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Speed Sensorless State Estimation for Induction Motors: A Moving Horizon Approach

Lei Zhou[1] and Yebin Wang[2]

***Abstract*—This paper investigates the speed sensorless state estimation problem for induction motors. Aiming at developing new state estimation means to improve the estimation bandwidth, this paper proposes various moving horizon estimation (MHE)-based state estimators. Applying MHE for induction motors is not straightforward due to the fast convergence requirement, external torque disturbances, parametric model errors, etc. To improve speed estimation transient performance, we propose an MHE based on the full induction motor model and an assumed load torque dynamics. We further formulate an adaptive MHE to jointly estimate parameters and states and thus improve robustness of the MHE with respect to parametric uncertainties. A dual-stage adaptive MHE, which performs parameter and state estimation in two steps, is proposed to reduce computational complexity. Under certain circumstances, the dual-stage adaptive MHE is equivalent to the case with a recursive least square algorithm for parameter estimation and a conventional MHE for state estimation. Implementation issues and tuning of the estimators are discussed. Numerical simulations demonstrate that the proposed MHE estimators can effectively estimate the induction motor states at a fast convergence rate, and the dual-stage adaptive MHE can provide converging state and parameter estimation despite the initial model parametric errors.**

I. INTRODUCTION

In the speed sensorless control of induction motors, where the motor speed and position are not measured, the convergence rate of the state estimation is the key limitation to the motor's tracking bandwidth. This fact motivates the development of new state estimation solutions for induction motor systems.

Speed sensorless state estimation for induction motors is a challenging problem since the motor dynamics is multivariable and nonlinear, and the motor parameters are often not exactly known. Through the years, numerous estimation schemes have been studied for induction motors. The classic *model reference adaptive system* (MRAS) approach treats the motor speed as a time-varying parameter to avoid nonlinearity [1]–[4], but it often suffers from slow converging due to the adaptive estimation. The *sliding mode observer* (SMO) treats the nonlinear terms as bounded uncertainties and has achieved robust performance [5], [6], but this often leads to an unnecessary worst case design and degraded estimation accuracy. The *extended/unscented Kalman filter* (EKF/UKF) schemes have also been studied [7], [8], where the rotor mechanical equation is often not included. This formulation allows state estimation without knowing the motor mechanical parameters, but often results in slow transient. Reference [9] performed EKF for induction motor with the mechanical dynamics included, which helps improving the transient performance and the estimation accuracy at low speed.

In this paper we study the *moving horizon estimation* (MHE) for speed sensorless state estimation of induction motors, targeting at improving the convergence rate of the speed estimation. The MHE has been initially introduced in [10] inspired by its widely used dual problem *receding horizon control* (RHC), and is receiving growing interest in the past decade due to the advances in numerical optimizations and computational capability of computers. References [11] and [12] have provided comprehensive studies of the MHE for general linear and nonlinear systems, respectively.

The MHE for induction motor state estimation has been explored in [13] to achieve better estimation accuracy and bandwidth against MRAS and EKF estimators. However in [13] the motor speed is assumed to be constant over the estimation horizon, which may limit the speed estimation convergence rate. Also [13] assumed exact knowledge of model parameters, which is not always available in practice.

In this paper, the MHE considering the full dynamics of the induction motor is being studied, where the rotor speed is estimated as a state using the rotor's equation of motion. Comparing with the constant speed assumption, the inclusion of the mechanical equation can improve the speed estimation convergence rate and can improve the estimation accuracy at low speed. This formulation, however, increases the estimator's sensitivity with respect to the mechanical uncertainties, such as load variations and friction torque disturbances. To address this, in our work the load torque is being estimated as a state variable with an assumed dynamics.

Another contribution of this paper is the inclusion of on-line parameter estimation. It is well known that the performance of MHE is significantly influenced by the model accuracy. In order to increase the estimator's robustness in terms of parametric uncertainties, the adaptive MHE is being studied, where the parameters are being estimated together with the states. Different formulations of the adaptive MHE for induction motors are introduced and discussed, and a dual-stage adaptive MHE that decomposes state and parameter estimations is proposed. Our simulation shows that the dual-stage estimator design can effectively lower the implementation difficulty of MHE and can achieve accurate estimation despite initial parameter errors.

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This paper is organized as follows. The induction motor model and the general MHE formulation are briefly introduced in Section II. The MHE for induction motor state estimation including the mechanical dynamics is presented in Section III. Several adaptive MHE formulations for induction motor are presented in Section IV. Section V discusses the design and tuning of the estimators, and Section VI verifies the performances of MHE and dual-stage adaptive MHE through numerical simulations. Conclusion is drawn in Section VII.

II. Induction Motor Model and General MHE

A. Induction Motor Model

The induction motor model in the stationary two-phase reference frame can be written as

$$\begin{aligned} \dot{i}_{ds} &= -\gamma i_{ds} + \alpha\beta\psi_{dr} + \beta\psi_{qr}\omega + u_{ds}/\sigma \\ \dot{i}_{qs} &= -\gamma i_{qs} - \beta\psi_{dr}\omega + \alpha\beta\psi_{qr} + u_{qs}/\sigma \\ \dot{\psi}_{dr} &= \alpha L_m i_{ds} - \alpha\psi_{dr} - \psi_{qr}\omega \\ \dot{\psi}_{qr} &= \alpha L_m i_{qs} + \psi_{dr}\omega - \alpha\psi_{qr} \\ \dot{\omega} &= \frac{\mu}{J}(-i_{ds}\psi_{qr} + \psi_{dr} i_{qs}) - \frac{T_L}{J} \\ y &= \left[i_{ds}, i_{qs}\right]^T, \end{aligned} \tag{1}$$

where ψ_{dr} and ψ_{qr} are the rotor fluxes, i_{ds} and i_{qs} are the stator currents, u_{ds} and u_{qs} are the stator voltages, all defined in the stationary d-q frame. ω is the rotor speed; J is the rotor inertia; T_L is the load torque, and y is the measurement. The rest variables in (1) denote model parameters, where $\sigma = L_s(1-\frac{L_m^2}{L_sL_r})$, $\alpha = \frac{R_r}{L_r}$, $\beta = \frac{L_m}{\sigma L_r}$, $\gamma = \frac{R_s}{\sigma} + \alpha\beta L_m$, $\mu = \frac{3}{2}\frac{L_m}{L_r}$. ($R_s$, L_s) and (R_r, L_r) are the resistance and inductance of the stator and the rotor, respectively, and L_m is the mutual inductance.

Speed sensorless estimation problem for induction motor is roughly formulated as: design a estimator to reconstruct the full state of the induction motor system (1) from measuring only the stator currents (i_{ds}, i_{qs}) and voltages (u_{ds}, u_{qs}).

B. General MHE formulation

This section briefly introduces the general MHE formulation to make this paper self-contained. Consider a nonlinear stochastic discrete-time system

$$\begin{aligned} x_{k+1} &= f_k(x_k, u_k) + w_k \\ y_k &= h_k(x_k) + v_k, \end{aligned} \tag{2}$$

where k is the time step, x_k is the state, u_k is the control input, y_k is the output, w_k is the process noises, and v_k is the measurement noises. The MHE at time T can be formulated as the following constrained optimization problem

$$\min_{z,\{w_k\}_{k=T-N}^{T-1}} \mathcal{Z}_{T-N}(z) + \sum_{k=T-N}^{T-1} L_k(w_k, v_k)$$

subject to

$$\begin{aligned} & x_{k+1} = f_k(x_k, u_k) + w_k, k = T-N, ..., T-1 \\ & v_k = y_k - h_k(x_k) \in \mathbb{V}_k,\ k = T-N, ..., T-1 \\ & x_k(k; z, \{w_j\}, u_k) \in \mathbb{X}_k,\ k = T-N, ..., T \\ & w_k \in \mathbb{W}_k,\ k = T-N, ..., T-1, \end{aligned} \tag{3}$$

where N is the length of the estimation horizon defined between $T-N$ and $T-1$, and $z = x_{T-N}$ is the state at the beginning of the estimation horizon. The sets $\mathbb{X}_k$, $\mathbb{W}_k$ and $\mathbb{V}_k$ denote the constraints on states, process noises, and measurement noises, respectively.

The cost function in (3) consists of two parts: the *arrival cost* $\mathcal{Z}_{T-N}(z)$ and the sum of the *stage costs* $L_k(w_k, v_k)$ over the horizon. The stage cost $L_k(w_k, v_k)$ penalizes on the estimation errors w_k and v_k at each time step inside the estimation horizon, and the arrival cost $\mathcal{Z}_{T-N}(z)$ summarizes the past data that are not explicitly accounted for in the objective function. A true arrival cost is defined as

$$\mathcal{Z}_{T-N}(z) = \min_{x_0,\{w_k\}_{k=0}^{T-N-1}} \sum_{k=0}^{T-N-1} L_k(w_k, v_k) + \Gamma(x_0) \tag{4}$$

and subject to constraints in (3) from 0 to $T-N$. Here $\Gamma(x_0)$ is the initial cost, penalizing on the deviation of the initial state estimate from its true value. The MHE with the true arrival cost ensures that it has the same solution with the full-information estimation.

Remark 2.1: When MHE is used for nonlinear or constrained systems, the exact expression for the true arrival cost cannot be established [12]. An approximation of the arrival cost, denoted by $\hat{\mathcal{Z}}_{T-N}(z)$, is usually used. The arrival cost approximation can significantly influence the estimation accuracy and the stability of the estimator.

III. MHE for Induction Motors

Work [13] considers the MHE for the induction motor with an assumed speed dynamics $\dot{\omega} = 0$. This treatment helps ameliorate numerical stability of the optimization problem derived from the MHE, however compromises estimation performance. In this section, we formulate the MHE using the induction motor model with the mechanical equation included.

Assuming that the load torque is slowly time-varying compared to the motor states, we have $\dot{T}_L = 0$. By combining Eq. (1) and $\dot{T}_L = 0$, we can obtain a 6th-order induction motor model with the state variables given by $x = [i_{ds}, i_{qs}, \psi_{dr}, \psi_{qr}, \omega, T_L]^T$. By discretizing the model and including the process and measurement noises, we can get a discrete-time stochastic model of the induction motor as

$$\begin{aligned} x_{k+1} &= f(x_k) + Bu_k + w_k \\ y_k &= Cx_k + v_k. \end{aligned} \tag{5}$$

Note that in (5) B and C are constant matrices, while $f(\cdot)$ is a smooth vector field.

Remark 3.1: The main goal of including the load torque as a state variable is to improve the estimator's robustness towards mechanical uncertainties. When the motor is running, the load torque may be time-varying, and the Coulomb friction is also known to deteriorate the estimator's performance especially during low speed operation. In order to maintain accurate estimation despite these uncertainties, the load torque is treated as a state variables. We selected an assumed dynamics $\dot{T}_L = 0$ since the motor load torque variation during operation are usually slow compared with the required speed bandwidth.

The MHE for the induction motor with rotor speed dynamics can be formulated as

$$\min_{z,\{w_k\}_{k=T-N}^{T-1}} \quad \Phi_T = \hat{\mathcal{Z}}_{T-N}(z) + \sum_{k=T-N}^{T-1} L_k(w_k, v_k) \qquad (6)$$

subject to the system dynamics (5). A quadratic stage cost is selected as $L_k(w_k, v_k) = w_k^T Q^{-1} w_k + v_k^T R^{-1} v_k$, where Q and R are positive definite matrices and can be regarded as design parameters of the estimator. Specifically, when w_k and v_k are zero mean, independent Gaussian variables, the matrices Q and R can be selected as their covariance matrices.

The induction motor model is nonlinear. As is mentioned in Remark 2.1, there does not exist a closed-form expression for the exact arrival cost. Here we use the *filtering* form of arrival cost approximation introduced in [12]. Define the cost for the initial estimation error as $\Gamma(x_0) = (x_0 - \hat{x}_0)^T \Pi_0^{-1}(x_0 - \hat{x}_0)$. The approximate arrival cost can be calculated by

$$\hat{\mathcal{Z}}_{T-N}(z) = (z - \hat{x}_{T-N})^T \Pi_{T-N}^{-1}(z - \hat{x}_{T-N}) + \Phi_{T-N}^*,$$

where Φ_{T-N}^* is computed optimal cost of the problem (6) at time $T-N$. The matrix Π_{T-N} is updated according to the following matrix Ricatti equation

$$\begin{aligned}\Pi_{k+1} =& Q + A_k \Pi_k A_k^T \\ &- A_k \Pi_k C^T (R + C^T \Pi_k C^T)^{-1} C \Pi_k A_k^T,\end{aligned}$$

in which $A_k = \partial f(\hat{x}_k)/\partial \hat{x}_k$.

Remark 3.2: The MHE (6) does not include inequality constraints for two reasons. First, this does not significantly improve the estimation performance, because the inequality constraints on induction motor states are loose and almost always satisfied. Second, removing the inequality constraints can simplify the optimization problem and greatly reduce the computational load.

IV. Adaptive MHE for Induction Motors

In induction motor systems, the model parameters are often not exactly known as well as time-varying during the operation. For example the electric heating incurs significant variations of both the stator and rotor resistance values. On the other hand, it is well known that the MHE is a model-based estimation scheme, and its performance highly relies on the model accuracy. In order to improve the estimator's robustness with respect to parametric model errors, we present an adaptive MHE for the speed sensorless estimation, where the system parameters are estimated together with states.

A. Augmented state MHE

One way to implement the adaptive MHE is through *augmented state* MHE, which is defined on the basis of an augmented system dynamics. Define the vector of model parameters as $p = [\alpha, \beta, \gamma, \sigma]^T$. We first expand the state x by including model parameters as an augmented state $x'^T = [x, p]^T$. Also define the augmented process noises $w' = [w, w_p]^T$, where w_p represent the mismatch between the true model parameters and its estimate $\hat{p}$. Consequently, assume the parameters are slowly time varying, we have the augmented system dynamics given by (5) and $\dot{p} = 0$. The augmented state MHE is therefore formulated as the following optimization problem:

$$\min_{z',\{w'_k\}_{k=T-N}^{T-1}} \quad \Phi'_T = \mathcal{Z}'_{T-N}(z') + \sum_{k=T-N}^{T-1} L'_k(w'_k, v'_k), \qquad (7)$$

and subject to the augmented system dynamics.

In the augmented state MHE, the stage and arrival costs are calculated using the same formulas as the non-adaptive MHE (6), except that the augmented state x' and process noises w' are used instead of x and w. The covariance matrix of the augmented process noises is defined as $Q' = \mathrm{diag}(Q, Q_p)$, where each diagonal component of the matrix $Q_p = \mathrm{diag}(Q_\alpha, Q_\beta, Q_\gamma, Q_\sigma)$ represents the weight on the estimation error of individual model parameter.

B. Dual-stage adaptive MHE

In the augmented state MHE, the inclusion of parameters in the states results in a higher order and highly non-convex optimization problem. This fact, however, adds significant difficulties to the optimization problem solving. In order to make the problem tractable, a *dual-stage* adaptive moving horizon estimator is proposed, where the parameter estimation and the state estimation are decomposed into two sequential steps. Comparing with the original augmented state MHE, the dual-stage MHE can effectively reduce the size and complexity of the optimization problems, and therefore makes them relatively easy to solve with established nonlinear programming (NLP) solvers.

In the dual-stage adaptive MHE, two optimization problems are solved sequentially at every time step for parameter and state estimation. The parameter estimation can be achieved by solving the optimization problem

$$\min_{p} \quad \Phi_T^p = \Phi_{T-N_p}^{p*} + \sum_{k=T-N_p}^{T-1} v_k^T R^{-1} v_k, \qquad (8)$$

where N_p is the length of the parameter estimation horizon. The arrival cost in (8) is selected as $\Phi_{T-N_p}^{p*}$, which implies the estimator is totally forgetting the initial guesses. This selection is made because there is no dynamics involved in the parameters to propagate the covariance of the parameter estimation error, i.e., $\dot{p} = 0$. The stage cost in (8) is selected as a quadratic form of the output error v_k, and the penalty on w_k is not included. This is because $\{w_k\}_{k=T-N_x}^{T-1}$ are the decision variables of the state estimation and thus are fixed in the parameter estimation, so the quadratic term $w_k^T Q^{-1} w_k$ does not directly penalize on the model parameters. In the formulation (8), the parameter vector p is constant in the parameter estimation horizon, therefore the size of the corresponding optimization problem is fixed and is independent to the horizon length N_p.

The state estimation problem is given by

$$\min_{z,\{w_k\}_{k=T-N_x}^{T-1}} \quad \Phi_T^x = \mathcal{Z}_{T-N_x}(z) + \sum_{k=T-N_x}^{T-1} L_k^x(w_k, v_k)$$

and subject to the induction motor model (5). Fig. 1 shows a block diagram of the data transmission in the dual-stage MHE.

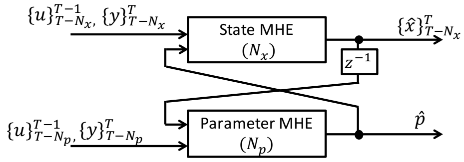

Fig. 1. Block diagram of the dual-stage MHE.

By separating out the parameter estimation, the state estimation in the dual-stage MHE is reduced to the conventional MHE for state estimation.

Remark 4.1: The parameter estimation can be simplified by redefining the parameter vector as $\theta = [\gamma, \alpha\beta, \beta, 1/\sigma]^T$ and using only the first two state equations in (5), i.e., the stator current dynamics. The reason is two fold. First, all four parameters are appearing in the stator current dynamics, therefore these two equations are sufficient to estimate all parameters. Second, the proposed parameters in θ are linearly involved in the induction model equations, therefore the parameter estimation can be an unconstrained linear estimation problem. With this modification, the complexity of solving the corresponding optimization problem can be significantly reduced.

C. RLS-based dual-stage adaptive MHE

The simplification to the parameter estimation in Remark 4.1 formulates the parameter estimation of the induction motor as an unconstrained linear system identification problem, in which case, the recursive least square (RLS) estimation method can be readily applied for the parameter estimation.

For a linear equation $y = \phi^T\theta$, where ϕ is the input vector and y is the vector of measurements, the RLS estimation gives the estimated parameter $\hat{\theta}$ that minimizes the accumulated mean squared error as

$$\min_{\theta} \quad \Phi_T^{RLS} = \frac{1}{T}\sum_{k=1}^{T}\left(y_{k|\theta} - y_k\right)^2. \tag{9}$$

To perform RLS-based parameter estimation for the induction motor, the discretized stator current equations can be written as the following linear regression form

$$\begin{bmatrix} \frac{i_{ds}^{k+1} - i_{ds}^{k}}{\mathrm{d}t} \\ \frac{i_{qs}^{k+1} - i_{qs}^{k}}{\mathrm{d}t} \end{bmatrix} = \begin{bmatrix} -\hat{i}_{ds}^{k} & \hat{\psi}_{ds}^{k} & \hat{\psi}_{qs}^{k}\hat{\omega}^{k} & u_{ds}^{k} \\ -\hat{i}_{qs}^{k} & \hat{\psi}_{qs}^{k} & -\hat{\psi}_{ds}^{k}\hat{\omega}^{k} & u_{qs}^{k} \end{bmatrix} \begin{bmatrix} \gamma \\ \alpha\beta \\ \beta \\ 1/\sigma \end{bmatrix} \tag{10}$$

where $\mathrm{d}t$ is the sampling interval. The RLS estimation algorithm can then be applied to (10) and identify the parameters.

Remark 4.2: By comparing the cost functions of the MHE parameter estimation formulation (8) and the RLS parameter estimation given in (9), we can conclude that with the simplification in Remark 4.1, the RLS parameter estimation is equivalent to the MHE parameter estimation with an infinite parameter estimation horizon length and with the matrix R in the cost function being an identity matrix.

V. Discussion

A. Arrival cost

The arrival cost in MHE plays a crucial role in determining the behavior of the overall estimation process. Since a closed-form expression for the true arrival cost does not exist for nonlinear or constrained systems, an approximation to the arrival cost need to be used. According to the stability analysis of MHE in [12], asymptotic convergence of the estimation error can be preserved if the approximated arrival cost is bounded by the true arrival cost.

Although this condition allows systematic stability analysis, a practical arrival cost synthetic method that meets this condition is hard to find. In our development of MHE for induction motors, the filtering arrival cost approximation in [12] and [14] is being used, as was discussed in Section III.

Another commonly used approximation of arrival cost is $Z_{T-N} = \Phi_{T-N}^{*}$. This arrival cost is independent of z and is totally ignoring the initial guesses. This arrival cost approximation satisfies the inequality conditions and therefore asymptotic convergence can be guaranteed. However, this selection does not necessarily give satisfactory performance [15]. With this approximation, the horizon length need to be sufficiently large to achieve faster convergence.

As an alternative to the filtering approximation, a *smoothing* arrival cost approximation was proposed in [11] and was further described in [14]. This arrival cost approximation helps to eliminate the periodic behavior of the estimator by including more data in the update of z, where the arrival cost covariance uses $\Pi_{T-N|T-1}$. In our implementation this approximation was not selected, mainly because the periodical behavior of the estimator does not significantly deteriorate the convergence rate of the estimation.

B. Horizon length

Another important design parameter of the MHE is the horizon length. Similar to its dual problem RHC, a large horizon length is preferable for the MHE. Nevertheless, a long estimation horizon will lead to a large scale optimization problem and overload the computational resources. Usually the horizon length is determined by balancing the trade-off between estimation performance and the computational time.

A longer estimation horizon allows the estimator to use more data, and therefore the estimation is less dependent on the approximation of the arrival cost. For unconstrained MHE, selecting a horizon length of $N = 1$ will reduce the MHE estimator to the EKF, where only the measurements at the current time step are used in the estimation process. Intuitively, one can deduce that comparing with unconstrained MHE with a horizon length larger than 1, the EKF is more sensitive to the initial error in states and covariances.

The horizon length may be dynamically changed in the estimation process. This method is particularly interesting for the dual-stage adaptive MHE, where the model accuracy changes along with the parameter estimation process. In our implementation, different horizon lengths are selected for the state estimation in the initial parameter estimation transient and in steady state. During the parameter estimation transient, a horizon length of 2 is selected for less trust to the model accuracy. After the parameter estimation converges, a longer horizon length is used to provide more precise estimation.

TABLE I
PARAMETERS OF INDUCTION MOTOR MODEL.

Parameter	Value
Stator resistance R_s	11.05Ω
Rotor resistance R_r	2.133Ω
Stator self-inductance L_s	0.23 H
Rotor self-inductance L_r	0.23 H
Mutual Inductance L_m	0.22 H
Rotor inertia J	$0.0012\ \mathrm{kgm}^2$
Number of pole pares p	2
Motor power	250 W

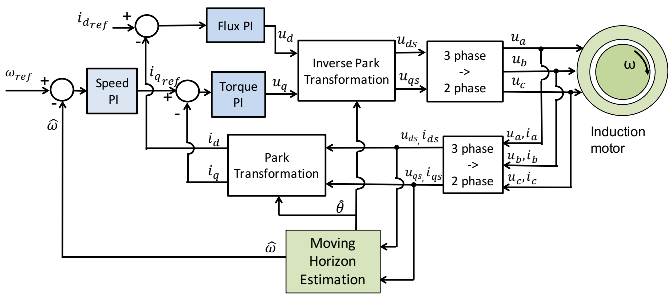

Fig. 2. Block diagram of induction motor vector control.

VI. NUMERICAL VALIDATION

A. Setup description

Numerical simulations are used to test the proposed MHE schemes. Table I shows the system parameters of the induction motor used in the simulations. The simulation runs at a sampling rate of 10 kHz, and the Matlab Optimization Toolbox$^{\mathrm{TM}}$ is used for solving the optimization problems. The process and measurement noises are assumed to be zero-mean Gaussian random processes, with the covariance matrices being $Q = \mathrm{diag}\big(1\times10^{-4}\mathrm{A}^2, 1\times10^{-4}\mathrm{A}^2, 1\times10^{-4}(\mathrm{V}\cdot\mathrm{s})^2, 1\times10^{-4}(\mathrm{V}\cdot\mathrm{s})^2, 1\times10^{-4}(\mathrm{rad/s})^2, Q_{T_L}(\mathrm{Nm})^2\big)$ and $R = \mathrm{diag}\big(1\times10^{-6}\mathrm{A}^6, 1\times10^{-4}\mathrm{A}^2\big)$, where Q_{T_L} can be selected according to the motor's operation conditions.

Fig. 2 shows a block diagram of the speed sensorless induction motor system that is used in the numerical evaluations. The controllers form a standard indirect field oriented control, and thus the details are omitted. The simulation is conducted with the control loops closed using the measured currents and the estimated speed, and the proportional-integral (PI) tracking controller gains are kept the same under different test cases.

B. MHE state estimation

The proposed MHE formulation for induction motor state estimation is compared with the transient performance of EKF. Note that both estimators have the mechanical equation included in the model. In this simulation, the induction motor parameters are assumed to be exactly known. The initial states are selected as $i_{ds} = i_{qs} = 1$ A, $\psi_{dr} = \psi_{qr} = 0\ \mathrm{V}\cdot\mathrm{s}$, $\omega = 5\ \mathrm{rad/s}$, $T_L = 0$ Nm. The initial values of the estimated states were selected to be all zero values. The covariance matrix for initial state estimation error is $\Pi_0 = I_{6\times6}\times10^{-3}$, and Q_{T_L} is selected as 1×10^{-4} for not including the torque estimation. An estimation horizon length of 20 time steps is selected for the MHE. The simulated operating condition is speed step responses, where the reference speed is 100 $\mathrm{rad/s}$ during the time interval $[0, 0.2s]$, and a reference speed step of 20 $\mathrm{rad/s}$ is added at $t = 0.2$ s.

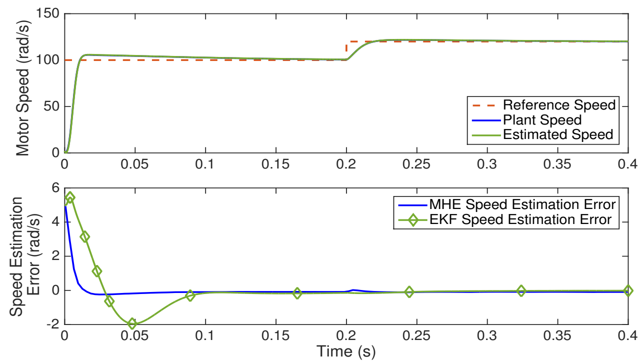

Fig. 3. State estimation for induction motor with MHE.

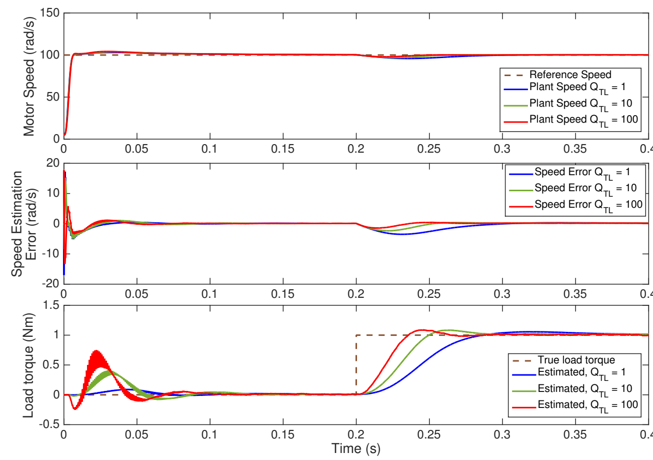

Fig. 4. MHE estimation under step torque disturbance.

Fig. 3 shows the simulation results of the MHE for induction motor. In Fig. 3, the top plot shows the reference, plant and estimated speed, and the bottom plot shows the estimation errors of the MHE and that of EKF with the same initial conditions. It can be seen that the MHE with mechanical equation included can correctly estimate the speed of the induction motor and demonstrated a faster convergence transient comparing with EKF. However when the reference speed step is happening, the estimation error of the MHE experiences a small transient (peak 0.25 rad/s), while the estimation error of the EKF barely deviates from zero.

The proposed MHE was also compared with the baseline MHE formulation in [13]. However with a step-type speed reference, our simulation shows that the baseline MHE has a relatively slow estimation transient, and consequently the tracking controllers have to be tuned slower than the proposed MHE to ensure system stability. This observation coincides with the fact that the baseline MHE will suffer slow transient due to inherent adaptation-based speed estimation.

C. Load torque estimation

We also simulate the proposed MHE with mechanical equation to verify its ability to sustain step-type load torque

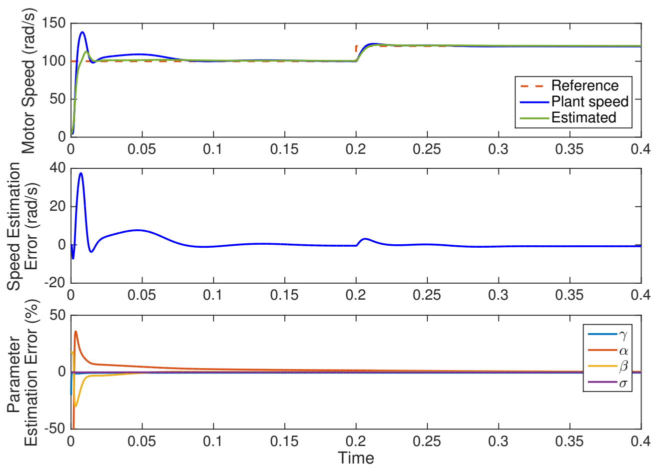

Fig. 5. Simulation results of the dual-stage adaptive MHE.

disturbances, and the results are shown in Fig. 4. In this test, an estimation horizon of 10 time steps is selected. The initial conditions for the state and its estimate are taken as $x = [1, 1, 0, 0, 5, 0]^T$ and $\hat{x} = [0, 0, 0, 0, 0, 0]^T$, and the initial guess on error covariance is $\Pi_0 = I_{6\times6} \times 10^{-3}$.

In Fig. 4, the top plot shows the reference and plant speed of the motor with estimators of different Q_{T_L} values being used for speed feedback control, the middle plot shows the corresponding speed estimation errors, and the bottom plot presents the true load torque and their estimates. The data show that the MHE formulation with load torque included in the state variables can successfully reject disturbances in the load torque, and the load torque model error covariance Q_{T_L} determines the convergence rate of the torque and speed estimation. This observation matches with the performance of the 6^{th} order EKF with load torque estimation included [16], where a larger error covariance term Q_{T_L} gives a faster estimation transient.

D. Dual-stage adaptive MHE

The RLS-based dual-stage adaptive MHE is simulated with the induction motor system. In this test case, the initial values of the parameter estimates are $\sigma_0 = 0.8\sigma$, $\gamma_0 = 0.8\gamma$, $\alpha_0 = 0.9\alpha$, $\beta_0 = 0.9\beta$. The horizon length of the MHE state estimator is selected as $N_x = 2$ when 0 s $\leq t \leq 0.1$ s, and $N_x = 10$ when 0.1 s $< t \leq 0.4$ s.

The simulation results of the RLS-based dual-stage adaptive MHE are shown in Fig. 5, where the top plot shows the plant and estimated speed, the middle plot shows the corresponding speed estimation error, and the parameter estimation percentage error is shown in the bottom plot. This simulation demonstrates that the dual-stage adaptive MHE can successfully incorporate the parameter estimation and give converging estimation despite the existence of initial model parametric errors, while the under these conditions non-adaptive MHE (6) and EKF fail to provide convergent state estimation.

VII. Conclusion and Future Work

In this work, a moving horizon estimation (MHE) scheme for induction motor state estimation with the rotor mechanical dynamics included was introduced, and a dual-stage adaptive MHE formulation that offers parameter on-line estimation was proposed. Simulation results show that the proposed MHE can provide a relatively fast converging estimation transient and can reject torque disturbances, which will allow the usage of high bandwidth tracking controllers and therefore improve the speed control bandwidth of the motor. The test results of the dual-stage adaptive MHE for induction motor show that the proposed estimation scheme can successfully achieve converging estimation performance when the model parameters are not exactly known initially. Future work should consider analysis and better tuning of the dual-stage adaptive MHE, which will significantly resolve the difficulties of the experimental implementation of MHE for induction motors.

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