MITSUBISHI ELECTRIC RESEARCH LABORATORIES
http://www.merl.com

Optimal Delta-Sigma Modulation Based Noise Shaping for Truly Aliasing-Free digital PWM

Ma, R.; Tanovic, O.; Teo, K.H.

TR2017-143 October 2017

Abstract

In recent years, carrier based pulse-width modulation (PWM) has been extensively used for reducing input signal resolution in all-digital transmitters, and is known to provide better coding efficiency than other power encoding methods, e.g. delta-sigma modulation (DSM). Unfortunately, high levels of in-band distortion introduced by fully digitally implemented PWM, limit its usage in transceivers realized for software defined radio. In this paper, we exploit time-domain characterization of digital PWM in-band distortion, and propose a multilevel power encoding scheme based on DSM-PWM hybrid architecture with carefully selected design parameters, which enable optimal in-band distortion cancellation with low hardware complexity. Experiments with an arbitrary waveform generator are used to demonstrate effectiveness of the proposed novel power encoding scheme, and are compared with a standard multilevel PWM. The novel power encoding scheme offers -20dB improvement in SNR, while maintaining similar coding efficiency of roughly 73%.

European Microwave Conference

Optimal Delta-Sigma Modulation Based Noise Shaping for Truly Aliasing-Free digital PWM

Omer Tanovic*†, Rui Ma* and Koon Hoo Teo*
* Mitsubishi Electric Research Laboratories
Cambridge, MA, 02139, USA, {rma, teo}@merl.com
†Laboratory for Information and Decision Systems, Massachusetts Institute of Technology
Cambridge, MA, 02139, USA, {otanovic}@mit.edu

***Abstract*—In recent years, carrier based pulse-width modulation (PWM) has been extensively used for reducing input signal resolution in all-digital transmitters, and is known to provide better coding efficiency than other power encoding methods, e.g. delta-sigma modulation (DSM). Unfortunately, high levels of in-band distortion introduced by fully digitally implemented PWM, limit its usage in transceivers realized for software defined radio. In this paper, we exploit time-domain characterization of digital PWM in-band distortion, and propose a multilevel power encoding scheme based on DSM-PWM hybrid architecture with carefully selected design parameters, which enable optimal in-band distortion cancellation with low hardware complexity. Experiments with an arbitrary waveform generator are used to demonstrate effectiveness of the proposed novel power encoding scheme, and are compared with a standard multilevel PWM. The novel power encoding scheme offers -20dB improvement in SNR, while maintaining similar coding efficiency of roughly 73%.**

I. Introduction

High performance standards of modern communication systems have led to the use of spectrally-efficient signal/modulation formats, which have high peak-to-average power ratio (PAPR) and thus result in low power efficiency with traditionally employed linear power amplifiers (PA). Due to their promising potential of achieving high power efficiency while maintaining good linearity, all-digital transmitters (ADT), which employ RF PAs in switched-mode operation (SMPA), have recently gained a lot of attention from researchers [1]- [4]. It is very changeling to drive SMPAs with high resolution RF input signals ($>$ 7 levels), and reduction of amplitude resolution of the SMPA input is needed in practice to adjust for switched-mode operation. This reduction of resolution is usually performed through a procedure called power encoding (see Fig. 1).

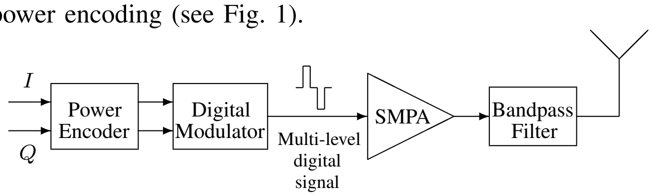

Fig. 1. An example of all-digital transmitter architecture.

Pulse-width modulation (PWM) is a very popular power encoding method, and has been traditionally implemented as analog (continuous-time) PWM. But forthcoming 5G communication standards envision transceivers implemented as software-defined radio [2], thus benefiting from available signal processing power in digital domain. This enables the use of PWM fully implemented in discrete-time (digital PWM), which can be described as a sampled version of its continuous-time counterpart, i.e. analog PWM. However, when the infinite bandwidth analog PWM output is sampled, the resulting signal (i.e. the output of digital PWM) will undergo substantial aliasing in spectral domain. This introduces a considerable amount of distortion in the band of the original input signal (called in-band distortion), which is the main limitation of practical usage of digital PWM. In-band distortion, and the spectral aliasing effects causing it, have been studied in the past [5]- [6], and various solutions have been proposed for its mitigation, with majority of those relying on noise shaping by delta-sigma modulation (DSM).

Recently, in [7], a PWM noise shaping through an additional DSM type of system has been proposed. Similarly, in [8], reduction of aliasing effects in radio-frequency (RF) PWM by employing feedback loop with DSM, has been reported. In the above papers, authors consider in-band distortion from time-domain perspective, but do not characterize the source of this distortion. In [9], authors propose to cancel distortion by realizing digital PWM in a way which is analogous to applying an anti-aliasing lowpass filter to the analog PWM output before it is sampled. Filtering of the analog PWM output is done by truncating its double Fourier series, causing no spectral overlap in the band of interest after sampling is performed. Now, due to Gibbs effect [10], the filtered output has amplitude ripples around the points of discontinuity, and hence the SMPA driving input is not a digital signal anymore. This introduces additional nonlinear distortion and decreases drain efficiency of the SMPA. In [11], DSM is used to quantize amplitude of a RF PWM input signal and limit minimal PWM duty-cycle, in order to prevent ”pulse swallowing”, which is a common issue observed in SMPAs [4]. In [12] DSM output signal of high resolution is outphased by *multidimensional power coding* scheme using many low resolution PWM blocks. Similarly, in [13], DSM output of high resolution is processed by a mixed-domain FIR filter employing many low level PWM blocks, having effects close to work in [12].

This work was done while Omer Tanovic was an intern at Mitsubishi Electric Research Laboratories (MERL).

In all of the above mentioned papers, authors propose various schemes for cancelling in-band distortion caused by spectral aliasing in digital PWM, but none of the results utilize time-domain characterization of this distortion, which is crucial if DSM-based noise cancellation is to be used. It can be shown that spectral aliasing effects manifest in time-domain as additional (hidden) quantization of the input baseband signal, where PWM does not perceive the true input signal but a uniformly quantized version of it. In this paper, we exploit this specific time-domain relationship between the input and output signals of a digital PWM, and propose a DSM-PWM power encoding architecture with carefully selected design parameters, which allow for an optimal in-band distortion cancellation with lowest hardware complexity. Moreover, we show that in order to achieve this optimality, it is necessary to select design parameters so to satisfy specific analytical relations. The novel power encoding scheme achieves linearity of a delta-sigma modulator with high number of output levels using a SMPA driving signal of (relatively) low resolution (i.e. small number of PWM output levels). In addition to providing high linearity, this considerably reduces design requirements for the SMPA (which can be significant in case of GaN based architectures [14]). It also potentially offers much lower hardware complexity than the state of the art e.g. [12]- [13], which use high number of SMPAs, while maintaining similar level of performance (both in terms of linearity and coding efficiency).

II. Analytical model of PWM

In this paper we consider carrier-based (CB) double-edge (DE) amplitude-slicing multilevel (ML) pulse-width modulation system [4]. In order to avoid cumbersome abbreviations, in the following sections we refer to it simply as PWM (with a prefix of analog or digital depending on the time-domain in which it is implemented). Let $a(t)$ be the baseband input signal into analog PWM, whose amplitude is normalized to be in the interval $(0, 1)$, and let $x(t)$ be the output of PWM (see Fig. 2a). We assume that PWM has $(M + 1)$ output levels, and its sawtooth carrier $c(t)$ has fundamental frequency f_p, with $f_p = 1/T_p$. An example of output signal generation for a 3-level PWM (i.e. $M = 2$) is shown in Fig. 3. Using double Fourier series and the fact that $c(t)$ is periodic, PWM output signal $x(t)$ can be expressed (see e.g. [9] and [15]) as

$$x(t) = a(t) + \sum_{k=1}^{\infty} \frac{2}{\pi M k} \sin(\pi M k a(t)) \cos(2k\pi f_p t). \quad (1)$$

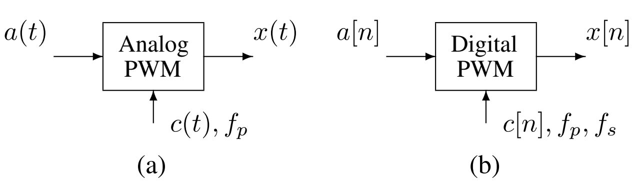

Fig. 2. Block diagram of (a) analog PWM and (b) digital PWM.

We can see from (1) that the analog PWM output signal $x(t)$ is a sum of a baseband component $a(t)$ and higher order

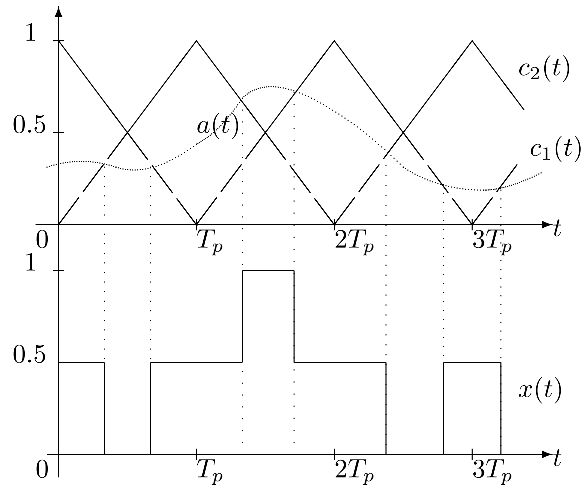

Fig. 3. PWM basic operation.

harmonics at multiples of the carrier frequency f_p, and is therefore of infinite bandwidth. In digital PWM, input signal is discrete in time, and the output is produced by comparing a discrete-time sawtooth carrier with the input. In other words, digital PWM can be seen as a sampled version of the above described analog PWM. Let f_s be the sampling frequency of digital PWM, and let $T_s = 1/f_s$. It is clear that the input and output signals of digital PWM satisfy $a[n] = a(nT_s)$ and $x[n] = x(nT_s)$, respectively (see Fig. 2b). The main parameter of interest, in analyzing digital PWM, is the oversampling (OSR) ratio f_s/f_p of PWM. Let us denote this OSR as N.

When signal $x(t)$ is sampled, regardless of how large the sampling frequency f_s is (or equivalently how large N is), spectral aliasing will occur due to the infinite bandwidth of $x(t)$. This aliasing is the main source of in-band distortion in digital PWM, and its main drawback in practical applications. One might ask the following question: is there a useful time-domain description of this in-band distortion caused by frequency-domain aliasing? If the answer is yes, then could it be used to improve in-band distortion of digital PWM? We give an answer to the first question below, and the second one is answered and discussed in the next section.

Without loss of generality we assume N to be an integer, since in practical applications that is indeed the case. Since $x[n] = x(nT_s)$, and using (1) together with $f_s = N f_p$, it can be shown that the digital PWM output signal $x[n]$ can be written as

$$x[n] = a_Q[n] + \sum_{k=1}^{\lfloor \frac{N}{2} \rfloor} \frac{2 \sin\left(\pi k M a_Q[n]\right)}{M N \sin\left(\frac{\pi k}{N}\right)} \cos\left(\frac{2\pi}{N} kn\right), \quad (2)$$

which represents a discrete-time equivalent of (1), (detailed derivation of (2) is out of scope of this article, and hence we present the final result only). Signal $a_Q[n]$ in (2) denotes uniformly quantized version of $a[n]$, with the number of quantization levels L of this uniform quantizer given by

$$L = \left\lfloor \frac{MN}{2} \right\rfloor, \tag{3}$$

where N is the oversampling ratio of PWM, and M is an integer such that $(M+1)$ is the number of output levels of PWM, as previously defined. Equation (2) suggests that digital PWM does not perceive the true input signal (i.e. $a[n]$) but a uniformly quantized version of it (i.e. $a_Q[n]$), with the resolution defined by (3). We call this effect a *hidden quantization*, to make distinction from the quantization (i.e. signal resolution reduction) which is achieved by the very operation of pulse-width modulation. Thus aliasing effects, which are caused by sampling of the infinite bandwidth analog PWM output, can be equivalently described in time-domain as this additional (hidden) quantization of the input signal.

It is important to emphasize that hidden quantization is an inherent property of all digital PWM schemes (as time-domain manifestation of spectral aliasing), and is due to the displacement of switching instants caused by sampling of a digitized pulse PWM output signal. Thus similar behavior can be expected of all other digital PWM schemes (single vs double edge, carrier vs threshold based, naturally vs uniformly sampled, etc).

III. Implications of Hidden Quantization: Truly Aliasing-Free PWM

Here we discuss implications of the result from previous section on power encoding schemes employing digital PWM.

Intuitively, it is clear that increasing the sampling rate and/or number of levels in PWM, the linearity performance increases, due to either less aliasing effects (higher sampling rate) or better amplitude resolution of the output signal (more levels of PWM). A common conclusion in literature is that in order to use digital PWM for power encoding in ADTs, the sampling frequency has to be orders of magnitude larger than the carrier frequency of PWM, in order to satisfy strict linearity constraints of digital communication standards [12]. Generally, this is true if signals taking arbitrary amplitude values in the interval $(0,1)$ are to be fed into digital PWM. As discussed in the previous section, the aliasing effects in frequency-domain (due to time-sampling), can be equivalently explained as hidden quantization in time-domain. This implies that in order to achieve truly aliasing-free (or distortion-free) digital PWM, its input signal has to be quantized before being fed into PWM, so that no additional in-band distortion is introduced by hidden quantization. It is necessary that this pre-quantization of the PWM input signal conforms to the relation given in (3): the input signal amplitudes have to correspond to output values of a uniform quantizer with the number of levels exactly matching that in (3).

However, pre-quantization of the true high resolution input signal would introduce its own distortion, with or without digital PWM. This would diminish possible gain achieved by using aliasing-free digital PWM, since the total distortion (quantization + PWM) would still be significant. But by

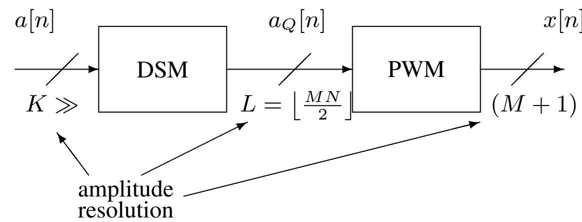

Fig. 4. Block diagram of the proposed DSM-PWM architecture.

applying a delta-sigma modulator on the true high resolution PWM input signal it is possible to shape noise to out-of-band frequencies and optimally control the level of in-band distortion, as long as the DSM and PWM parameters conform to relation (3) (see Fig. 4). In this architecture, the use of DSM is twofold: it shapes quantization noise to out-of-band frequency region, and, with carefully chosen design parameters, ensures that no aliasing distortion is generated in digital PWM. Relation (3) is crucial in achieving optimal in-band signal-to-noise ratio (SNR). Indeed, if (3) is not satisfied, additional in-band distortion will be generated by PWM, as confirmed by Matlab simulation results shown in Fig. 5. These plots depict baseband spectrum of the output of a DSM-PWM power encoder with $M = 2$ and $N = 6$, driven by the same high resolution input signal with four different DSM configurations: uniformly quantized with the number of quantization levels L equal to 100, 15, 6 and 3. Clearly $L = 6$ satisfies relation (3), and gives optimal in-band SNR.

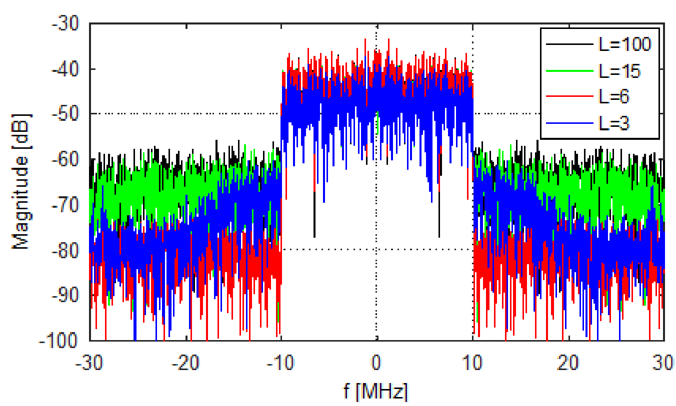

Fig. 5. Baseband spectrum of DSM-PWM output for various selection of DSM quantization levels.

IV. Experimental Performance Evaluation

In order to evaluate performance of the proposed DSM-PWM hybrid power coding scheme, measurements have been carried. Performance is measured in terms of achieved coding efficiency and in-band signal-to-noise ratio (SNR), and proposed hybrid DSM-PWM power encoder is compared to the one employing only digital PWM, while keeping parameters of PWM fixed in both cases.

As input signal we take amplitude of a 64QAM signal with 20MHz bandwidth. PWM carrier frequency is set to 1GHz, and OSR is 8, thus giving the sampling frequency of

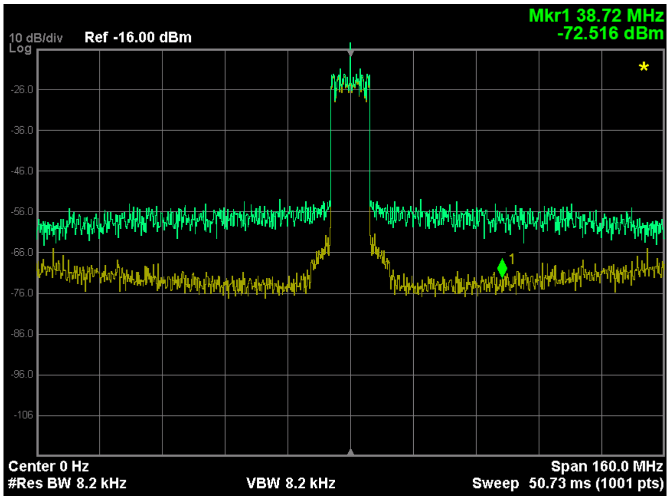

Fig. 6. Measured baseband spectrum of the PWM (green) and DSM-PWM (yellow) outputs.

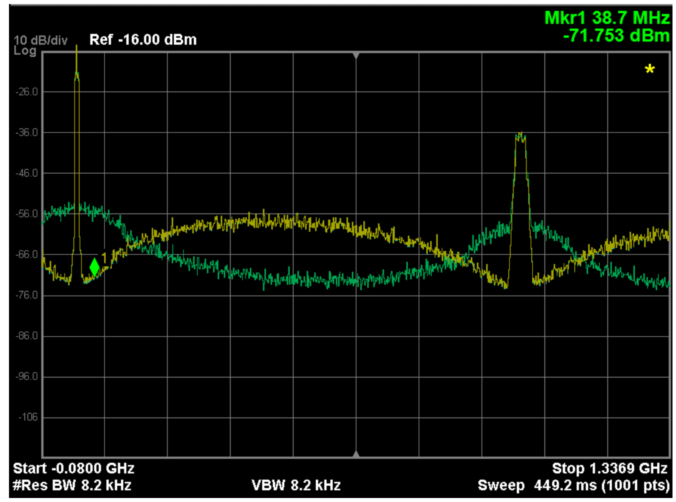

Fig. 7. Measured wideband spectrum of the PWM (green) and DSM-PWM (yellow) outputs.

8GS/s. Number of output levels of PWM is set to 5, which implies the number of hidden quantization levels equal to $L = 8 \cdot (5 - \ 1)/2 = 16$. We therefore employ DSM with 16 output levels. The order of DSM is 1, so to simplify the power encoder design.

Matlab simulated output signals file is loaded into an arbitrary waveform generator (AWG-34G from Micram Instruments) to generate the signals. Keysight EXA 9010A signal analyzer is used to measure the spectrum. Fig. 6 compares the measured spectrum for PWM (green) and DSM-PWM (yellow) output signals. Measured coding efficiencies are 75.95% and 72.7% respectively. It is expected that digital PWM alone can achieve better coding efficiency than hybrid DSM-PWM, due to noise shaping property of DSM as shown in Fig. 7, but the effective number of quantization levels is large, and thus these coding efficiencies differ by insignificant amount (3%). Meanwhile, it can be seen that the latter one offers considerably improved in-band SNR of -54.8dB, compared to -33.4dB for PWM.

V. Conclusion

A highly linear (distortion-free) hybrid DSM-PWM multilevel power coding scheme, which exploits time domain relationship between the baseband input signal and the pulsed output signal, has been demonstrated. By selecting parameters of the DSM according to the derived formula, an improvement in output amplitude resolution, and correspondingly the output in-band SNR, is achieved, as compared to a power encoder employing only digital PWM. Future work will include incorporating this multilevel power coding scheme in novel ADT architectures, as well as investigating hidden quantization and potential of DSM-PWM hybrids in regards to radio-frequency pulse-width modulation (RF PWM).

References

[1] F. H. Raab, P. Asbeck, S. Cripps, P. B. Kenington, Z. B. Popovich, N. Pothecary, J. F. Sevic and N. O. Sokal, "RF and microwave power amplifier and transmitter technologies - Part 2. *High-Freq. Electron.*, vol. 2, no. 3, pp. 2236, May-Nov. 2003.

[2] F. Ghannouchi, "Power Amplifier and Transmitter Architectures for Software Defined Radio Systems," *IEEE Circuits Syst. Mag.*, vol. 10, no. 4, pp. 56-63, Nov. 2010

[3] F. H. Raab,"Class-D power amplifier with RF pulse-width modulation," *Dig. IEEE Microw. Theory Tech. Sym.*, pp. 924-927, June 2010.

[4] P.-J. Nuyts, P. Reynaert and W. Dehaene, *Continuous-Time Digital Front-Ends for Multistandard Wireless Transmission*, Analog Circuits and Signal Processing, Springer International Publishing Switzerland; 2014.

[5] S. Santi, R. Rovatti, and G. Setti, "Spectral aliasing effects of PWM signals with time-quantized switching instants," *in Proc. 2004 Int. Symp. Circuits Syst. (ISCAS2004)*, pp. 689692, May 2004.

[6] A. Amin, M. El-Korfolly and S. Mohammed, "Exploring aliasing distortion effects on regularly-sampled PWM signals," *in Proc. 3rd IEEE Conf. Ind. Electron. Appl.*, pp. 20362041, Jun. 2008.

[7] H. Gheidi and P. M. Asbeck, "An Improved Algorithm for Waveform Generation for Digitally-Driven Switching-Mode Power Amplifiers," *in Proc. 2016 IEEE Radio and Wireless Symposium (RWS2016)*, pp. 187189, Austin, TX, Jan. 2016.

[8] D. Seebacher, P. Singerl, C. Schubert, F. Dielacher, P. Reynaert and W. Bösch, "Reduction of Aliasing Effects of RF PWM Modulated Signals by Cross Point Estimation," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 61, no. 11, pp. 3184-3192, Nov. 2014

[9] K. Hausmair, S. Chi, P. Singerl and C. Vogel, "Aliasing-Free Digital Pulse-Width Modulation for Burst-Mode RF Transmitters," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 60, no. 2, pp. 415-427, Feb. 2013

[10] A. V. Oppenheim, A. S. Willsky and S. H. Nawad, *Signals and Systems*, 2nd ed. Prentice-Hall, Inc. Upper Saddle River, NJ, USA, 1996

[11] D. Markert, C. Haslach, H. Heimpel, A. Pascht and G. Fischer, "Phase-modulated DSM-PWM hybrids with pulse length restriction for switch-mode power amplifiers," *in Proc. 2014 44th Europ. Microw. Conf. (EuMC)*, pp. 1364-1367, Rome, Oct. 2014.

[12] S. Chung, R. Ma, S. Shinjo, H Nakamizo, K. Parsons and K. H. Teo, "Concurrent Multiband Digital Outphasing Transmitter Architecture Using Multidimensional Power Coding," *IEEE Trans. Microw. Theory Techn.*, vol 63, no. 2, pp. 598-613, Feb. 2015.

[13] R. Hezar, L. Ding, A. Banerjee, J. Hur and B. Haroun, "A PWM Based Fully Integrated Digital Transmitter/PA for WLAN and LTE Applications," *IEEE J. Solid -State Circuits*, vol 50, no. 5, pp. 1117-1125, May 2015.

[14] R. Ma, " A review of recent development on digital transmitters with integrated GaN switch-mode amplifiers," *Radio-Frequency Integration Technology (RFIT), 2015 IEEE International Symposium on*, pp. 73-75, Sendai, Aug. 2015.

[15] H. Enzinger and C. Vogel, "Analytical description of multilevel carrier-based PWM of arbitrary bounded input signals," *in Proc. 2014 Int. Symp. Circuits Syst. (ISCAS2014)*, pp. 1030-1033, Melbourne VIC, Jun. 2014.