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Subbaraman, A.; Benosman, M.

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Extremum Seeking-based Iterative Learning Model Predictive Control (ESILC-MPC)

Anantharaman Subbaraman *

* *Anantharaman Subbaraman (anantharaman@umail.ucsb.edu) is with the Department of Electrical and Computer Engineering, University of California at Santa Barbara, USA.*

Mouhacine Benosman **

** *Mouhacine Benosman (m_benosman@ieee.org) is with Mitsubishi Electric Research Laboratories, Cambridge, MA 02139, USA.*

Abstract: In this paper, we study a tracking control problem for linear time-invariant systems with model parametric uncertainties under input and states constraints. We apply the idea of modular design introduced in Benosman [2014], to solve this problem in the model predictive control (MPC) framework. We propose to design an MPC with input-to-state stability (ISS) guarantee, and complement it with an extremum seeking (ES) algorithm to iteratively learn the model uncertainties. The obtained MPC algorithms can be classified as iterative learning control (ILC)-MPC.

1. INTRODUCTION

Model predictive control (MPC), e.g., Mayne et al. [2000], is a model-based framework for optimal control of constrained multi-variable systems. MPC is based on the repeated, receding horizon solution of a finite-time optimal control problem formulated from the system dynamics, constraints on system states, inputs, outputs, and a cost function describing the control objective. However, the performance of MPC based controllers inevitably depends on the quality of the prediction model used in the optimal control computation. In contrast, extremum seeking (ES) control is a well known approach where the extremum of a cost function associated with a given process performance (under some conditions) is found without the need for detailed modelling information, see, e.g., Ariyur and Krstic [2003, 2002], Nesic [2009]. Several ES algorithms (and associated stability analysis) have been proposed, Krstic [2000], Tan et al. [2006], Nesic [2009], Ariyur and Krstic [2003], Guay et al. [2013], and many applications of ES have been reported Hudon et al. [2008], Zhang and Ordóñez [2012], Benosman and Atinc [2013].

The idea that we want to theoretically analyze in this paper, is that the performance of a model-based MPC controller can be combined with the robustness of a model-free ES learning algorithm for simultaneous identification and control of linear time-invariant systems with structural uncertainties. We refer the reader to Benosman [2014], Benosman and Atinc [2013], Atinc and Benosman [2013] where this idea of learning-based modular adaptive control has been introduced in a more general setting of nonlinear dynamics. We aim at proposing an alternative approach to realize an iterative learning-based adaptive MPC. We introduce an approach for an ES-based iterative learning MPC that merges a model-based linear MPC algorithm with a model-free ES algorithm to realize an iterative learning MPC that adapts to structured model uncertainties. Due to the iterative nature of the learning model improvement, we first review some existing Iterative learning control (ILC) MPC methods. Indeed, ILC method introduced in Arimoto [1990] is a control technique which focuses on improving tracking performance of processes that repeatedly execute the same operation over time. It is of particular importance in robotics and in chemical process control of batch processes. We refer the reader to e.g., Wang et al. [2009], and Ahn et al. [2007] for more details on ILC and its applications. At the intersection of learning based control and constrained control is the ILC-MPC concept. For instance, ILC-MPC for chemical batch processes are studied in Wang et al. [2008], Cueli and Bordons [2008], and Shi et al. [2007]. As noted in Cueli and Bordons [2008] one of the shortcomings of the current literature is a rigorous justification of feasibility, and Lyapunov-based stability analysis for ILC-MPC . For example, in Wang et al. [2008] the goal is to reduce the error between the reference and the output over multiple trials while satisfying only input constraints. However, the reference signals is arbitrary and the MPC scheme for tracking such signals is not rigorously justified. Furthermore, the MPC problem does not have any stabilizing conditions (terminal cost or terminal constraint set). In Cueli and Bordons [2008], an ILC-MPC scheme for a general class of nonlinear systems with disturbances is proposed. The proof is presented only for MPC without constraints. In Shi et al. [2007], the ILC update law is designed using MPC. State constraints are not considered in Shi et al. [2007]. In Lee et al. [1999] a batch MPC (BMPC) is proposed, which integrates conventional MPC scheme with an iterative learning scheme. A simplified static input-output map is considered in the paper as opposed to a dynamical system. Finally, the work of Aswani et al. [2013, 2012a,b], studies similar control objectives as the one targeted in this paper using a learning-based MPC approach. The main differences are in the control/learning design methodology and the proof techniques. In summary, we think that there is a need for more rigorous theoretical justification attempted in this paper. Furthermore, to the best of our knowledge, the literature on ILC-MPC schemes do not consider state constraints, do not treat robust feasibility issues in the MPC tracking problem, rigorous justification of reference tracking proofs for the MPC is not present in the literature and stability proofs for the combination of the ILC and MPC schemes are not established in a systematic manner.

The main contribution of this work is to present a rigorous proof of an ILC-MPC scheme using existing Lyapunov function based stability analysis established in Limon et al. [2010] and extremum seeking algorithms in Khong et al. [2013b], to justify the ILC-MPC method in Benosman et al. [2014], where an ES-based modular approach to design ILC-MPC schemes for a class of constrained linear systems is proposed.

The rest of the paper is organized as follows. Section 2 contains some useful notations and definitions. The MPC control problem formulation is presented in Section 3. Section 4 is dedicated to a rigorous analysis of the proposed ES-based ILC-MPC. Finally, simulation results and concluding comments are presented in Section 5 and Section 6, respectively.

2. NOTATION AND BASIC DEFINITIONS

Throughout this paper, $\mathbb{R}$ denotes the set of real numbers and $\mathbb{Z}$ denotes the set of integers. State constraints and input constraints are represented by $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{U} \subset \mathbb{R}^m$, respectively. $\mathbb{B}$ refers to a closed unit ball in $\mathbb{R}^n$. The optimization horizon for MPC is denoted by $N \in \mathbb{Z}_{\geq 1}$. The feasible region for the MPC optimization problem is denoted by $\mathcal{X}_N$. A continuous function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ with $\alpha(0) = 0$ belongs to class $\mathcal{K}$ if it is increasing and bounded. A function β belongs to class $\mathcal{K}_\infty$ if it belongs to class $\mathcal{K}$ and is unbounded. A function $\beta(s,t) \in \mathcal{KL}$ if $\beta(\cdot, t) \in \mathcal{K}$ and $\lim_{t\to\infty} \beta(s,t) = 0$. Given two sets A and B, such that $A \subset \mathbb{R}^n$, $B \subset \mathbb{R}^n$, the Minkowski sum is defined as $A \oplus B := \{a + b | a \in A, b \in B\}$. The Pontryagin set difference is defined as $A \ominus B := \{x | x \oplus B \in A\}$. Given a matrix $M \in \mathbb{R}^{m\times n}$, the set $MA \subset \mathbb{R}^m$, is defined as $MA \triangleq \{Ma \ : \ a \in A\}$. A positive definite matrix is denoted by $P > 0$. The standard Euclidean norm is represented as $|x|$ for $x \in \mathbb{R}^n$, $|x|_P := \sqrt{x^T P x}$ for a positive definite matrix P, $|x|_{\mathcal{A}} := \inf_{y\in\mathcal{A}} |x - y|$ for a closed set $\mathcal{A} \subset \mathbb{R}^n$ and $\|A\|$ represents an appropriate matrix norm where A is a matrix. $\mathbb{B}$ represents the closed unit ball in the Euclidean space. Also, a matrix $M \in \mathbb{R}^{n\times n}$ is said to be Schur iff all its eigenvalues are inside the unitary disk.

3. PROBLEM FORMULATION

We consider linear systems of the form

$$x(k+1) = (A + \Delta A)x(k) + (B + \Delta B)u(k), \quad (1)$$

$$y(k) = Cx(k) + Du(k), \quad (2)$$

where ΔA and ΔB represent the uncertainty in the system model. We will assume that the uncertainties are bounded as follows:

Assumption 1. The uncertainties $\|\Delta A\| \leq \ell_A$ and $\|\Delta B\| \leq \ell_B$ for some $\ell_A, \ell_B > 0$.

Next, we impose some assumptions on the reference signal r.

Assumption 2. The reference signal $r : [0,T] \to \mathbb{R}$ is a piecewise constant trajectory for some $T > 0$.

Due to the iterative control design methodology, the initial condition x_0 for the system is fixed over multiple trials and at the end of each trial the state is reset to the initial condition. The goal is design the sequence of control inputs $\{u(k)\}_{k=0}^{T-1}$ using MPC to track the reference trajectory r while satisfying the state and input constraints, and the update laws for parameter estimation of the uncertainties $\Delta A, \Delta B$ after each trial or iteration. We also implicitly assume that the reference signal r is slowly varying and the time T is sufficiently large to allow learning from previous trials. Next, we will explain in detail the optimization problem associated with the MPC based controller. The results stated here are from Limon et al. [2010]. We exploit the analysis results in Limon et al. [2010] to establish that the closed-loop system has an ISS property with respect to the parameter estimation error. We first observe that since the reference trajectory r is a piecewise constant trajectory, the problem of tracking the signal r is simplified to the problem of tracking multiple constant and feasible set points during successive time intervals in $[0,T]$ in the presence of uncertainties.

Since the value of ΔA and ΔB are not known a priori, the MPC uses a model of the plant based on the current estimate $\hat{\Delta}A$ and $\hat{\Delta}B$.

We will now formulate the MPC problem with a given estimate of the uncertainty for a particular iteration of the learning process. We will rewrite the system dynamics as

$$x(k+1) = f(x,u) + g(x,u,\Delta) = F(x,u,\Delta), \quad (3)$$

where $f(x,u) = Ax + Bu$ and $g(x,u,\Delta) = \Delta Ax + \Delta Bu$.

Assumption 3. The state constraint set $\mathcal{X} \subset \mathbb{R}^n$ and control constraint set $\mathcal{U} \subset \mathbb{R}^m$ are compact, convex polyhedral sets.

The MPC model is generated using an estimate $\hat{\Delta}A$, $\hat{\Delta}B$ and is expressed as

$$x(k+1) = f(x,u) + g(x,u,\hat{\Delta}) = F(x,u,\hat{\Delta}). \quad (4)$$

We can now rewrite the actual model as

$$x(k+1) = f(x,u) + g(x,u,\hat{\Delta}) + (\Delta A - \hat{\Delta}A)x + (\Delta B - \hat{\Delta}B)u. (5)$$

This system can now be compared to the model in Limon et al. [2010]. So we have

$$x(k+1) = F(x(k), u(k), \hat{\Delta}) + w(k), \quad (6)$$

where

$$w(k) = (\Delta A - \hat{\Delta}A)x(k) + (\Delta B - \hat{\Delta}B)u(k), \quad (7)$$

and $x(k) \in \mathcal{X}, u(k) \in \mathcal{U}$. The following assumption will be justified in the next section.

Assumption 4. The estimates of the uncertain parameters are bounded with $\|\hat{\Delta}A\| \leq \ell_A$ and $\|\hat{\Delta}B\| \leq \ell_B$ for all iterations of the extremum seeking algorithm.

We now impose certain conditions on the disturbance $w(k)$ and system matrices in accordance with [Limon et al., 2010, Assumption 1].

Assumption 5. The pair $(A + \hat{\Delta}A, B + \hat{\Delta}B)$ is controllable for every realization of $\hat{\Delta}A$ and $\hat{\Delta}B$.

We will denote the actual model using (x,u) and the MPC model through $(\bar{x}, \bar{u})$. Hence we have

$$x(k+1) = F(x, u, \hat{\Delta}) + w,$$
$$\bar{x}(k+1) = F(\bar{x}, \bar{u}, \hat{\Delta}).$$

between the states of the true model and MPC model by $e(k) = x(k) - \bar{x}(k)$. We want the error to be bounded during tracking. The error dynamics is then given by

$$e(k+1) = (A + \hat{\Delta}A + (B + \hat{\Delta}B)K)e(k) + w(k), \quad (8)$$

where $u = \bar{u} + Ke$ and the matrix K is such that $A_K := (A + \hat{\Delta}A + (B + \hat{\Delta}B)K)$ is Schur.

We first recall the definition of a robust positive invariant set (RPI), e.g., Limon et al. [2010].

Definition 1. A set Φ_K is called an RPI set for the uncertain dynamics (8), if $A_K \Phi_k \oplus \mathcal{W} \subseteq \Phi_K$.

So, we let Φ_K be an RPI set associated with the error dynamics (8), i.e., $A_K \Phi_K \oplus \mathcal{W} \subseteq \Phi_K$.

Now we follow Limon et al. [2010] and tighten the constraints for the MPC model so that we achieve robust constraint satisfaction for the actual model with uncertainties. Let $\mathcal{X}_1 = X \ominus \Phi_K$ and $\mathcal{U}_1 = \mathcal{U} \ominus K\Phi_K$. The following result is from Alvarado et al. [2007b].

Proposition 1. Let Φ_K be RPI for the error dynamics. If $e(0) \in \Phi_K$, then $x(k) \in \bar{x}(k) \oplus \Phi_K$ for all $k \geq 0$ and $w(k) \in \mathcal{W}$. If in addition, $\bar{x}(k) \in \mathcal{X}_1$ and $\bar{u}(k) \in \mathcal{U}_1$ then with the control law $u = \bar{u} + Ke$, $x(k) \in \mathcal{X}$ and $u(k) \in \mathcal{U}$ for all $k \geq 0$.

As in Alvarado et al. [2007b] and Limon et al. [2010], we will characterize the set of nominal steady states and inputs so that we can relate them later to the tracking problem. Let $z_s = (\bar{x}_s, \bar{u}_s)$ be the steady state for the MPC model. Then,

$$\begin{bmatrix} A + \hat{\Delta}A - I & B + \hat{\Delta}B \\ C & D \end{bmatrix} \begin{bmatrix} \bar{x}_s \\ \bar{u}_s \end{bmatrix} = \begin{bmatrix} 0 \\ \bar{y}_s \end{bmatrix}. \quad (9)$$

From the controllability assumption on the system matrices, the admissible steady states can be characterized by a single parameter $\bar{\theta}$ as

$$\bar{z}_s = M_\theta \bar{\theta}, \quad (10)$$
$$\bar{y}_s = N_\theta \bar{\theta}, \quad (11)$$

for some $\bar{\theta}$ and matrices M_θ and $N_\theta = [C \quad D]M_\theta$. We let $\mathcal{X}_s$, $\mathcal{U}_s$ denote the set of admissible steady states that are contained in $\mathcal{X}_1, \mathcal{U}_1$ and satisfy (9). $\mathcal{Y}_s$ denotes the set of admissible output steady states. Now we will define an invariant set for tracking which will be utilized as a terminal constraint for the optimization problem.

Definition 2. [Limon et al., 2010, Definition 2] An invariant set for tracking for the MPC model is the set of initial conditions, steady states and inputs (characterized by $\bar{\theta}$) that can be stabilized by the control law $\bar{u} = \bar{K}\bar{x} + L\bar{\theta}$ with $L := [-\bar{K} \quad I]M_\theta$ while $(\bar{x}(k), \bar{u}(k)) \in \mathcal{X}_1 \times \mathcal{U}_1$ for all $k \geq 0$.

We choose the matrix $\bar{K}$ such that $A_{\bar{K}} := (A + \hat{\Delta}A + (B + \hat{\Delta}B)\bar{K})$ is Schur. We refer the reader to Alvarado et al. [2007b] and Limon et al. [2010] for more details on computing the invariant set for tracking. We will refer to the invariant set for tracking as $\Omega_{\bar{K}}$. We say a point $(\bar{x}(0), \bar{\theta}) \in \Omega_{\bar{K}}$ if with the control law $u = \bar{K}(\bar{x} - \bar{x}_s) + \bar{u}_s = \bar{K}\bar{x} + L\bar{\theta}$, the solutions of the MPC model from $\bar{x}(0)$ satisfy $\bar{x}(k) \in \mathrm{Proj}_x(\Omega_{\bar{K}})$ for all $k \geq 0$. As stated in Limon et al. [2010] the set can be taken to be a polyhedral.

3.1 MPC problem formulation

We are ready now to define the MPC optimization problem that will be solved at every instant to determine the control law for the actual plant dynamics. For a given target set-point y_t and initial condition x, the optimization problem $\mathcal{P}_N(x, y_t)$ is defined as,

$$\begin{aligned} &\min_{\bar{x}(0), \bar{\theta}, \bar{\mathbf{u}}} V_N(x, y_t, \bar{x}(0), \bar{\theta}, \bar{\mathbf{u}}) \\ &\text{s.t } \bar{x}(0) \in x \oplus (-\Phi_K) \\ &\bar{x}(k+1) = (A + \hat{\Delta}A)\bar{x}(k) + (B + \hat{\Delta}B)\bar{u}(k) \\ &\bar{x}_s = M_\theta \bar{\theta} \\ &\bar{y}_s = N_\theta \bar{\theta} \\ &(\bar{x}(k), \bar{u}(k)) \in \mathcal{X}_1 \times \mathcal{U}_1, k \in \mathbb{Z}_{\leq N-1} \\ &(\bar{x}(N), \bar{\theta}) \in \Omega_{\bar{K}}, \end{aligned}$$

where the cost function is defined as follows

$$V_N(x, y_t, \bar{x}(0), \bar{\theta}, \bar{\mathbf{u}}) = \sum_{k=0}^{N-1} |\bar{x}(k) - \bar{x}_s|^2_{\tilde{Q}}$$
$$+ |\bar{u}(k) - \bar{u}_s|^2_R + |\bar{x}(N) - \bar{x}_s|^2_P + |\bar{y}_s - y_t|^2_T. \quad (12)$$

Such cost function is frequently used in MPC literature for tracking except for the additional term in the end which penalizes the difference between the artificial stead state and the actual target value. We refer the reader to Alvarado et al. [2007a], Alvarado et al. [2007b] and Limon et al. [2010] for more details.

Assumption 6. The following conditions are satisfied by the optimization problem

(1) The matrices $\tilde{Q} > 0, R > 0, T > 0$.
(2) $(A + \hat{\Delta}A + (B + \hat{\Delta}B)K)$ is Schur matrix, Φ_K is a RPI set for the error dynamics, and $\mathcal{X}_1, \mathcal{U}_1$ are non-empty.
(3) The matrix $\bar{K}$ is such that $A + \hat{\Delta}A + (B + \hat{\Delta}B)\bar{K}$ is Schur and $P > 0$ satisfies:
$$P - (A + \hat{\Delta}A + (B + \hat{\Delta}B)\bar{K})^T P$$
$$(A + \hat{\Delta}A + (B + \hat{\Delta}B)\bar{K}) = \tilde{Q} + \bar{K}^T R \bar{K}.$$
(4) The set $\Omega_{\bar{K}}$ is an invariant set for tracking subject to the tightened constraints $\mathcal{X}_1, \mathcal{U}_1$.

As noted in Limon et al. [2010], the feasible set $\mathcal{X}_N$ does not vary with the set points y_t and the optimization problem is a Quadratic programming (QP) problem. The optimal values are given by $\bar{x}_s^*, \bar{u}^*(0), \bar{x}^*$. The MPC control law writes then as: $u = \kappa_N(x) = K(x - \bar{x}^*) + \bar{u}^*(0)$. The MPC law κ_N implicitly depends on the current estimate of the uncertainty $\hat{\Delta}$. Also it follows from the results in Bemporad et al. [2002] that the control law for the MPC problem is continuous [1].

4. DIRECT EXTREMUM SEEKING-BASED ITERATIVE LEARNING MPC

4.1 DIRECT-based iterative learning MPC

In this section we will explain the assumptions regarding the learning cost function [2] used for identifying the true parameters of the uncertain system via nonlinear programming based method called DIRECT which uses a sampling

[1] The authors would like to thank Dr. S. Di Cairano for pointing out to us the paper Bemporad et al. [2002].
[2] Not to be confused with the MPC cost function.

based methodology to achieve extremum seeking. Due to space constraints we refer the reader to Jones et al. [1993] for details on the DIRECT algorithm. Let Δ be a vector that contains the entries in ΔA and ΔB. Similarly the estimate will be denoted by $\hat{\Delta}$. Then $\Delta, \hat{\Delta} \in \mathbb{R}^{n(n+m)}$.

Since we do not impose the presence of attractors for the closed-loop system as in Popovic et al. [2006] or Khong et al. [2013a], the cost function that we utilize $Q : \mathbb{R}^{n(n+m)} \to \mathbb{R}_{\geq 0}$ depends on x_0. For iterative learning methods, the same initial condition x_0 is used to learn the uncertain parameters and hence we refer to $Q(x_0, \hat{\Delta})$ as only $Q(\hat{\Delta})$ since x_0 is fixed.

Assumption 7. The learning cost function $Q : \mathbb{R}^{n(n+m)} \to \mathbb{R}_{\geq 0}$ is

(1) Lipshitz in the compact set of uncertain parameters
(2) The true parameter Δ is such that $Q(\Delta) < Q(\hat{\Delta})$ for all $\hat{\Delta} \neq \Delta$.

One example of a learning cost functions is identification-type cost function, where the error between outputs measurements from the system are compared to the MPC model outputs. Another example of a learning cost function, can be a performance-type cost function, where a measured output of the system is directly compared to a desired reference trajectory. Later in Section 5, we will test the former learning cost function.

We then use the DIRECT optimization algorithm introduced in Jones et al. [1993] for finding the global minimum of a Lipschitz function without knowledge of the Lipschitz constant. The algorithm is implemented in MATLAB using Finkel [2003]. We will utilize a modified termination criterion introduced in Khong et al. [2013a] for the DIRECT algorithm to make it more suitable for extremum seeking applications. As we will mention in the next section, the DIRECT algorithm has nice convergence properties which will be used to establish our main results. The combined algorithm for the ILC-MPC scheme using the DIRECT algorithm is stated below. We note that the index 't' for the estimates refers to the trail number and the index 'k' is used to keep track of the solutions for a particular trial. The estimate is also fixed for the entire duration of a trial.

Algorithm 1 Extremum Seeking ILC-MPC

Require: $r : [0, T] \to \mathbb{R}$, x_0, Q, $\hat{\Delta}_0$
Set $t = 0$
Initial trial point of DIRECT: $\hat{\Delta}_0$
Form MPC model with $\hat{\Delta}_0$
while $t \in \mathbb{Z}_{\geq 0}$ **do**
  Set $x(0) = x_0$
  $k = 0$
  for $k \in [0, T]$ **do**
    Compute MPC law at $x(k)$, update $x(k+1)$
    $k = k + 1$
  Compute $Q(\hat{\Delta}_t)$ from $\{x(k), u(k)\}_{k=0}^{T}$
  Find $\hat{\Delta}_{t+1}$ using DIRECT with $\{\hat{\Delta}_0, ...\hat{\Delta}_t\}$
  Update MPC model and $\mathcal{P}_N$ using $\hat{\Delta}_{t+1}$
  $t = t + 1$

4.2 Main results: Proof of the MPC ISS and the learning convergence

We will now present the main results of this paper, namely the stability analysis of the proposed ESILC-MPC algorithm 1, using the existing results for MPC tracking and DIRECT algorithm established in Limon et al. [2010] and Khong et al. [2013b], respectively.

First, we define the value function

$$V_N^*(x, y_t) = \min_{\bar{x}(0), \theta, \bar{\mathbf{u}}} V_N(x, y_t, \bar{x}(0), \theta, \bar{\mathbf{u}})$$

for a fixed target y_t. Also, we let $\tilde{\theta} := \arg\min_{\bar{\theta}} |N_\theta \bar{\theta} - y_t|$, $(\tilde{x}_s, \tilde{u}_s) = M_\theta \tilde{\theta}$ and $\tilde{y}_s = C\tilde{x}_s + D\tilde{u}_s$. If the target steady state y_t is not admissible, the MPC tracking scheme drives the output to converge to the point $\tilde{y}_s$ which is a steady state output that is admissible and also minimizes the error with the target steady state. The proof of the following result follows from [Limon et al., 2010, Theorem 1] and classically uses $V_N^*(x, y_t)$ as a Lyapunov function for the closed-loop system.

Proposition 2. Let y_t be given. For all $x(0) \in \mathcal{X}_N$, the MPC problem is recursively feasible. The state $x(k)$ converges to $\tilde{x}_s \oplus \Phi_K$ and the output $y(k)$ converges to $\tilde{y}_s \oplus (C + DK)\Phi_K$.

The next result states the convergence properties of the modified DIRECT algorithm, which we will used in establishing the main result. This result is stated as [Khong et al., 2013b, Assumption 7] and it follows from the analysis of the modified DIRECT algorithm in Khong et al. [2013a].

Proposition 3. For any sequence of updates $\hat{\Delta}_t$, $t = 1, 2, ...$ from the modified DIRECT algorithm and $\varepsilon > 0$, there exists a $\widetilde{N} > 0$ such that $|\Delta - \hat{\Delta}_t| \leq \varepsilon$ for $t \geq \widetilde{N}$.

Remark 1. Note that the results in Khong et al. [2013a] also include a robustness aspect of the DIRECT algorithm. This can be used to account for measurement noises and computational error associated with the learning cost Q.

We now state the main result of the section that combines the ISS MPC formulation and the extremum seeking algorithm.

Theorem 1. Under Assumptions 1-7, given an initial condition x_0, let the output target target y_t be such that y_t is constant over $[0, T^*]$ for some $T^* \in \mathbb{Z}_{>0}$ sufficiently large. Then, for every $\varepsilon > 0$, there exists $N_1, N_2 \in \mathbb{Z}_{>0}$ such that $|y(k) - \tilde{y}_s| <= \varepsilon$ for $k \in [N_1, T^*]$ after N_2 iterations of the ILC-MPC scheme.

Proof. It can observed that the size of Φ_K grows with the size of $\mathcal{W}$ and $\Phi_K = \{0\}$ for the case without disturbances. Also, since the worst case disturbance depends directly on the estimation error, without loss of generality we have that $\Phi_K \subseteq \gamma(|\Delta - \hat{\Delta}|)\mathbb{B}$ for some $\gamma \in \mathcal{K}$. It follows from Proposition 2 that $\lim_{k\to\infty} |x(k)|_{\tilde{x}_s \oplus \Phi_K} = 0$. Then,

$$\begin{aligned}\lim_{k\to\infty} |x(k) - \tilde{x}_s| &\leq \max_{x \in \Phi_K} |x| \\ &\leq \gamma(|\Delta - \hat{\Delta}|).\end{aligned}$$

We observe that the above set of equations state that the closed-loop system with the MPC controller has the asymptotic gain property and it is upper bounded by the size of the parameter estimation error. Note that the estimate $\hat{\Delta}$ is constant for a particular iteration of the process. Also, for the case of no uncertainties we have $0-$stability (Lyapunov stability for the case of zero uncertainty). This can be proven by using the cost function $V_N^*(x, y_t)$ as the Lyapunov function, such that $V_N^*(x(k+1), y_t) \leq V_N^*(x(k), y_t)$ and $\lambda_{\min}(\tilde{Q})|x - \tilde{x}_s|^2 \leq V_N^*(x, y_t) \leq \lambda_{\max}(P)|x - \tilde{x}_s|^2$, see Limón et al. [2008]. Furthermore, here the stability and asymptotic gain property can be interpreted with respect to the compact set $\mathcal{A} := \{\tilde{x}_s\}$. Since the MPC control law is continuous, the closed-loop system for a particular iteration of the ILC-MPC scheme is also continuous with respect to the state. Then, from [Cai and Teel, 2009, Theorem 3.1] we can conclude that the closed-loop system is ISS with respect to the parameter estimation error and hence satisfies,

$$|x(k) - \tilde{x}_s| \leq \beta(|x(0) - \tilde{x}_s|, k) + \hat{\gamma}(|\Delta - \hat{\Delta}|),$$

where $\beta \in \mathcal{KL}$ and $\hat{\gamma} \in \mathcal{K}$. Now, let $\varepsilon_1 > 0$ be small enough such that $\hat{\gamma}(\varepsilon_1) \leq \varepsilon/2$. From Proposition 3, it follows that there exists a $\tilde{N}_2 > 0$ such that $|\Delta - \hat{\Delta}_t| \leq \varepsilon_1$ for $t \geq \tilde{N}_2$, where t is the iteration number of the ILC-MPC scheme. Also, there exists $\tilde{N}_1 > 0$ such that $\beta(|x(0) - \tilde{x}_s|, k) \leq \varepsilon/2$ for $k \geq \tilde{N}_1$. Then, we have that for $k \in [\tilde{N}_1, T^*]$ and for $t \geq \tilde{N}_2$,

$$|x(k) - \tilde{x}_s| \leq \varepsilon/2 + \varepsilon/2 \leq \varepsilon.$$

A similar analysis for the output establishes that for any $\varepsilon > 0$, there exists $N_1, N_2 > 0$ such that such that for $k \in [N_1, T^*]$ and for $t \geq N_2$

$$|y(k) - \tilde{y}_s| \leq \varepsilon.$$

Remark 2. We note that a result similar to Theorem 1 can be easily applied to the case of piecewise constant signals $r : [0, T] \to \mathbb{R}$ provided the signal r varies sufficiently slowly to better facilitate tracking using the ILC-MPC algorithm.

5. NUMERICAL EXAMPLE

We consider the following simple system dynamics

$$\begin{aligned}
x_1(k+1) &= x_1(k) + (-1 + 3/(k_1 + 1))x_2(k) + u(k) \\
x_2(k+1) &= -k_2 x_2(k) + u(k) \\
y(k) &= x_1(k).
\end{aligned}$$

Together with the state constraints $|x_i| \leq 50$ for $i \in \{1, 2\}$ and the input constraint $|u| \leq 10$. We assume that the nominal values for the parameters are $k_1 = -0.3$ and $k_2 = 1$. Furthermore, we assume that the uncertainties are bounded by $-0.9 \leq k_1 \leq 0$ and $0 \leq k_2 \leq 4$. The robust invariant sets for tracking are determined using the algorithms in Borrelli et al. [2012]. Next, we define an identification-type cost function: For a given initial condition x_0, and a piecewise constant reference trajectory of length T sufficiently large, the learning cost function Q is defined as

$$Q(\hat{\Delta}) := \alpha \sum_{k=1}^{T} |x(k) - \tilde{x}(k)|^2,$$

where $\alpha > 0$ is a scaling factor, $x(k)$ is the trajectory of the actual system (assuming the availability of the full states measurements), and $\tilde{x}(k)$ is the trajectory of the MPC model with the same inputs. In this example we can observe from the plot of Q in Figure 1 that the unique minimum occurs at the true value of the parameters. Figure 3 shows the identification of the uncertainties obtained by the DIRECT algorithm. Finally, we see in Figure 2, that the ILC-MPC scheme successfully achieves tracking of the piecewise constant reference trajectory.

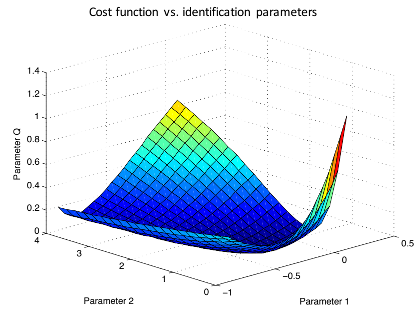

Fig. 1. ESILC-MPC scheme - Cost function $Q(\Delta)$ as function of the uncertain parameters

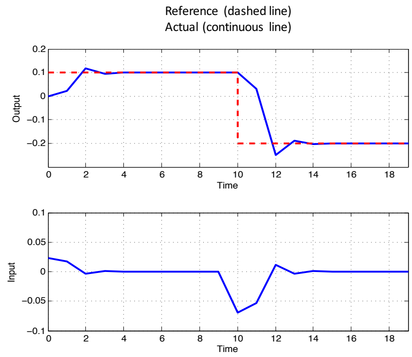

Fig. 2. ESILC-MPC scheme - output tracking performance

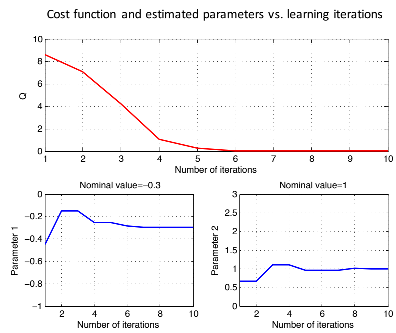

Fig. 3. ESILC-MPC scheme - Learning const function and uncertainties estimation over learning iterations

6. CONCLUSION

In this paper, we have reported some results about extremum seeking (ES)-based ILC-MPC algorithms. We have argued that it is possible to merge together a model-based linear MPC algorithm with a model-free ES algorithm to iteratively learn structural model uncertainties and thus improve the overall performance of the MPC controller. We have presented the analysis of this modular design technique for ESILC-MPC where we addressed feasibility, learning and tracking performance. Future work can include extending this method to a wider class of nonlinear systems, tracking a more richer class of signals, employing different non-smooth optimization techniques for the extremum seeking algorithm, and comparing this type of ESILC-MPC to other adaptive MPC methods in terms of performances and convergence speed, etc.

REFERENCES

Hyo-Sung Ahn, YangQuan Chen, and Kevin L Moore. Iterative learning control: brief survey and categorization. *IEEE Transactions on Systems Man and Cybernetics*, 37 (6):1099, 2007.

I Alvarado, D Limon, T Alamo, and EF Camacho. Output feedback robust tube based MPC for tracking of piece–wise constant references. In *Proceedings of the 46th IEEE Conference on Decision and Control*, pages 2175–2180. IEEE, 2007a.

I Alvarado, D Limon, T Alamo, M Fiacchini, and EF Camacho. Robust tube based MPC for tracking of piece–wise constant references. In *Decision and Control, 2007 46th IEEE Conference on*, pages 1820–1825. IEEE, 2007b.

Suguru Arimoto. Robustness of learning control for robot manipulators. In *Proceedings of the IEEE International Conference on Robotics and Automation*, pages 1528–1533. IEEE, 1990.

K. B. Ariyur and M. Krstic. Multivariable extremum seeking feedback: Analysis and design. In *Proc. of the Mathematical Theory of Networks and Systems*, South Bend, IN, August 2002.

Kartik B Ariyur and Miroslav Krstic. *Real–time optimization by extremum–seeking control*. John Wiley & Sons, 2003.

Anil Aswani, Patrick Bouffard, and Claire Tomlin. Extensions of learning–based model predictive control for real–time application to a quadrotor helicopter. In *American Control Conference (ACC), 2012*, pages 4661–4666. IEEE, 2012a.

Anil Aswani, Humberto Gonzalez, S Shankar Sastry, and Claire Tomlin. Provably safe and robust learning–based model predictive control. *Automatica*, 49(5):1216–1226, 2013.

Anil Aswani, Neal Master, Jay Taneja, David Culler, and Claire Tomlin. Reducing transient and steady state electricity consumption in hvac using learning–based model–predictive control. *Proceedings of the IEEE*, 100 (1):240–253, 2012b.

G. Atinc and M. Benosman. Nonlinear learning–based adaptive control for electromagnetic actuators with proof of stability. In *IEEE, Conference on Decision and Control*, pages 1277–1282,, Florence, 2013.

Alberto Bemporad, Manfred Morari, Vivek Dua, and Efstratios N Pistikopoulos. The explicit linear quadratic regulator for constrained systems. *Automatica*, 38(1): 3–20, 2002.

M. Benosman. Learning–based adaptive control for nonlinear systems. In *IEEE European Control Conference*, pages 920–925, Strasbourg, FR, 2014.

M. Benosman and G. Atinc. Nonlinear learning–based adaptive control for electromagnetic actuators. In *IEEE, European Control Conference*, pages 2904–2909, Zurich, 2013.

M. Benosman, S. Di Cairano, and A. Weiss. Extremum seeking–based iterative learning linear MPC. In *IEEE Multi–conference on Systems and Control*, pages 1849–1854, 2014.

F Borrelli, A Bemporad, and M Morari. Predictive control. *Draft Available at: http://www. mpc. berkeley. edu/mpc–course–material*, 2012.

Chaohong Cai and Andrew R Teel. Characterizations of input–to–state stability for hybrid systems. *Systems & Control Letters*, 58(1):47–53, 2009.

José R Cueli and Carlos Bordons. Iterative nonlinear model predictive control. stability, robustness and applications. *Control Engineering Practice*, 16(9):1023–1034, 2008.

Daniel E Finkel. DIRECT optimization algorithm user guide. *Center for Research in Scientific Computation, North Carolina State University*, 2, 2003.

M. Guay, S. Dhaliwal, and D. Dochain. A time-varying extremum-seeking control approach. In *IEEE, American Control Conference*, pages 2643–2648, 2013.

N. Hudon, M. Guay, M. Perrier, and D. Dochain. Adaptive extremum–seeking control of convection-reaction distributed reactor with limited actuation. *Computers & Chemical Engineering*, 32(12):2994–3001, 2008.

Donald R Jones, Cary D Perttunen, and Bruce E Stuckman. Lipschitzian optimization without the Lipschitz constant. *Journal of Optimization Theory and Applications*, 79(1):157–181, 1993.

Sei Zhen Khong, Dragan Nešić, Chris Manzie, and Ying Tan. Multidimensional global extremum seeking via the DIRECT optimisation algorithm. *Automatica*, 49(7): 1970–1978, 2013a.

Sei Zhen Khong, Dragan Nešić, Ying Tan, and Chris Manzie. Unified frameworks for sampled–data extremum seeking control: Global optimisation and multi–unit systems. *Automatica*, 49(9):2720–2733, 2013b.

M. Krstic. Performance improvement and limitations in extremum seeking. *Systems & Control Letters*, 39:313–326, 2000.

Kwang S Lee, In-Shik Chin, Hyuk J Lee, and Jay H Lee. Model predictive control technique combined with iterative learning for batch processes. *AIChE Journal*, 45(10):2175–2187, 1999.

D Limon, I Alvarado, T Alamo, and EF Camacho. Robust tube–based MPC for tracking of constrained linear systems with additive disturbances. *Journal of Process Control*, 20(3):248–260, 2010.

Daniel Limón, Ignacio Alvarado, T Alamo, and Eduardo F Camacho. MPC for tracking piecewise constant references for constrained linear systems. *Automatica*, 44(9): 2382–2387, 2008.

D. Q. Mayne, J. B. Rawlings, C. V. Rao, and P. O. M. Scokaert. Constrained model predictive control: Stability and optimality. *Automatica*, 36:789–814, 2000.

D. Nesic. Extremum seeking control: Convergence analysis. *European Journal of Control*, 15(34):331–347, 2009. ISSN 0947–3580.

Dobrivoje Popovic, Mrdjan Jankovic, Steve Magner, and Andrew R Teel. Extremum seeking methods for optimization of variable cam timing engine operation. *IEEE Transactions on Control Systems Technology*, 14 (3):398–407, 2006.

Jia Shi, Furong Gao, and Tie-Jun Wu. Single–cycle and multi–cycle generalized 2D model predictive iterative learning control (2D–GPILC) schemes for batch processes. *Journal of Process Control*, 17(9):715–727, 2007.

Y. Tan, D. Nesic, and I. Mareels. On non–local stability properties of extremum seeking control. *Automatica*, (42):889–903, 2006.

Youqing Wang, Furong Gao, and Francis J Doyle III. Survey on iterative learning control, repetitive control, and run–to–run control. *Journal of Process Control*, 19 (10):1589–1600, 2009.

Youqing Wang, Donghua Zhou, and Furong Gao. Iterative learning model predictive control for multi–phase batch processes. *Journal of Process Control*, 18(6):543–557, 2008.

C. Zhang and R. Ordóñez. *Extremum–Seeking Control and Applications*. Springer–Verlag, 2012.