any bounded input $u(t)$, the solution $x(t)$ exists for all $t \geq t_0$ and satisfies*

$$\|x(t)\| \leq \beta(\|x(t_0)\|, t - t_0) + \gamma(\sup_{t_0 \leq \tau \leq t} \|u(\tau)\|).$$

Theorem 2.1. ([15, 22]) *Let $V : [0, \infty) \times \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable function such that*

$$\begin{aligned} \alpha_1(\|x\|) \leq & V(t, x) \leq \alpha_2(\|x\|) \\ \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x, u) \leq & -W(x), \quad \forall \|x\| \geq \rho(\|u\|) > 0 \end{aligned} \tag{2.2}$$

for all $(t, x, u) \in [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m$, where α_1, α_2 are class $\mathcal{K}_\infty$ functions, ρ is a class $\mathcal{K}$ function, and $W(x)$ is a continuous positive definite function on $\mathbb{R}^n$. Then, the system (2.1) is input-to-state stable (ISS).

REMARK 1. *Note that other equivalent definitions for ISS have been given in [22, pp. 1974-1975]. For instance, Theorem 2.1 holds with all the assumptions are the same except that the inequality (2.2) is replaced by*

$$\frac{\partial V}{\partial t} + \frac{\partial V}{\partial x} f(t, x, u) \le -\mu(\|x\|) + \Omega(\|u\|)$$

where $\mu \in \mathcal{K}_\infty \bigcap C^1$ *and* $\Omega \in \mathcal{K}_\infty$.

3 Problem Formulation

3.1 Nonlinear system model We consider here affine uncertain nonlinear systems of the form:

$$\begin{aligned} \dot{x} &= f(x) + \Delta f(x) + g(x)u, \quad x(0) = x_0 \\ y &= h(x) \end{aligned} \tag{3.3}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, $y \in \mathbb{R}^m$ $(p \geq m)$, represent respectively the state, the input and the controlled output vectors, x_0 is a given initial condition, $\Delta f(x)$ is a vector field representing additive model uncertainties. The vector fields f, Δf, columns of g and function h satisfy the following assumptions.

ASSUMPTION 1. *The function* $f : \mathbb{R}^n \to \mathbb{R}^n$ *and the columns of* $g : \mathbb{R}^n \to \mathbb{R}^p$ *are* $\mathbb{C}^\infty$ *vector fields on a bounded set* X *of* $\mathbb{R}^n$ *and* $h : \mathbb{R}^n \to \mathbb{R}^m$ *is a* $\mathbb{C}^\infty$ *vector on* X. *The vector field* $\Delta f(x)$ *is* $\mathbb{C}^1$ *on* X.

ASSUMPTION 2. *System (3.3) has a well-defined (vector) relative degree* $\{r_1,\ r_2,\ \cdots,\ r_m\}$ *at each point* $x^0 \in X$, *and the system is linearizable, i.e.* $\sum_{i=1}^m r_i = n$.

ASSUMPTION 3. *The desired output trajectories* y_{id} $(1 \le i \le m)$ *are smooth functions of time, relating desired initial points* $y_{id}(0)$ *at* $t = 0$ *to desired final points* $y_{id}(t_f)$ *at* $t = t_f$.

3.2 Control objectives Our objective is to design a state feedback adaptive controller so that the tracking error is uniformly bounded, whereas the tracking upper-bound can be made smaller over the ES learning iterations. We stress here that the goal of the ES is not stabilization but rather performance optimization, i.e. estimating online the uncertain part of the model and thus improving the performance of the overall controller. To achieve this control objective, we proceed as follows. First, we design a robust controller which can guarantee the input-to-state stability (ISS) of the tracking error dynamics w.r.t the estimation errors input. Then, we combine this controller with a model-free extremum-seeking algorithm to iteratively estimate the uncertain parameters, to optimize online a desired performance cost function.

4 Nominal Controller Design

Under Assumption 2 and nominal conditions, i.e. when $\Delta f(x) = 0$, system (3.3) can be written as

$$y^{(r)}(t) = b(\xi(t)) + A(\xi(t))u(t), \tag{4.4}$$

where

$$\begin{aligned} y^{(r)}(t) &= [y_1^{(r_1)}(t),\ y_2^{(r_2)}(t),\ \cdots,\ y_m^{(r_m)}(t)]^T \\ \xi(t) &= [\xi^1(t),\ \cdots,\ \xi^m(t)]^T \\ \xi^i(t) &= [y_i(t),\ \cdots,\ y_i^{(r_i-1)}(t)], \quad 1 \le i \le m \end{aligned} \tag{4.5}$$

The functions $b(\xi)$, $A(\xi)$ can be written as functions of f, g and h, and $A(\xi)$ is non-singular in $\tilde{X}$, where $\tilde{X}$ is the image of the set of X by the diffeomorphism $x \to \xi$ between the states of system (3.3) and the linearized model (4.4).

At this point, we introduce one more assumption on system (3.3).

ASSUMPTION 4. *The additive uncertainties* $\Delta f(x)$ *in (3.3) appear as additive uncertainties in the input-output linearized model (4.4)-(4.5) as follows (see also [16])*

$$y^{(r)}(t) = b(\xi(t)) + A(\xi(t))u(t) + \Delta b(\xi(t)), \tag{4.6}$$

where $\Delta b(\xi(t))$ *is* $\mathbb{C}^1$ *on* $\tilde{X}$.

If we consider the nominal model (4.4)first, then we can define a virtual input vector $v(t)$ as

$$v(t) = b(\xi(t)) + A(\xi(t))u(t). \tag{4.7}$$

Combining (4.4) and (4.7), we can obtain the following input-output mapping

$$y^{(r)}(t) = v(t). \tag{4.8}$$

Based on the linear system (4.8), it is straightforward to apply a stabilizing controller for the nominal system (4.4) as

$$u_n = A^{-1}(\xi)\left[v_s(t, \xi) - b(\xi)\right], \tag{4.9}$$

where v_s is a $m \times 1$ vector and the i-th $(1 \le i \le m)$ element v_{si} is given by

$$v_{si} = y_{id}^{(r_i)} - K_{r_i}^i(y_i^{(r_i-1)} - y_{id}^{(r_i-1)}) - \cdots - K_1^i(y_i - y_{id}). \tag{4.10}$$

Denote the tracking error as $e_i(t) \triangleq y_i(t) - y_{id}(t)$, we obtain the following tracking error dynamics

$$e_i^{(r_i)}(t) + K_{r_i}^i e^{(r_i-1)}(t) + \cdots + K_1^i e_i(t) = 0, \tag{4.11}$$

where $i \in \{1,\ 2,\ \cdots,\ m\}$. By selecting the gains K_j^i where $i \in \{1,\ 2,\ \cdots,\ m\}$ and $j \in \{1,\ 2,\ \cdots,\ r_i\}$, we can obtain global asymptotic stability of the tracking errors $e_i(t)$. To formalize this condition, we make the following assumption.

ASSUMPTION 5. *There exists a non-empty set $\mathcal{A}$ where $K_j^i \in \mathcal{A}$ such that the polynomials in (4.11) are Hurwitz, where $i \in \{1, 2, \cdots, m\}$ and $j \in \{1, 2, \cdots, r_i\}$.*

To this end, we define $z = [z^1, z^2, \cdots, z^m]^T$, where $z^i = [e_i, \dot{e}_i, \cdots, e_i^{(r_i-1)}]$ and $i \in \{1, 2, \cdots, m\}$. Then, from (4.11), we can obtain

$$\dot{z} = \tilde{A}z,$$

where $\tilde{A} \in \mathbb{R}^{n\times n}$ is a diagonal block matrix given by

$$\tilde{A} = \mathrm{diag}\{\tilde{A}_1, \tilde{A}_2, \cdots, \tilde{A}_m\}, \tag{4.12}$$

and $\tilde{A}_i$ $(1 \leq i \leq m)$ is a $r_i \times r_i$ matrix given by

$$\tilde{A}_i = \begin{bmatrix} 0 & 1 & & & \\ 0 & & 1 & & \\ 0 & & & \ddots & \\ \vdots & & & & 1 \\ -K_1^i & -K_2^i & \cdots & \cdots & -K_{r_i}^i \end{bmatrix}.$$

As discussed above, the gains K_j^i can be chosen so that the matrix $\tilde{A}$ is Hurwitz. Thus, there exists a positive definite matrix $P > 0$ such that (see e.g. [15])

$$\tilde{A}^T P + P\tilde{A} = -I. \tag{4.13}$$

5 Robust Controller Design

We now consider the uncertain model (3.3), i.e. when $\Delta f(x) \neq 0$. The corresponding linearized model is given by (4.6) where $\Delta b(\xi(t)) \neq 0$. The global asymptotic stability of the error dynamics (4.11) cannot be guaranteed anymore due to the additive uncertainty $\Delta b(\xi(t))$. We use Lyapunov reconstruction techniques to design a new controller so that the tracking error is guaranteed to be bounded given that the estimate error of $\Delta b(\xi(t))$ is bounded. The new controller for the uncertain model (4.6) is defined as

$$u_f = u_n + u_r, \tag{5.14}$$

where the nominal controller u_n is given by (4.9) and the robust controller u_r will be given later on based on particular forms of the uncertainty $\Delta b(\xi(t))$. By using the controller (5.14), from (4.6) we obtain

$$\begin{aligned} y^{(r)}(t) &= b(\xi(t)) + A(\xi(t))u_f + \Delta b(\xi(t)) \\ &= b(\xi(t)) + A(\xi(t))u_n + A(\xi(t))u_r + \Delta b(\xi(t)) \\ &= v_s(t,\xi) + A(\xi(t))u_r + \Delta b(\xi(t)), \end{aligned} \tag{5.15}$$

where (5.15) holds from (4.9). Thus, we have

$$\begin{aligned} & e_i^{(r_i)}(t) + K_{r_i}^i e^{(r_i-1)}(t) + \cdots + K_1^i e_i(t) \\ &= A(\xi(t))u_r + \Delta b(\xi(t)) \end{aligned} \tag{5.16}$$

Further, the dynamics for z is given by

$$\dot{z} = \tilde{A}z + \tilde{B}\delta, \tag{5.17}$$

where $\tilde{A}$ is defined in (4.12), δ is a $m \times 1$ vector given by

$$\delta = A(\xi(t))u_r + \Delta b(\xi(t)), \tag{5.18}$$

and the matrix $\tilde{B} \in \mathbb{R}^{n\times m}$ is given by

$$\tilde{B} = \begin{bmatrix} \tilde{B}_1 \\ \tilde{B}_2 \\ \vdots \\ \tilde{B}_m \end{bmatrix}, \tag{5.19}$$

with $\tilde{B}_i$ $(1 \leq i \leq m)$ given by a $r_i \times m$ matrix such that

$$\tilde{B}_i(l,q) = \begin{cases} 1 & \text{if } l = r_i \text{ and } q = i \\ 0 & \text{otherwise} \end{cases}$$

If we apply $V(z) = z^T P z$ as a Lyapunov function for the dynamics (5.17), where P is the solution of the Lyapunov equation (4.13), then we obtain

$$\begin{aligned} \dot{V}(t) &= \frac{\partial V}{\partial z}\dot{z} \\ &= z^T(\tilde{A}^T P + P\tilde{A})z + 2z^T P\tilde{B}\delta \\ &= -\|z\|^2 + 2z^T P\tilde{B}\delta, \end{aligned} \tag{5.20}$$

where δ given by (5.18) depends on the robust controller u_r.

Next, we will design the controller u_r based on the particular forms of the uncertainties that appear in (4.6), i.e. $\Delta b(\xi(t))$. For notational convenience, the unknown parameter vector/matrix is denoted by Δ and the estimate for the unknowns is denoted by $\widehat{\Delta}(t)$. Further, the estimation error vector/matrix is given by $e_\Delta(t) = \Delta - \widehat{\Delta}(t)$, where the dimensions of Δ (and in turn, $\widehat{\Delta}(t)$ and $e_\Delta(t)$ will be clear from the context.

5.1 The case of bounded state-dependent uncertainties We consider the case where the unknown $\|\Delta b(\xi(t))\|$ is upper bounded by a function of the state $\xi(t)$, i.e.

$$\|\Delta b(\xi(t))\| \leq \|\Delta\|\|L(\xi)\|, \tag{5.21}$$

where $\Delta \in \mathbb{R}^{m\times m}$ is constant, and $L(\xi)$ is a known bounded state function. Assume, for now, that we can obtain the estimate of $\Delta(i,j)$, which may be time-varying and is denoted by $\widehat{\Delta}(i,j)$, for $i,j = 1,2,\ldots,m$.

Let $\widehat{\Delta}(t)$ be the matrix with the element $\widehat{\Delta}(i,j)$. We use the following robust controller

$$u_r = -A^{-1}(\xi)\tilde{B}^T Pz\|L(\xi)\|^2 - A^{-1}(\xi)\|\widehat{\Delta}(t)\|\|L(\xi)\|\text{sign}(\tilde{B}^T Pz). \tag{5.22}$$

Similar to the previous case, the closed-loop error dynamics can be written in the form of

$$\dot{z} = f(t, z, e_\Delta), \tag{5.23}$$

where $e_\Delta(t)$ is the system input and $z(t)$ is the system state.

THEOREM 5.1. *Consider the system (3.3), under Assumptions 1-5 and the assumption that $\Delta b(\xi(t))$ satisfies (5.21), with the feedback controller (5.14), where u_n is given by (4.9) and u_r is given by (5.22). Then, the closed-loop system (5.23) is ISS from the estimation errors input $e_\Delta(t) \in \mathbb{R}^{m\times m}$ to the tracking errors state $z(t) \in \mathbb{R}^n$.*

Proof. By substitution (5.22) into (5.18), we obtain $\delta = -\tilde{B}^T Pz\|L(\xi)\|^2 - \|\widehat{\Delta}(t)\|\|L(\xi)\|\text{sign}(\tilde{B}^T Pz) + \Delta b(\xi(t))$. We consider $V(z) = z^T Pz$ as a Lyapunov function for the error dynamics (5.17), where $P > 0$ is a solution of (4.13). We can derive that

$$\lambda_{\min}(P)\|z\|^2 \le V(z) \le \lambda_{\max}(P)\|z\|^2, \tag{5.24}$$

where $\lambda_{\min}(P) > 0$, $\lambda_{\max}(P) > 0$ denote respectively the minimum and the maximum eigenvalues of the matrix P. Then, from (5.20), we obtain

$$\begin{aligned}\dot{V} &= -\|z\|^2 + 2z^T P\tilde{B}\Delta b(\xi(t)) \\ &\quad - 2\|z^T P\tilde{B}\|^2\|L(\xi)\|^2 - 2\|z^T P\tilde{B}\|_1\|\widehat{\Delta}(t)\|\|L(\xi)\|.\end{aligned}$$

Since $\|z^T P\tilde{B}\| \le \|z^T P\tilde{B}\|_1$, we have

$$\begin{aligned}\dot{V} &\le -\|z\|^2 + 2z^T P\tilde{B}\Delta b(\xi(t)) \\ &\quad - 2\|z^T P\tilde{B}\|^2\|L(\xi)\|^2 - 2\|z^T P\tilde{B}\|\|\widehat{\Delta}(t)\|\|L(\xi)\|.\end{aligned}$$

Then based on the assumption (5.21) and the fact that $z^T P\tilde{B}\Delta b(\xi(t)) \le \|z^T P\tilde{B}\|\|\Delta b(\xi(t))\|$, we obtain

$$\begin{aligned}\dot{V} &\le -\|z\|^2 + 2\|z^T P\tilde{B}\|\|\Delta\|\|L(\xi)\| \\ &\quad - 2\|z^T P\tilde{B}\|^2\|L(\xi)\|^2 - 2\|z^T P\tilde{B}\|\|\widehat{\Delta}(t)\|\|L(\xi)\| \\ &= -\|z\|^2 - 2\|z^T P\tilde{B}\|^2\|L(\xi)\|^2 \\ &\quad + 2\|z^T P\tilde{B}\|\|L(\xi)\|(\|\Delta\| - \|\widehat{\Delta}(t)\|).\end{aligned}$$

Because $\|\Delta\| - \|\widehat{\Delta}(t)\| \le \|e_\Delta\|$, we obtain

$$\begin{aligned}\dot{V} &\le -\|z\|^2 - 2\|z^T P\tilde{B}\|^2\|L(\xi)\|^2 \\ &\quad + 2\|z^T P\tilde{B}\|\|L(\xi)\|\|e_\Delta\|.\end{aligned}$$

Further, we can obtain

$$\begin{aligned}\dot{V} &\le -\|z\|^2 - 2(\|z^T P\tilde{B}\|\|L(\xi)\| - \frac{1}{2}\|e_\Delta\|)^2 + \frac{1}{2}\|e_\Delta\|^2 \\ &\le -\|z\|^2 + \frac{1}{2}\|e_\Delta\|^2.\end{aligned}$$

Thus, we have the following relation

$$\dot{V} \le -\frac{1}{2}\|z\|^2, \quad \forall \|z\| \ge \|e_\Delta\| > 0,$$

Then from (5.24), we obtain that system (5.23) is ISS from input e_Δ to state z as guaranteed by Theorem 2.1.

6 Multi-parametric ES-based uncertainties estimation

Let us define now the following cost function

$$J(\widehat{\Delta}) = F(z(\widehat{\Delta})) \tag{6.25}$$

where $F : \mathbb{R}^n \to \mathbb{R}$, $F(0) = 0$, $F(z) > 0$ for $z \neq 0$. We need the following assumptions on J.

ASSUMPTION 6. *The cost function J has a local minimum at $\widehat{\Delta}^* = \Delta$.*

ASSUMPTION 7. *The initial error $e_\Delta(t_0)$ is sufficiently small, i.e. the original parameter estimate vector or matrix $\widehat{\Delta}$ are close enough to the actual parameter vector or matrix Δ.*

ASSUMPTION 8. *The cost function J is analytic and its variation with respect to the uncertain parameters is bounded in the neighborhood of $\widehat{\Delta}^*$, i.e. $\|\frac{\partial J}{\partial \widehat{\Delta}}(\tilde{\Delta})\| \le \xi_2$, $\xi_2 > 0$, $\tilde{\Delta} \in \mathcal{V}(\widehat{\Delta}^*)$, where $\mathcal{V}(\widehat{\Delta}^*)$ denotes a compact neighborhood of $\widehat{\Delta}^*$.*

REMARK 2. *Assumption 6 simply states that the cost function J has at least a local minimum at the true values of the uncertain parameters.*

REMARK 3. *Assumption 7 indicates that out results will be of local nature, i.e. our analysis holds in a small neighborhood of the actual values of the uncertain parameters.*

We can now state the following result.

LEMMA 6.1. *Consider the system (3.3) with the cost function (6.25), under Assumptions 1-8 and the assumption that $\Delta b(\xi(t)) = [\Delta_1, \dots, \Delta_m]^T$, with the feedback controller (5.14), where u_n is given by (4.9) and u_r is given by (5.22), and $\widehat{\Delta}(t)$ is estimated through the MES algorithm*

$$\begin{aligned}\dot{x}_i &= a_i \sin(\omega_i t + \frac{\pi}{2})J(\widehat{\Delta}) \\ \widehat{\Delta}_i(t) &= x_i + a_i \sin(\omega_i t - \frac{\pi}{2}), \quad i \in \{1, 2, \dots, m\}\end{aligned} \tag{6.26}$$

with $\omega_i \neq \omega_j$, $\omega_i + \omega_j \neq \omega_k$, $i, j, k \in \{1, 2, \ldots, m\}$, and $\omega_i > \omega^*$, $\forall\ i \in \{1, 2, \ldots, m\}$, with ω^* large enough, ensures that the norm of the error vector $z(t)$ admits the following bound

$$\|z(t)\| \leq \beta(\|z(0)\|, t) + \gamma(\tilde{\beta}(\|e_\Delta(0)\|, t) + \|e_\Delta\|_{\max})$$

where $\|e_\Delta\|_{\max} = \frac{\xi_1}{\omega_0} + \sqrt{\sum_{i=1}^{m} a_i^2}$, $\xi_1 > 0$, $e_\Delta(0) \in \mathcal{D}_e$, $\omega_0 = \max_{i\in\{1,2,\ldots,m\}} \omega_i$, $\beta \in \mathcal{KL}$, $\tilde{\beta} \in \mathcal{KL}$ and $\gamma \in \mathcal{K}$.

Proof. Based on Theorem 5.1, we know that the tracking error dynamics (5.23) is ISS from the input $e_\Delta(t)$ to the state $z(t)$. Thus, by Definition 2.1, there exist a class $\mathcal{KL}$ function β and a class $\mathcal{K}$ function γ such that for any initial state $z(0)$, any bounded input $e_\Delta(t)$ and any $t \geq 0$,

$$\|z(t)\| \leq \beta(\|z(0)\|, t) + \gamma(\sup_{0\leq\tau\leq t} \|e_\Delta(\tau)\|). \tag{6.27}$$

Now, we need to evaluate the bound on the estimation vector $\widehat{\Delta}(t)$, to do so we use the results presented in [10]. First, based on Assumption 8, the cost function is locally Lipschitz, i.e. there exists $\eta_1 > 0$ such that $|J(\Delta_1) - J(\Delta_2)| \leq \eta_1 \|\Delta_1 - \Delta_2\|$, for all $\Delta_1, \Delta_2 \in \mathcal{V}(\widehat{\Delta}^*)$. Furthermore, since J is analytic, it can be approximated locally in $\mathcal{V}(\widehat{\Delta}^*)$ by a quadratic function, e.g. Taylor series up to the second order. Based on this and on Assumptions 6 and 7, we can obtain the following bound ([10, p. 436-437],[14])

$$\|e_\Delta(t)\| - \|d(t)\| \leq \|e_\Delta(t) - d(t)\| \leq \tilde{\beta}(\|e_\Delta(0), t\|) + \frac{\xi_1}{\omega_0},$$

where $\tilde{\beta} \in \mathcal{KL}$, $\xi_1 > 0$, $t \geq 0$, $\omega_0 = \max_{i\in\{1,2,\ldots,m\}} \omega_i$, and $d(t) = [a_1 \sin(\omega_1 t + \frac{\pi}{2}), \ldots, a_m \sin(\omega_m t + \frac{\pi}{2})]^T$. We can further obtain that

$$\begin{aligned}\|e_\Delta(t)\| \leq&\ \tilde{\beta}(\|e_\Delta(0), t\|) + \frac{\xi_1}{\omega_0} + \|d(t)\| \\ \leq&\ \tilde{\beta}(\|e_\Delta(0), t\|) + \frac{\xi_1}{\omega_0} + \sqrt{\sum_{i=1}^{m} a_i^2}.\end{aligned}$$

Together with (6.27) yields the desired result.

7 Mechatronic Example

We consider here a two-link robot manipulator example. The dynamics for the manipulator in the nominal case, is given by (see e.g. [19])

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) \ = \ \tau, \tag{7.28}$$

where $q \triangleq [q_1, q_2]^T$ denotes the two joint angles and $\tau \triangleq [\tau_1, \tau_2]^T$ denotes the two joint torques. The matrix H is assumed to be non-singular and is given by

$$H \ \triangleq \ \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$$

Table 1: System Parameters for the manipulator example.

Parameter	Value
I_2	$\frac{5}{12}$ $[kg \cdot m^2]$
m_1	10 $[kg]$
m_2	5 $[kg]$
ℓ_1	1 $[m]$
ℓ_2	1 $[m]$
ℓ_{c_1}	0.5 $[m]$
ℓ_{c_2}	0.5 $[m]$
I_1	$\frac{10}{12}$ $[kg \cdot m^2]$
g	9.8 $[m/s^2]$

where

$$\begin{aligned} H_{11} &= m_1\ell_{c_1}^2 + I_1 + m_2[\ell_1^2 + \ell_{c_2}^2 + 2\ell_1\ell_{c_2}\cos(q_2)] + I_2 \\ H_{12} &= m_2\ell_1\ell_{c_2}\cos(q_2) + m_2\ell_{c_2}^2 + I_2 \\ H_{21} &= H_{12} \\ H_{22} &= m_2\ell_{c_2}^2 + I_2 \end{aligned} \tag{7.29}$$

The matrix $C(q, \dot{q})$ is given by

$$C(q, \dot{q}) \ \triangleq \ \begin{bmatrix} -h\dot{q}_2 & -h\dot{q}_1 - h\dot{q}_2 \\ h\dot{q}_1 & 0 \end{bmatrix},$$

where $h = m_2\ell_1\ell_{c_2}\sin(q_2)$. The vector $G = [G_1, G_2]^T$ is given by

$$\begin{aligned} G_1 &= m_1\ell_{c_1}g\cos(q_1) + m_2 g[\ell_2\cos(q_1 + q_2) + \ell_1\cos(q_1)] \\ G_2 &= m_2\ell_{c_2}g\cos(q_1 + q_2) \end{aligned} \tag{7.30}$$

In our simulations, we assume that the parameters take values according to [19] summarized in Table 1. The system dynamics (7.28) can be rewritten as

$$\ddot{q} \ = \ H^{-1}(q)\tau - H^{-1}(q)\left[C(q, \dot{q})\dot{q} + G(q)\right], \tag{7.31}$$

Thus, the nominal controller is given by

$$\begin{aligned} \tau_n \ =&\ [C(q, \dot{q})\dot{q} + G(q)] \\ +&\ H(q)\left[\ddot{q}_d - K_2(\dot{q} - \dot{q}_d) - K_1(q - q_d)\right], \end{aligned} \tag{7.32}$$

where $q_d = [q_{1d}, q_{2d}]^T$, denotes the desired trajectory and the feedback gains $K_1 > 0$, $K_2 > 0$, are chosen such that the tracking error will go to zero asymptotically. For simplicity, we use the feedback gains $K_j^i = 1$ in (4.10) for $i = 1, 2$ and $j = 1, 2$ in our simulations. The reference trajectory is given by the following function from the initial time $t_0 = 0$ to the final time t_f, where

$$q_{id}(t) \ = \ \frac{1}{1 + \exp(-t)}, \quad i \ = \ 1, 2$$

Table 2: Parameters used in MES

Q_1	Q_2	a_1	a_2	ω_1	ω_2	t_f
5	5	0.05	0.04	7.4	7.5	4

Now we introduce an uncertain term to the nonlinear model (7.28). In particular, we assume that there exist additive uncertainties in the model (7.31), i.e.

(7.33)
$$\ddot{q} = H^{-1}(q)\tau - H^{-1}(q)\left[C(q,\dot{q})\dot{q} + G(q)\right] + \Delta b(q).$$

We assume additive uncertainties on the gravity vector, such that

$$\Delta b(q) = \Delta \times G(q), \quad (7.34)$$

so that we have $\|\Delta b(q)\| \leq \|\Delta\| \|G(q)\|$. For simplicity, we assume that Δ is a diagonal matrix given by $\Delta = \mathrm{diag}\{\Delta_1, \Delta_2\}$. The robust controller term τ_r is designed according to (5.22), where $H = A^{-1},\ L = G$, and finally the two unknown parameters Δ_1 and Δ_2 are estimated by the MES, as shown in the next section.

7.1 MES Based uncertainties estimation First, we choose the following performance cost function

$$J = Q_1 \int_0^{t_f} (q - q_d)^T (q - q_d)\mathrm{d}t + Q_2 \int_0^{t_f} (\dot{q} - \dot{q}_d)^T (\dot{q} - \dot{q}_d)\mathrm{d}t, \quad (7.35)$$

where $Q_1 > 0$ and $Q_2 > 0$ denote the weighting parameters. Then, the two unknown parameters Δ_1 and Δ_2 are estimated by the MES (which is a discrete version of (6.26))

(7.36)
$$\begin{aligned} x_i(k+1) &= x_i(k) + a_i t_f \sin(\omega_i t_f k + \frac{\pi}{2})J \\ \widehat{\Delta}_i(k+1) &= x_i(k+1) + a_i \sin(\omega_i t_f k - \frac{\pi}{2}), \quad i = 1, 2 \end{aligned}$$

where $k = 0,\ 1,\ 2, \cdots$ denotes the iteration index, x_i and $\widehat{\Delta}_i$ $(i = 1, 2)$ start from zero initial conditions. We simulate the system with $\Delta_1 = -1$ and $\Delta_2 = -3$. The parameters that were used in the cost function (7.35) and the MES (7.36) are summarized in Table 2. As shown in Fig. 7.1, the ISS-based controller combined with ES greatly improves the tracking performance. Fig. 2 shows that the cost function starts at an initial value around 6 and decreases below 0.5 within 100 iterations and the value of the cost function is decreasing over the iterations. Moreover, the estimate of the unknown parameters converge to a neighborhood of the true parameter values, as shown in Fig. 3.

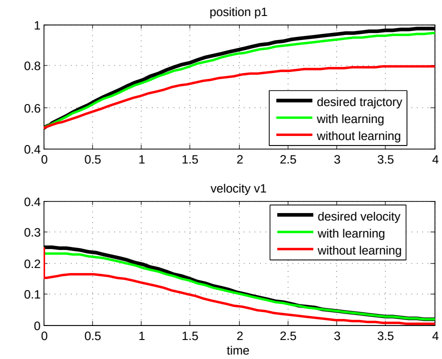

Figure 1: Obtained trajectories vs. Reference Trajectories for q_1 (in the top plot) and $\dot{q}_1$ (in the bottom plot)

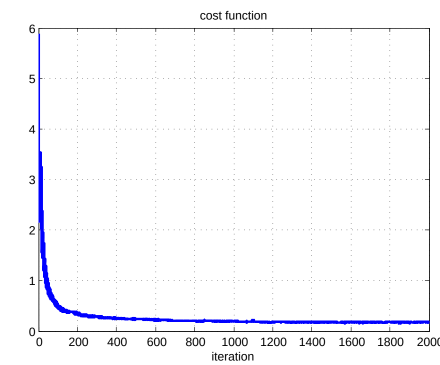

Figure 2: The cost function vs. the number of iterations

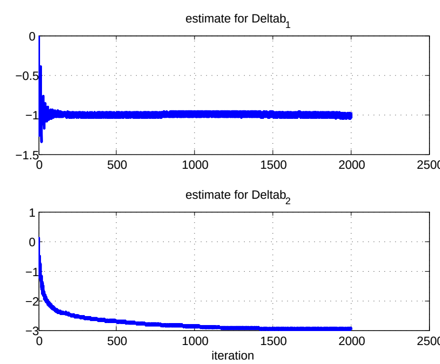

Figure 3: Estimate of parameter Δ_1 (in the top plot) and parameter Δ_2 (in the bottom plot)

8 Conclusion

In this paper, we studied the problem of extremum seeking-based indirect adaptive control for nonlinear systems affine in the control with bounded additive state-dependent uncertainties. We have proposed a robust controller which renders the feedback dynamics ISS w.r.t the parameter estimation errors. Then we have combined the ISS feedback controller with a model-free ES algorithm to obtain a learning-based adaptive controller, where the ES is used to estimate the uncertain part of the model. We have presented the stability proof of this controller and have shown a detailed application of this approach on a two-link robot manipulator example. Future works will deal with the case where the uncertainties to be estimated are time-varying.

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