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Abstract

This work investigates the problem of speed sensorless state estimation for induction motors. We first exploit a state transformation for the induction motor model. Based on the new state coordinates, we design a new Luenberger observer, which can provide better dynamic performance compared to baseline algorithm. To address the parameter variation problem, the Lyapunov redesign method is used to achieve an adaptation with respect to the parameter alpha. It is shown that the proposed observer can achieve guaranteed asymptotic stability and readily extend to the time-varying speed case. Advantages of the proposed observer include guaranteed asymptotic stability of estimation errors, parameter alpha adaptation, and better dynamic performance. Simulation results are presented to validate the proposed method.

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An Adaptive Luenberger Observer for Speed-Sensorless Estimation of Induction Machines

Jie You, Wencen Wu, and Yebin Wang

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I. Introduction

Due to the simplicity, efficiency, and ruggedness, the induction motor drivers have been widely used at variable speed and torque control [1]. To improve reliability and reduce the system cost, it's preferable that the motor drives can remove the rotor shaft encoder, which are often viewed as speed-sensorless motor drives [2]. However, without an encoder, speed-sensorless motors suffer performance degradation. Hence, to overcome this bottleneck, the observer design problem for the speed-sensorless motors has received a remarkable attention over last years [1]–[3].

Many contributions exist on the speed-sensorless estimation for induction motors in the past decades. These contributions present a wide variety of approaches such as adaptive estimators [4]–[7], high gain observers [3], [8], sliding mode observers [9]–[11], extended/unscented Kalman filter [12]–[15], moving horizon estimator [16], [17], etc. The comprehensive analysis and performance limitations of the various approaches have been discussed in [18], [19] and references therein. In all of these estimation methods, the adaptive estimators were initially exploited, and have a relatively simple structure. The typical types of the adaptive estimators include the classic adaptive/baseline estimators [4], model reference adaptive systems method [6], [7], and the adaptive full-order observer [5]. Among these, the classic adaptive/baseline estimator has been one of the most prevalent and successful for the speed-sensorless estimation [8]. The adaptive idea here is treating the rotor speed as an unknown parameter, which can significantly simplify the estimator design. Then, the speed is estimated by means of a Luenberger observer defined by the mechanical characters of the induction motor model. The advantage of this estimator lies in avoiding dealing with nonlinear dynamics and making the structure robustness and simplicity. Even though researchers have been keeping exploring possible improvements to the classic adaptive/baseline estimators, there are still a number of inherent limitations. First of all, a major drawback is the lack of guaranteed stability. Second, the performance of the resultant observer heavily depends on the mechanical characteristics, which requires good knowledge of the machine parameters. Actually, the parameter variations have significant effects on the estimation accuracy.

These limitations motivate us to study the improvements toward the class adaptive full-order state observer for induction motors. These improvements include the better dynamic performance and the robustness to parameter variations. We first employ a state transformation for the induction motor model. Based on the new state coordinates, we propose a new adaptive Luenberger observer for speed-sensorless estimation. It is shown that the state transformation endows the observer with more freedoms for the parameter adaptation as well as the asymptotic stability. For the parameter robustness purpose, we further make our structure adapt to the parameter α. By considering the α adaption, better estimation performance could be obtained to make the speed-sensorless observer robust to parameter variations. With a persistence of excitation (PE) condition, theoretical justifications are provided for asymptotic convergence analysis of state estimation errors. Simulation results are given to demonstrate the effectiveness of the proposed adaptive observer.

This paper is organized as follows. Section II introduces the problem formulation. Section III presents the design of the proposed observer. Section IV illustrates the convergence analysis. Section V introduces the simulation result, and Section VI draws conclusions and discusses future work.

II. Problem Formulation

A. The Induction Machine Model

Consider the following the induction motor model in the frame rotating at an angular speed ω_1:

$$\begin{aligned}
\dot{i}_{ds} &= -\gamma i_{ds} + \omega_1 i_{qs} + \beta(\alpha\phi_{dr} + \omega\phi_{qr}) + \frac{u_{ds}}{\sigma}, \\
\dot{i}_{qs} &= -\gamma i_{qs} - \omega_1 i_{ds} + \beta(\alpha\phi_{qr} - \omega\phi_{dr}) + \frac{u_{qs}}{\sigma}, \\
\dot{\phi}_{dr} &= -\alpha\phi_{dr} + (\omega_1 - \omega)\phi_{qr} + \alpha L_m i_{ds}, \\
\dot{\phi}_{qr} &= -\alpha\phi_{qr} - (\omega_1 - \omega)\phi_{dr} + \alpha L_m i_{qs}, \\
\dot{\omega} &= \mu(\phi_{dr} i_{qs} - \phi_{qr} i_{ds}) - \frac{T_l}{J},
\end{aligned} \tag{1}$$

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TABLE I

NOTATIONS

Notation	Description
i_{ds}, i_{qs}	stator currents in arbitrary rotational d- and q- axis
u_{ds}, u_{qs}	stator voltages in arbitrary rotational d- and q- axis
ϕ_{dr}, ϕ_{qr}	rotor fluxes in arbitrary rotational d- and q- axis
ω	rotor angular speed
ω_1	angular speed of a rotating frame
T_l	load torque
J	rotor inertia
L_s, L_M, L_r	stator, mutual, rotor inductances
R_s, R_r	stator and rotor resistances
σ	$(L_sL_r - L_m^2)/L_r$
α	R_r/L_r
β	$L_m/(\sigma L_r)$
γ	$R_s/\sigma + \alpha\beta L_m$
μ	$3L_m/(2L_r)$
z_1	$i_{ds} + \beta\phi_{dr}$
z_2	$i_{qs} + \beta\phi_{qr}$

where the notation is denoted in Table I. Throughout this paper, we take the angular speed of the rotating frame $\omega_1 = 0$, which is typically called the stationary frame. For more details about the induction motor model, please refer to [20]. The objective of speed sensorless state estimation problem for the induction motor is : design an estimator to reconstruct the full state of the induction motor model by using the measurements of stator currents $(i_{ds},\, i_{qs})$ and stator voltages $(u_{ds},\, u_{qs})$.

B. Baseline adaptive observer

The baseline adaptive observer [4], [5] for speed sensorless state estimation is provided to suffice self-containedness. The basic idea is to treat the rotor speed ω as a constant parameter, i.e. $\dot{\omega} = 0$. Then, the original nonlinear system dynamics in Equation (1) can be reduced to a linear system with a parameter ω, which can be rewritten as follows

$$\begin{aligned} \dot{x}' &= A'(\omega)x' + B'u \\ y &= C'x', \end{aligned} \tag{2}$$

where $x' = [i_{ds}, i_{qs}, \phi_{dr}, \phi_{qr}]^T$ and $u = [u_{ds}, u_{qs}]^T$. $A'(\omega) = \begin{bmatrix} -\gamma & 0 & \alpha\beta & \beta\omega \\ 0 & -\gamma & -\beta\omega & \alpha\beta \\ \alpha L_m & 0 & -\alpha & -\omega \\ 0 & \alpha L_m & \omega & -\alpha \end{bmatrix}$, $B^{'} = \begin{bmatrix} \frac{1}{\sigma} & 0 \\ 0 & \frac{1}{\sigma} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T$, $C^{'} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T$. Based on the system (2), the Luenberger observer is designed as follow,

$$\begin{aligned} \dot{\hat{x}}' &= A'(\hat{\omega})\hat{x}' + B'u + L(y - \hat{y}) \\ \hat{y} &= C'\hat{x}', \end{aligned} \tag{3}$$

where L is the observer gain matrix, $\hat{x}'$ denotes the estimated value of x', and $A'(\hat{\omega})$ is the matrix $A'(\omega)$ with ω being replaced by $\hat{\omega}$. Speed estimation of the baseline algorithm is given by

$$\dot{\hat{\omega}} = h\beta(\tilde{i}_{ds}\hat{\phi}_{qr} - \tilde{i}_{qs}\hat{\phi}_{dr}), \tag{4}$$

where $h > 0$ is constant, $\tilde{i}_{ds} = i_{ds} - \hat{i}_{ds}$, and $\tilde{i}_{qs} = i_{qs} - \hat{i}_{qs}$. Even though the baseline algorithm works well in practice, there are still two major limitations. As shown in the literatures [4], [5], the zero solution of the estimation error dynamics result from the baseline observer in Equation (4) can not achieve asymptotic convergence. The other shortcoming is that the performance of the observer is highly dependent on the accuracy of the model parameters. Hence, this paper is to construct a new adaptive observer to overcome the above drawbacks and improve the performance of the baseline observer. Since the convergence of the observer (4) can not be established in the original state coordinates, we first exploit a state transformation. Based on the new state coordinates, we design an adaptive observer, which guarantees the asymptotic convergence of the resultant estimation error dynamics. To further improve the model-based observer, we make the structure of the proposed observer adapt to the parameter α. Lastly, we show that our proposed observer can readily extend to the case that ω is treated as a slowly time-varying parameter with bounded derivative.

III. PROPOSED LUENBERGER OBSERVER FOR SPEED-SENSORLESS ESTIMATION

In this section, we first propose a state transformation and design a new Luenberger observer based on the new state coordinates. Next, the proposed speed and α adaptation laws are designed based on the Lyapunov redesign method. Main techniques used across this section can be found in [21], which however concentrates on induction machines with encoder.

A. Design of the Luenberger observer

The basic idea is to introduce the variables

$$\begin{aligned} z_1 &= i_{ds} + \beta\phi_{dr}, \\ z_2 &= i_{qs} + \beta\phi_{qr}. \end{aligned} \tag{5}$$

Denote the new states as i_{ds}, i_{qs}, z_1, and z_2. By a change of state coordinates, the system (1) is transformed to a linear parametric-varying system as follows,

$$\begin{aligned} \dot{i}_{ds} &= -\frac{R_s}{\sigma}i_{ds} - \alpha(1+\beta L_m)i_{ds} - \omega i_{qs} + \alpha z_1 + \omega z_2 + \frac{u_{ds}}{\sigma}, \\ \dot{i}_{qs} &= -\frac{R_s}{\sigma}i_{qs} - \alpha(1+\beta L_m)i_{qs} + \omega i_{ds} + \alpha z_2 - \omega z_1 + \frac{u_{qs}}{\sigma}, \\ \dot{z}_1 &= -\frac{R_s}{\sigma}i_{ds} + \frac{u_{ds}}{\sigma}, \\ \dot{z}_2 &= -\frac{R_s}{\sigma}i_{qs} + \frac{u_{qs}}{\sigma}. \end{aligned} \tag{6}$$

Based on the system (6), we perform the Luenberger observer design. We denote $\hat{z}_1 = \hat{i}_{ds} + \beta\hat{\phi}_{dr}$ and $\hat{z}_2 = \hat{i}_{qs} + \beta\hat{\phi}_{qr}$ as the estimates of z_1 and z_2, respectively, and consider the following Luenberger observer

$$\begin{aligned} \dot{\hat{i}}_{ds} = &-\frac{R_s}{\sigma}\hat{i}_{ds} - \alpha(1+\beta L_m)i_{ds} - \hat{\omega}\hat{i}_{qs} + \alpha\hat{z}_1 + \hat{\omega}\hat{z}_2 \\ &+ k_1(i_{ds} - \hat{i}_{ds}) + \frac{u_{ds}}{\sigma}, \end{aligned}$$

$$\dot{\hat{i}}_{qs} = -\frac{R_s}{\sigma}\hat{i}_{qs} - \alpha(1+\beta L_m)i_{qs} + \hat{\omega}\hat{i}_{ds} + \alpha\hat{z}_2 - \hat{\omega}\hat{z}_1 + k_1(i_{qs} - \hat{i}_{qs}) + \frac{u_{qs}}{\sigma}, \tag{7}$$

$$\dot{\hat{z}}_1 = -\frac{R_s}{\sigma}i_{ds} - k_2\hat{\omega}(i_{ds} - \hat{i}_{ds}) + \frac{u_{ds}}{\sigma},$$

$$\dot{\hat{z}}_2 = -\frac{R_s}{\sigma}i_{qs} + k_2\hat{\omega}(i_{qs} - \hat{i}_{qs}) + \frac{u_{qs}}{\sigma},$$

where k_1 and k_2 are the observer gains. Note that the dynamics of z_1 and z_2 only depend on the measurements of stator currents (i_{ds}, i_{qs}) and stator voltages (u_{ds}, u_{qs}).

B. Design of ω and α adaption

The above subsection illustrates the design of a Luenberger observer based on the new states. In this subsection, we show that this observer enables us to rebuild the ω and α by adaption. To enable adaption with respect to α, we introduce two additional state variables $\hat{\xi}_1$ and $\hat{\xi}_2$. In fact, $(\hat{\xi}_1, \hat{\xi}_2)$ and $(\hat{z}_1, \hat{z}_2)$ denote two different estimates of the unmeasured variables (z_1, z_2). Thus, the dynamics of ξ_1 and ξ_2 are the same as that of z_1 and z_2 in (6). By including the dynamics of the states $(\hat{\xi}_1, \hat{\xi}_2)$ into Equation (7), the proposed adaptive Luenberger observer is shown as follow,

$$\dot{\hat{i}}_{ds} = -\frac{R_s}{\sigma}\hat{i}_{ds} - \hat{\alpha}(1+\beta L_m)i_{ds} - \hat{\omega}\hat{i}_{qs} + \hat{\alpha}\hat{\xi}_1 + \hat{\omega}\hat{z}_2 + k_1(i_{ds} - \hat{i}_{ds}) + \frac{u_{ds}}{\sigma},$$

$$\dot{\hat{i}}_{qs} = -\frac{R_s}{\sigma}\hat{i}_{qs} - \hat{\alpha}(1+\beta L_m)i_{qs} + \hat{\omega}\hat{i}_{ds} + \hat{\alpha}\hat{\xi}_2 - \hat{\omega}\hat{z}_1 + k_1(i_{qs} - \hat{i}_{qs}) + \frac{u_{qs}}{\sigma},$$

$$\dot{\hat{z}}_1 = -\frac{R_s}{\sigma}i_{ds} - k_2\hat{\omega}(i_{ds} - \hat{i}_{ds}) + \frac{u_{ds}}{\sigma},$$

$$\dot{\hat{z}}_2 = -\frac{R_s}{\sigma}i_{qs} + k_2\hat{\omega}(i_{qs} - \hat{i}_{qs}) + \frac{u_{qs}}{\sigma},$$

$$\dot{\hat{\xi}}_1 = -\frac{R_s}{\sigma}i_{ds} + k_3(i_{ds} - \hat{i}_{ds}) + \frac{u_{ds}}{\sigma},$$

$$\dot{\hat{\xi}}_2 = -\frac{R_s}{\sigma}i_{qs} + k_3(i_{qs} - \hat{i}_{qs}) + \frac{u_{qs}}{\sigma}.$$

where $k_3 > 0$ is the feedback gain for the states variables $\hat{\xi}_1$ and $\hat{\xi}_2$. Write the above equations in a matrix from

$$\begin{aligned} \dot{\hat{x}} &= A(\hat{\omega})\hat{x} + Bu + L(\hat{\omega})(y - \hat{y}) \\ \hat{y} &= C\hat{x}. \end{aligned} \tag{8}$$

where $x = [i_{ds}, i_{qs}, z_1, z_2, \xi_1, \xi_2]^T$ and $\hat{x}$ is the estimate of x, the matrices $A(\hat{\omega})$, B, $L(\hat{\omega})$ and C are defined as

$$A(\hat{\omega}) = \begin{bmatrix} -\frac{R_s}{\sigma} & -\hat{\omega} & 0 & \hat{\omega} & \alpha & 0 \\ \hat{\omega} & -\frac{R_s}{\sigma} & -\hat{\omega} & 0 & 0 & \alpha \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{\sigma} & 0 \\ 0 & \frac{1}{\sigma} \\ \frac{1}{\sigma} & 0 \\ 0 & \frac{1}{\sigma} \\ \frac{1}{\sigma} & 0 \\ 0 & \frac{1}{\sigma} \end{bmatrix}^T,$$

$$L(\hat{\omega}) = \begin{bmatrix} k_1 & 0 & 0 & -k_2\hat{\omega} & k_3 & 0 \\ 0 & k_1 & k_2\hat{\omega} & 0 & 0 & k_3 \end{bmatrix}^T, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^T.$$

Based on the above system, the α adaption scheme is selected as

$$\dot{\hat{\alpha}} = \mathrm{Proj}(g(\tilde{i}_{ds}[\hat{\xi}_1 - (1+\beta L_m)i_{ds}] + \tilde{i}_{qs}[\hat{\xi}_2 - (1+\beta L_m)i_{qs}]), \hat{\alpha}). \tag{9}$$

where $g > 0$ is a constant and Proj(.) is the smooth projection operator defined by

$$\mathrm{Proj}\{\rho, \hat{\alpha}\} = \begin{cases} \rho & \text{if } p(\hat{\alpha}) \le 0 \\ \rho & \text{if } p(\hat{\alpha}) \ge 0 \text{ and } \rho \ge 0 \\ [1 - p(\hat{\alpha})]\rho & \text{otherwise} \end{cases} \tag{10}$$

in which $p(\hat{\alpha}) = \frac{\alpha_l^2 - \hat{\alpha}^2}{2\delta\alpha_l - \delta^2}$. We employ a projection operator to ensure $\hat{\alpha} > \alpha_l > 0$, where α_l is a lower bound of $\hat{\alpha}$.

The adaptation law for estimating ω is

$$\begin{aligned} \dot{\hat{\omega}} &= K_p \cdot \mathrm{Sign}(\omega) \cdot \mathrm{Proj}(-\hat{i}_{qs}\tilde{i}_{ds} + \hat{i}_{ds}\tilde{i}_{qs} + \hat{z}_2\tilde{i}_{ds} - \hat{z}_1\tilde{i}_{qs}) \\ &= K_p \cdot \mathrm{Sign}(\omega) \cdot \mathrm{Proj}\left(\beta(\tilde{i}_{ds}\hat{\phi}_{qr} - \tilde{i}_{qs}\hat{\phi}_{dr})\right), \end{aligned} \tag{11}$$

where $K_p > 0$ is a constant selected by the user, Sign(.) is the sign operator, and the fluxes $\hat{\phi}_{dr}$, $\hat{\phi}_{qr}$ can be reconstructed by using the following equation, $\begin{bmatrix} \hat{\phi}_{dr} \\ \hat{\phi}_{qr} \end{bmatrix} = \frac{1}{\beta}\begin{bmatrix} \hat{\alpha} & \hat{\omega} \\ -\hat{\omega} & \hat{\alpha} \end{bmatrix}^{-1}\begin{bmatrix} \hat{\alpha}\hat{\xi}_1 + \hat{\omega}\hat{z}_2 \\ \hat{\alpha}\hat{\xi}_2 - \hat{\omega}\hat{z}_1 \end{bmatrix} - \frac{1}{\beta}\begin{bmatrix} i_{ds} \\ i_{qs} \end{bmatrix}$. Note that we develop the adaptive signals (9) and (11) based on a Lyapunov redesign algorithm [22].

Remark III.1 *A key assumption is the adaptive law* (11) *and the stability analysis below is that we know the rotational direction of the motor. This will not incur any problem when the motion works in mid or high speeds. We expect the proposed estimation algorithm results in improved performance and reliability versus the baseline algorithm. While the motor runs at low speeds, the proposed estimation algorithm may not enjoy such advantages.*

IV. Stability Analysis

In this section, we show that by running the proposed observer (8) with the update law (9) and (11), the full state estimation errors are bounded. Furthermore, if a PE condition is met, the estimation errors can converge to zero.

A. Stability analysis of the proposed observer

To prove the convergence of parameter estimation error dynamics, we need the Barbalat's lemma [22] as follows

Lemma IV.1 *Let $f : [0, \infty] \to \mathbf{R}$ be a uniformly continuous function and suppose that $\lim_{t\to\infty}\int_0^t f(\tau)d\tau$ exists and is finite. Then $f(t) \to 0$ as $t \to \infty$.*

We have the following proposition about the boundedness of the state estimation errors

Proposition IV.2 *Consider the induction motor model in Equation* (1)*. By implementing the Luenberger observer* (8) *with the adaption laws* (9) *and* (11)*, the stator current estimations errors* $\tilde{i}_{ds} = i_{ds} - \hat{i}_{ds}$ *and* $\tilde{i}_{qs} = i_{qs} - \hat{i}_{qs}$ *achieve asymptotic convergence and the full state estimation errors are bounded.*

Proof: Let us first introduce the error signals as $\tilde{z}_1 = z_1 - \hat{z}_1$, $\tilde{z}_2 = z_2 - \hat{z}_2$, $\tilde{\xi}_1 = z_1 - \hat{\xi}_1$, $\tilde{\xi}_2 = z_2 - \hat{\xi}_2$, $\tilde{\alpha} = \alpha - \hat{\alpha}$, and $\tilde{\omega} = \omega - \hat{\omega}$. By combining the system (6) and the Luenberger observer (8), the dynamics of the error signals can be obtained as follow,

$$
\begin{aligned}
\dot{\tilde{i}}_{ds} &= -(\frac{R_s}{\sigma} + k_1)\tilde{i}_{ds} - \tilde{\alpha}(1+\beta L_m)i_{ds} - \tilde{\omega}\hat{i}_{qs} - \omega\tilde{i}_{qs} \\
&\quad + \alpha\tilde{\xi}_1 + \tilde{\alpha}\hat{\xi}_1 + \omega\tilde{z}_2 + \tilde{\omega}\hat{z}_2 \\
\dot{\tilde{i}}_{qs} &= -(\frac{R_s}{\sigma} + k_1)\tilde{i}_{qs} - \tilde{\alpha}(1+\beta L_m)i_{qs} - \tilde{\omega}\hat{i}_{ds} - \omega\tilde{i}_{ds} \\
&\quad + \alpha\tilde{\xi}_2 + \tilde{\alpha}\hat{\xi}_2 - \omega\tilde{z}_1 - \tilde{\omega}\hat{z}_1 \\
\dot{\tilde{z}}_1 &= k_2\hat{\omega}\tilde{i}_{qs}, \;\; \dot{\tilde{\xi}}_1 = -k_3\tilde{i}_{ds}, \\
\dot{\tilde{z}}_2 &= -k_2\hat{\omega}\tilde{i}_{ds}, \;\; \dot{\tilde{\xi}}_2 = -k_3\tilde{i}_{qs}.
\end{aligned}
\tag{12}
$$

The stability of the estimation error dynamics can be established using the following Lyapunov function,

$$
\begin{aligned}
V &= \frac{1}{2}(\tilde{i}_{ds}^2 + \tilde{i}_{qs}^2) + \frac{\omega}{2k_2\hat{\omega}}(\tilde{z}_1^2 + \tilde{z}_2^2) + \frac{\alpha}{2k_2}(\tilde{\xi}_1^2 + \tilde{\xi}_2^2) \\
&\quad + \frac{1}{2g}\tilde{\alpha}^2 + \frac{1}{2K_p}\tilde{\omega}^2.
\end{aligned}
\tag{13}
$$

The derivative of the Lyapunov function V can be written as

$$
\begin{aligned}
\dot{V} &= -(k_1 + \frac{R_s}{\sigma})(\tilde{i}_{ds}^2 + \tilde{i}_{qs}^2) + \frac{-\dot{\hat{\omega}}}{\hat{\omega}^2}\frac{\omega}{2k_2}(\tilde{z}_1^2 + \tilde{z}_2^2) \\
&\quad + \underbrace{\tilde{i}_{ds}\tilde{\alpha}(\hat{\xi}_1 - (1+\beta L_m)i_{ds}) + \tilde{i}_{qs}\tilde{\alpha}[\hat{\xi}_2 - (1+\beta L_m)i_{ds}]}_{\rho_1} \\
&\quad + \underbrace{\alpha\tilde{i}_{ds}\tilde{\xi}_1 + \tilde{i}_{ds}\omega\tilde{z}_2 + \alpha\tilde{i}_{qs}\tilde{\xi}_2 - \tilde{i}_{qs}\omega\tilde{z}_1}_{\rho_2} \\
&\quad \underbrace{-\tilde{\omega}\hat{i}_{qs}\tilde{i}_{ds} + \tilde{\omega}\hat{i}_{ds}\tilde{i}_{qs} + \tilde{\omega}\hat{z}_2\tilde{i}_{ds} - \tilde{\omega}\hat{z}_1\tilde{i}_{qs}}_{\rho_3} + \frac{\omega}{k_2\hat{\omega}}(z_1\dot{\tilde{z}}_1 + z_2\dot{\tilde{z}}_2) \\
&\quad \frac{\alpha}{k_3}(\xi_1\dot{\tilde{\xi}}_1 + \xi_2\dot{\tilde{\xi}}_2) + \frac{1}{g}\tilde{\alpha}\dot{\tilde{\alpha}} + \frac{1}{g}\tilde{\omega}\dot{\tilde{\omega}}.
\end{aligned}
\tag{14}
$$

From (12), we know that

$$
\begin{aligned}
\frac{\omega}{k_2\hat{\omega}}(z_1\dot{\tilde{z}}_1 + z_2\dot{\tilde{z}}_2) &= -\tilde{i}_{ds}\omega\tilde{z}_2 + \tilde{i}_{qs}\omega\tilde{z}_1, \\
\frac{\alpha}{k_3}(\xi_1\dot{\tilde{\xi}}_1 + \xi_2\dot{\tilde{\xi}}_2) &= -\alpha\tilde{i}_{ds}\tilde{\xi}_1 - \alpha\tilde{i}_{qs}\tilde{\xi}_2,
\end{aligned}
\tag{15}
$$

which can cancel the ρ_2 terms in Equation (14).

Substituting the adaptive law Equation (9) into Equation (14), the properties of the projection operator guarantee that

$$
\tilde{i}_{ds}\tilde{\alpha}(\hat{\xi}_1 - (1+\beta L_m)i_{ds}) + \tilde{i}_{qs}\tilde{\alpha}[\hat{\xi}_2 - (1+\beta L_m)i_{ds}] + \frac{1}{g}\tilde{\alpha}\dot{\tilde{\alpha}} \leq 0
$$

Similarly, with the adaptive ω update law in (11), the ρ_3 term in Equation (14) can be canceled. Meanwhile, we can guarantee the term $\frac{-\dot{\hat{\omega}}}{\hat{\omega}^2}\frac{\omega}{2k_2}(\tilde{z}_1^2 + \tilde{z}_2^2) \leq 0$ by using the property of the sign operator. Therefore, substituting Equations(11) and (9) into (14), we have the inequality

$$
\dot{V} \leq -(k_1 + \frac{R_s}{\sigma})(\tilde{i}_{ds}^2 + \tilde{i}_{qs}^2). \tag{16}
$$

Equation (16) implies $V(t) < V(0)$, from which we conclude that the estimation errors $\tilde{i}_{ds}$, $\tilde{i}_{qs}$, $\tilde{z}_1$, $\tilde{z}_2$, $\tilde{\xi}_1$ and $\tilde{\xi}_2$ are bounded.

We can further apply Barbalat's Lemma to prove the asymptotic convergence of current estimation errors. By assuming the states $i_{ds}, i_{qs}, \phi_{dr}, \phi_{qr}$, ω to be bounded, from the error dynamics in Equation (12), we observe that $\dot{\tilde{i}}_{ds}$ and $\dot{\tilde{i}}_{qs}$ are bounded, and thus $\tilde{i}_{ds}$ and $\tilde{i}_{qs}$ are uniformly continuous. Then, if we can prove that $\lim_{t\to\infty}\int_0^t \tilde{i}_{ds}^2 + \tilde{i}_{qs}^2 dt$ exists and finite, then Barbalat's lemma would imply $\tilde{i}_{ds}$ and $\tilde{i}_{qs}$ go to zero. Given the results of (16), we have

$$
-\int_0^t \dot{V}(\tau)d\tau \geq (k_1 + \frac{R_s}{\sigma})\int_0^t (\tilde{i}_{ds}^2(\tau) + \tilde{i}_{qs}^2(\tau))d\tau. \tag{17}
$$

Then, we have

$$
\int_0^t (\tilde{i}_{ds}^2(\tau) + \tilde{i}_{qs}^2(\tau))d\tau \leq (k_1 + \frac{R_s}{\sigma})V(0), \tag{18}
$$

which implies that the existence of $\lim_{t\to\infty}\int_0^t \tilde{i}_{ds}^2 + \tilde{i}_{qs}^2 dt$. Hence, by using the Lemma IV.1, we can conclude that $\lim_{t\to\infty}[i_{ds}(t) - \hat{i}_{ds}(t)] = 0, \lim_{t\to\infty}[i_{qs}(t) - \hat{i}_{qs}(t)] = 0$ This completes the proof. ■

As discussed previously, the state estimation errors $\tilde{z}_1$, $\tilde{z}_2$, $\tilde{\xi}_1$ and $\tilde{\xi}_2$ are bounded. It is known that the convergence of the estimates to the true values relies on the existence of the persistency of excitation conditions [23]. To guarantee the asymptotic convergence, we further provide the persistency of excitation conditions and have the following proposition.

Proposition IV.3 *Consider the induction motor model in* (1)*. By implementing the Luenberger observer* (8) *with the adaption update laws* (9) *and* (11)*, the full state estimation errors* $\tilde{z}_1, \tilde{z}_2, \tilde{\xi}_1, \tilde{\xi}_2, \tilde{\alpha}$*, and* $\tilde{\omega}$ *achieve asymptotic convergence if there exist two positive real numbers* $T > 0$ *and* $m > 0$ *such that for all* $t > 0$ *the following inequality holds:*

$$
\int_t^{t+T} B^{*T}(\tau)B^*(\tau)d\tau \geq mI_{6\times 6}, \tag{19}
$$

where the matrix B^* *is defined as,*

$$
B^* = \begin{bmatrix} \alpha & 0 & 0 & \omega & \hat{\xi}_1 - (1+\beta L_m)i_{ds} & \hat{z}_2 - \hat{i}_{qs} \\ 0 & \alpha & -\omega & 0 & \hat{\xi}_2 - (1+\beta L_m)i_{qs} & \hat{z}_1 - \hat{i}_{ds} \end{bmatrix}.
$$

Proof: To construct the persistence of excitation condition, we rewrite the error systems in Equations (9), (11), and (12) in the following compact form

$$
\begin{aligned}
\dot{S} &= A^*S + B^*Z \\
\dot{Z} &= D^*S,
\end{aligned}
\tag{20}
$$

where $S = [\tilde{i}_{ds}, \tilde{i}_{qs}]^T$, $Z = [\tilde{\xi}_1, \tilde{\xi}_2, \tilde{z}_1, \tilde{z}_2, \tilde{\alpha}, \tilde{\omega}]^T$, D^* is a suitable matrix, and the matrix A^* is denoted as $A^* =$

$\begin{bmatrix} -\frac{R_s}{\sigma}-k_1 & -\omega \\ \omega & -\frac{R_s}{\sigma}-k_1 \end{bmatrix}$. According to the PE condition in (19), if i_{ds}, i_{qs}, ϕ_{dr}, ϕ_{qr}, ω are bounded, then the error $\tilde{z}_1, \tilde{z}_2, \tilde{\xi}_1, \tilde{\xi}_2, \tilde{\alpha}$, and $\tilde{\omega}$ exponentially converge to zero from arbitrary initial conditions [21, Lem. A.3]. ■

B. Extension to the time-varying ω case

We previously assume ω as a constant parameter. In the following, we further show that the proposed observer design can deal with a more general case, where ω is a slowly time-varying parameter with a bound derivative. When $\dot{\omega} \neq 0$, the error systems can be rewritten as follows,

$$\begin{aligned} \dot{S} &= A^* S + B^* Z, \\ \dot{Z} &= D^* S + E^* \dot{\omega}, \end{aligned} \tag{21}$$

where the matrix E^* is defined as, $E^* = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}^T$. Based on the new error system (21), we establish a set of sufficient conditions for the system such that the estimation errors are uniformly bounded. We have the proposition as follow.

Proposition IV.4 *If i) there exist positive constants T and m such that $\forall t \geq 0$ the persistency of excitation condition* (19) *is satisfied, ii) there exists a positive constant M_1 such that $\forall t \geq 0$*

$$\|\dot{\omega}\|_\infty \leq M_1 < \infty, \tag{22}$$

then there exist finite constants K_1 and K_2 such that

$$\|\tilde{\omega}(t)\| \leq K_1 \|\tilde{\omega}(0)\| + K_2, \forall t \geq 0, \tag{23}$$

Proof: It follows from condition i) that the homogeneous part of the error systems (21) is exponentially stable. One can readily verify that the result (23) follows from [23, Thm. 3.1, p.105] using (22). Detailed proof is omitted here. ■

V. Simulation Validation

In this section, we demonstrate the performance of the proposed observer by simulation in a Matlab/Simulink platform. In simulation, all observer initial conditions are set to zero. Table II compares the eigenvalues of the baseline and the proposed observers under different operating speed. Since the proposed observer has similar negative eigenvalues, it can have a fast convergence rate. To achieve a fair validation, all the model parameters for both the baseline algorithm and the proposed observer are unbiased. The induction motor parameters in simulation are shown in Table III. In Fig. 1 and Fig. 2, we show the estimated speed under step-type speed reference from 50 rad/s and 100 rad/s and 20 rad/s and 40 rad/s, respectively. The comparison between the baseline observer and the proposed observer at different speeds are shown in Fig. 2(b) and Fig. 3(b). It can been seen that in the speed estimation error plot, the proposed method can achieve better dynamic performance under different operating speeds. In Fig. 3, we further show the Luenberger observer performance with respect to the variance of parameter α. In this case, we set the nominal value to $\alpha = 10$ and the bound on $\hat{\alpha}$ from below to $\alpha_l = 0.05$. The initial values $\hat{\alpha}$ for both the baseline algorithm and the proposed algorithm are set to 6.6. When the parameter of α is biased, the baseline algorithm becomes much worse than the proposed method in Fig. 3(b). We can see from the Fig. 4 that the final value of $\hat{\alpha}$ is 10.5, which is slightly different from the nominal value 9.9354.

TABLE II

Comparison of Eigenvalues between the baseline and the proposed observers.

Speed	The proposed observer	The baseline observer
$\hat{\omega} = 30$	-347.97 + 13.97i	-479.11 + 14.55i
	-347.97 - 13.97i	-479.11 - 14.55i
	-147.42 + 16.03i	-16.28 + 15.45i
	-147.42 - 16.03i	-16.28 - 15.45i
$\hat{\omega} = 60$	-247.11 + 384.43i	-474.37 + 29.09i
	-247.11 - 384.43i	-474.37 - 29.09i
	-248.28 + 324.43i	-21.03 + 30.91i
	-248.28 - 324.43i	-21.03 - 30.91i
$\hat{\omega} = 100$	-247.18 + 718.75i	-466.28 - 48.42i
	-247.18 - 718.75i	-466.28 - 48.42i
	-248.21 + 618.75i	-29.11 + 51.58i
	-248.21 - 618.75i	-29.11 - 51.58i

TABLE III

Induction Motor Parameters in Simulation.

Parameter	Value	Description
p	2	Pole pair number
J	0.0163 kgm^2	Rotor inertia
R_s	0.439 Ω	Stator resistance
R_r	0.41 Ω	Rotor resistance
L_m	60.1e-3 H	Mutual inductance
σ_s	1.4e-3 H	Stator leak inductance
σ_r	1.8e-3 H	Rotor leak inductance
α	9.9354	R_r/L_r
ϕ^*	0.5307	reference rotor flux
ω^*	NA	reference rotor speed

VI. Conclusions and Future Work

This paper developed a new adaptive Luenberger observer for speed-sensorless induction motors. The proposed observer is designed on a new state coordinates, which provides better dynamic performance. Applying the α adaptation results in the robustness to parameter variations. Theoretical justifications of the proposed observer were provided by performing convergence analysis. Simulation results were provided to demonstrate the proposed approach. Future work includes experimental validation of the proposed algorithm and construct a systematic scheme for tuning the parameters.

References

[1] J. Chiasson, "Dynamic feedback linearization of the induction motor," *IEEE Trans. Automat. Control*, vol. 38, no. 10, pp. 1588–1594, 1993.

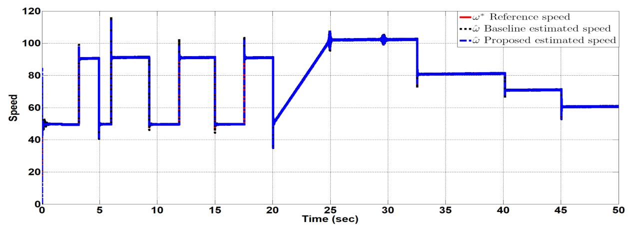
[2] L. Harnefors, "Globally stable speed-adaptive observers for sensorless induction motor drives," *IEEE Trans. Ind. Electron*, vol. 54, no. 2, pp. 1243–1245, 2007.

[3] M. Montanari, S. Peresada, and A. Tilli, "A speed-sensorless indirect field-oriented control for induction motors based on high gain speed estimation," *Automatica*, vol. 42, no. 10, pp. 1637–1650, 2006.

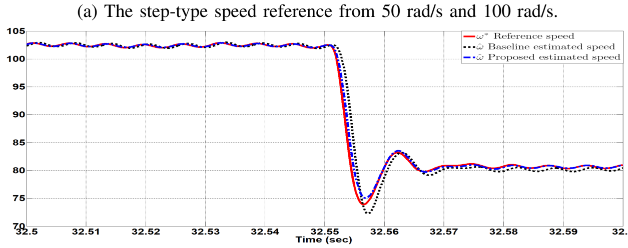
[4] H. Kubota, K. Matsuse, and T. Nakano, "Dsp-based speed adaptive flux observer of induction motor," *IEEE Trans. Ind. Appl.*, vol. 29, no. 2, pp. 344–348, 1993.

[5] H. Kubota and K. Matsuse, "Speed sensorless field-oriented control of induction motor with rotor resistance adaptation," *IEEE Trans. Ind. Appl.*, vol. 30, no. 5, pp. 1219–1224, 1994.

[6] S. Tamai, H. Sugimoto, and M. Yano, "Speed sensorless vector control of induction motor with model reference adaptation system," in *Conf. Rec. of 1987 IEEE IAS Annual Meeting*, 1987, pp. 189–195.

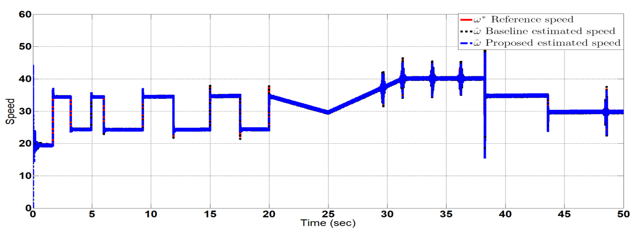


(a) The step-type speed reference from 50 rad/s and 100 rad/s.

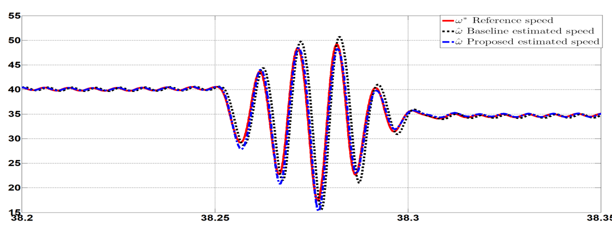


(b) Step response comparisons between baseline and the proposed observers in the time range of $[32.5, 32.6]$.

Fig. 1. Estimated speed under step-type speed reference from 50 rad/s and 100 rad/s based on the baseline and the proposed observers.

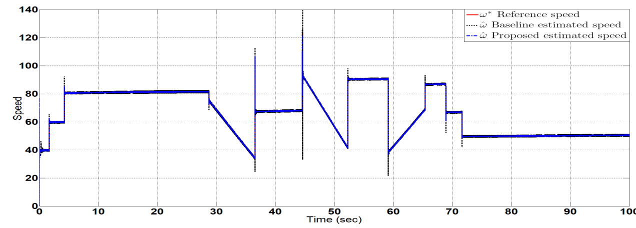


(a) Step-type speed reference from 20 rad/s and 40 rad/s.

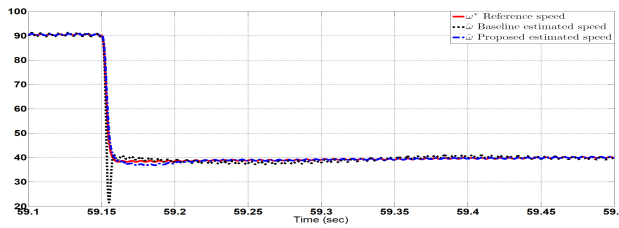


(b) Step response comparisons between baseline and the proposed observers in the time range of $[38.2, 38.35]$.

Fig. 2. Estimated speed under step-type speed reference from 20 rad/s and 40 rad/s based on the baseline and the proposed observers.



(a) The cross validation of the adaptive performance.



(b) Step response comparisons between baseline and the proposed observers in the time range of $[59.1, 59.5]$.

Fig. 3. Estimated speed under step-type speed reference from 40 rad/s and 100 rad/s based on the baseline and the proposed observers.

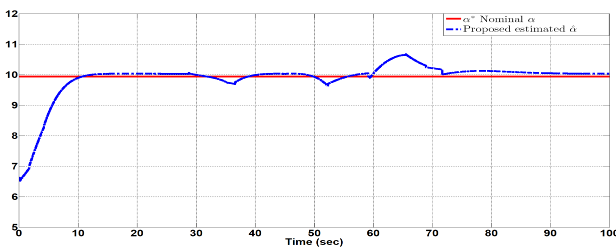


Fig. 4. Estimated parameter $\hat{\alpha}$ based on the proposed observer.

[7] C. Schauder, "Adaptive speed identification for vector control of induction motor with rotational transducers," *IEEE Trans. Ind. Appl.*, vol. 28, no. 5, pp. 1054–1061, 1992.

[8] Y. Wang, L. Zhou, S. A. Bortoff, A. Satake, and S. Frutani, "High gain observer for speed-sensorless motor drives : algorithm and experiments," in *IEEE International Conference on Advanced Intelligent Mechatronics*, 2016.

[9] A. Benchaib, A. Rachid, E. Audrezet, and M. Tadjine, "Real-time sliding-mode observer and control of an induction motor," *IEEE Trans. Ind. Electron*, vol. 46, no. 1, pp. 128–139, 1999.

[10] M. Tursini, R. Petrella, and F. Parasiliti, "Adaptive sliding-mode observer for speed-sensorless control of induction motors," *IEEE Trans. Ind. Appl.*, vol. 36, no. 5, pp. 1380–1387, 2000.

[11] L. Zhao, J. Huang, H. Liu, B. Li, and W. Kong, "Second-order sliding-mode observer with online parameter identification for sensorless induction motor drives," *IEEE Trans. Ind. Electron*, vol. 61, no. 10, pp. 5280–5289, 2014.

[12] Y.-R. Kim, S.-K. Sul, and M.-H. Park, "Speed sensorless vector control of induction motor using extended kalman filter," *IEEE Trans. Ind. Appl.*, vol. 30, no. 5, pp. 1225–1233, 1994.

[13] M. Barut, S. Bogosyan, and M. Gokasan, "Speed sensorless estimation for induction motors using extended kalman filters," *IEEE Trans. Ind. Electron*, vol. 54, no. 1, pp. 272–280, 2007.

[14] F. Alonge, F. D'Ippolito, and A. Sferlazza, "Sensorless control of induction-motor drive based on robust kalman filter and adaptive speed estimation," *IEEE Trans. Ind. Electron*, vol. 61, no. 3, pp. 1444–1453, 2014.

[15] F. Alonge, T. Cangemi, F. D'Ippolito, A. Fagiolini, and A. Sferlazza, "Convergence analysis of extended kalman filter for sensorless control of induction motor," *IEEE Trans. Ind. Electron*, vol. 62, no. 4, pp. 2341–2352, 2015.

[16] D. Frick, A. Domahidi, M. Vukov, S. Mariethoz, M. Diehl, and M. Morari, "Moving horizon estimation for induction motors," in *Proc. 3rd IEEE Int. Sym. on Sensorless Control for Electrical Drives*, 2012.

[17] L. Zhou and Y. Wang, "Speed sensorless state estimation for induction motors: A moving horzon approach," in *Proc. 2016 ACC*, 2016, pp. 2229–2234.

[18] R. Blasco-Gimenez, G. Asher, M. Summer, and K. Bradley, "Dynamic performance limitations for mras based sensorless induction motor drives. i. stability analysis for the closed loop drive," *IEE Proceedings-Electric Power Applications*, vol. 143, no. 2, pp. 113–122, 1996.

[19] H. Tajima, G. Guidi, and H. Umida, "Consideration about problems and solutions of speed estimation method and parameter tuing for speed-sensorless vector control of induction motor drives," *IEEE Trans. Ind. Appl.*, vol. 38, no. 5, pp. 1282–1289, 2002.

[20] W. Leonhard, *Control of Electrical Drives*. Springer, 2001.

[21] R. Marino, P. Tomei, and C. M. Verrelli, *Induction Motor Control Design*. London, UK: Springer, 2010.

[22] H. K. Khalil, *Nonlinear systems*. 3rd ed. Prentice Hall, 2002.

[23] J. Willems, *Stability Theory of Dynamical Systems*. London: Nelson, 1970.