Moving Horizon Sensor Selection for Reducing Communication Costs with Applications to Internet of Vehicles

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Heejin Ahn and Claus Danielson

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I. Introduction

In applications of the Internet of Vehicles (IoV), a large amount of data is available through communication. Using available data, the conventional Kalman filter estimates the dynamical state of systems within given error bounds with a certain probability, but requires high costs of transmitting and processing all the data. Moreover, constraints on the communication bandwidth or computation power can limit the amount of data used in state estimation. The objective of this paper is to address this issue by communicating with a subset of sensors in real-time state estimation, thereby reducing communication costs, while satisfying time-varying constraints on estimation errors.

In this paper, we propose a moving horizon sensor selection problem formulation based on the Kalman filter theory, where the problem finds an optimal set of sensors over a finite horizon. Common approaches to sensor selection problems are search tree algorithms or greedy algorithms. The work [1] exhaustively searches for an optimal solution by evaluating all of the combinations of sensors, and [2] provides efficient pruning strategies of a search tree to reduce the computation time required to obtain an optimal solution. The works [3]–[7] present algorithms that greedily choose one sensor at a time based on some metrics. For specific problems with submodular cost functions over matroids, greedy algorithms achieve performances within quantified bounds of the optimal solutions [5]–[7]. In this paper, we present a greedy algorithm that is tailored to solve our sensor selection problem. Because the goal of this paper is to reduce the data-usage, we focus on showing that our algorithm is an improvement over the currently used Kalman filter, rather than bounding the sub-optimality of our algorithm.

The sensor selection problem proposed in this paper is motivated by applications of the IoV, and differs from other sensor selection problems in the literature. Our problem is subject to constraints on error covariance matrices and minimizes the communication costs, whereas other problems typically optimize the trace, determinant, or maximum eigenvalue of error covariance matrices by using a fixed number of sensors without imposing additional constraints. Also, the constraints on estimation errors in our problem are time-varying to enable the design of a robust and less conservative controller; for instance, in path planning, if a vehicle is far away from other vehicles, the controller can tolerate large estimation errors without causing a collision. We present a heuristic algorithm that approximately solves the sensor selection problem in a greedy manner, and validate the algorithm on two motivating examples of the IoV through computer simulations. Because our approach considers linear time-invariant systems subject to Gaussian noise, we compare the performances of our heuristic with the conventional Kalman filter.

The rest of the paper is organized as follows. In Section II, we provide two examples in the IoV that motivate this paper. We formulate the moving horizon sensor selection problem in Section III and provide approximate solutions in Section IV. In Section V, we present the simulation results.

II. Motivating Examples

In this section, we provide two motivating examples in the IoV where multiple data from different sensors are available for state estimation.

A. Rear-end Collision Avoidance

Consider an ego vehicle that is controlled to maintain minimum safety distance $d_{\min}$ from a vehicle immediately in front as shown in Fig. 1(a). Let $\Delta p(t)$ and $\Delta v(t)$ be the difference in longitudinal position and speed, respectively, between the two vehicles at time instance t. The dynamics are expressed as the following linear system

$$x(t+1) = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \Delta t \end{bmatrix} u(t) + w(t), \qquad (1)$$

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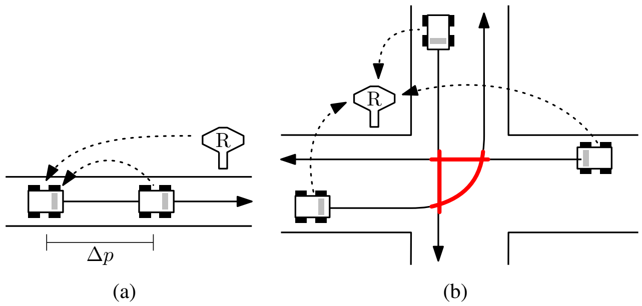


Fig. 1. (a) Estimating $(\Delta p, \Delta v)$ to prevent rear-end collisions. (b) Estimating the dynamical states of vehicles to prevent side collisions at road intersections.

where Δt is sampling-time, $x(t) = (\Delta p(t), \Delta v(t))$ is the state, $u(t)$ is the input (acceleration difference between the two vehicles), and $w(t)$ is the process noise.

The ego vehicle has access to measurements from 3 different sources:

$$y_i(t) = C_i x(t) + v_i(t), \qquad i = 1, 2, 3, \tag{2}$$

where C_i is the output matrix and $v_i(t)$ is the noise of measurement i. Measurement $i = 1$ is provided from onboard sensors. Measurements $i = 2$ and $i = 3$ are transmitted from sensors located on the preceding vehicle and on the roadside unit, respectively, accessible through communication. This is depicted in Fig. 1(a). Although our approach can handle different data types measured from different sensors as long as the system is observable, we assume in this example for simplicity that all sensors measure the state, that is, C_i is the identity matrix. Each sensor is associated with a cost ℓ_i that represents how expensive it is to transmit and process data of the sensor. For instance, since sensor $i = 1$ is located onboard, its cost is zero, $\ell_1 = 0$. The off-board sensors $i = 2$ and $i = 3$ have transmission costs $\ell_2 = 3$ and $\ell_3 = 5$, respectively.

We can easily compute a control input $u(t)$ that ensures that the ego vehicle avoids rear-end collisions and is robust to state estimation errors within given error bounds $\Omega(t)$. Error bounds are time-varying because for instance, if the ego vehicle is far from the lead vehicle (i.e., $\Delta p \gg d_{\min}$), less accuracy in state estimation is required. Because we just need the state estimation error within the error bounds, we can improve the total cost $\sum \ell_i$ by using only a subset of the sensor data.

B. Intersection Collision Avoidance

Consider a roadside unit at an intersection that coordinates three vehicles to prevent them from colliding as shown in Fig. 1(b). Let $p_i(t)$ and $v_i(t)$ are the longitudinal position and speed of vehicle i at time instance t. With the state $x_i(t) = (p_i(t), v_i(t))$, the longitudinal dynamics of vehicle i are expressed as the following linear system

$$x_i(t+1) = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} x_i(t) + \begin{bmatrix} 0 \\ \Delta t \end{bmatrix} u_i(t) + w_i(t),$$

where $u_i(t)$ is the input that the roadside unit provides to vehicle i and $w_i(t)$ is the process noise. The roadside unit has access to 12 measurements

$$y_{i,j} = C_{i,j} x_i(t) + v_{i,j}(t)$$

of the states of vehicles $i = 1, 2, 3$ measured by sensor systems $j = 1, 2, 3, 4$ located on the three vehicles and the roadside unit. Here, $v_{i,j}(t)$ is the sensor noise. Also in this example, we let $C_{i,j}$ be the identity matrix.

We can compute a control input that guarantees no side collisions at the intersection for any state estimation errors within given error bounds [8], [9]. As in the rear-end collision avoidance example, the error bounds $\Omega(t)$ are time-varying because large error bounds can be allowed when vehicles are far from the intersection. We can improve the communication costs by selecting a minimal number of the sensor data that enable the state estimation within the error bounds.

III. Problem Statement

In this section, we formulate a moving horizon sensor selection problem, following the problem setup and formulation of estimation error constraints.

A. Problem Setup

Consider the problem of estimating $\hat{x}(t)$ of the state $x(t)$ of the following linear system

$$x(t+1) = Ax(t) + Bu(t) + w(t) \tag{3a}$$

$$y_i(t) = C_i x(t) + v_i(t) \qquad i = 1, \ldots, M \tag{3b}$$

using a subset of the M sensors, where $x(t) \in \mathbb{R}^n$ is the state, $w(t) \in \mathbb{R}^n$ is the process noise, and $v_i(t) \in \mathbb{R}^{m_i}$ is the noise on the i-th sensors. The process $w(t)$ and sensor noises $v_i(t)$ are independent identically distributed, zero-mean Gaussian variables with covariances $\mathbf{E}(w(t)w(t)^\intercal) = W$ and $\mathbf{E}(v_i(t)v_i(t)^\intercal) = V_i$. Each of the measurement $y_i = C_i x + v_i$ has an associated cost ℓ_i that represents the costs of transmitting and processing data. For instance, this cost would be small for sensors located on the system and large for a high resolution image from a camera located far from the system.

At each time instance, the estimation error $e(t) = x(t) - \hat{x}(t)$ must satisfy the error bounds $e(t) \in \Omega(t) \subset \mathbb{R}^n$ (see Fig. 2). We assume that it is possible to achieve this objective by communicating with all the surrounding sensors. Since future estimation errors $e(t+k)$ will depend on the current estimation error $e(t)$, we use a moving horizon estimator to plan which sensor data will be requested. At each time instance t, we solve the following conceptual sensor selection problem

$$\min \quad \textstyle\sum_{k=0}^{T-1}\sum_{i=1}^{M} \ell_i \mu_{i,t+k} \tag{4a}$$

$$\text{s.t.} \quad e_{t+k|t} \in \Omega(t+k) \tag{4b}$$

where the binary variables $\mu_{i,t+k} \in \{0, 1\}$ indicate whether the i-th sensor data will be acquired at time $t+k$, and $e_{t+k|t}$ denotes future estimation errors at time $t+k$ predicted at time t. The cost function (4a) is the total cost, including the

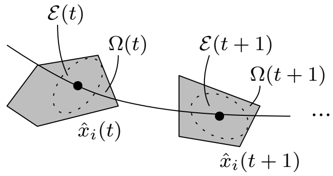


Fig. 2. Chance constraints $\mathbf{P}\big(e(t) \in \Omega(t)\big) \geq p$ given in (5) are guaranteed by the set-inclusion constraints $\mathcal{E}(t) \in \Omega(t)$ given in (6).

costs of transmitting and processing data, over the horizon T. The constraints (4b) ensure that the estimation error $e_{t+k|t}$ satisfies the future bounds $e_{t+k|t} \in \Omega(t+k)$. Notice that planning ahead is required to ensure that enough data is gathered to satisfy possibly more restrictive future bounds $\Omega(t+k) \subset \Omega(t)$. The implementation of the moving horizon estimator (4) is only conceptual since we omit how the sensor data are used to bound the estimation error $e_{t+k|t}$. One particular method is described in the following subsection.

B. Chance Constraint Formulation

One of the most common methods for state estimation is the Kalman filter. However, the Gaussian estimation errors produced by the Kalman filter are inherently unbounded. Thus, the hard constraints $e(t) \in \Omega(t)$ are often replaced by chance constraints

$$\mathbf{P}(e(t) \in \Omega(t)) \geq p, \tag{5}$$

which ensure that the constraints $e(t) \in \Omega(t)$ are satisfied with at least probability p. For the Gaussian estimation errors produced by the Kalman filter, the chance constraints $\mathbf{P}\big(e(t) \in \Omega(t)\big) \geq p$ are guaranteed by the set-inclusion constraints

$$\mathcal{E}(t) := \big\{e : e^\intercal P(t)^{-1} e \leq \alpha(p)\big\} \subseteq \Omega(t), \tag{6}$$

as illustrated in Fig. 2. Here, $P(t) = \mathbf{E}\big(e(t)e(t)^\intercal\big)$ is the covariance of the estimation error $e(t)$, and $\alpha(p)$ is obtained using the χ^2 distribution with n-degrees of freedom, which is the distribution of $\sum_{i=1}^n Z_i^2$ where $Z_1, \dots, Z_n$ are independent Gaussian random variables [10]. For instance, if $n = 2$, $\alpha(0.9) = 4.605, \alpha(0.95) = 5.991$, and $\alpha(0.99) = 9.210$. The estimation errors produced by the Kalman filter have zero mean $\mathbf{E}\big(e(t)\big) = 0$.

To easily check the satisfaction of the set-inclusion constraints (6), we provide equivalent constraints in the following proposition.

Proposition 1: If the error sets are polytopes

$$\Omega(t) = \big\{e : h_j^\intercal e \leq k_j \ \forall j \in \mathcal{J}(t)\big\},$$

then the set-inclusion constraints (6) are equivalent to the linear matrix inequalities (LMI)

$$P(t)^{-1} \succeq \alpha(p)\frac{h_j h_j^\intercal}{k_j^2}, \ \forall j \in \mathcal{J}(t). \tag{7}$$

Proof: Suppose $P(t)$ does not satisfy the LMI (7), that is, there is $j \in \mathcal{J}(t)$ and a vector e such that

$$e^\intercal P(t)^{-1} e < \frac{\alpha(p)}{k_j^2} e^\intercal h_j h_j^\intercal e.$$

Any scaled vector $\bar{e} = ce$ with a constant c also satisfies the above inequality. Moreover, for $e \in \mathcal{E}$, we can find a scaled vector $\bar{e}$ satisfying $\bar{e}^\intercal P(t)^{-1}\bar{e} = \alpha(p)$. Then, $k_j^2 < \bar{e}^\intercal h_j h_j^\intercal \bar{e}$, which implies $h_j^\intercal \bar{e} > k_j$ or $h_j^\intercal \bar{e} < -k_j$. This means that $\bar{e} \notin \Omega(t)$; for $\bar{e} \in \mathcal{E}(t)$, $h_j^\intercal \bar{e} \leq k_j$ also implies $h_j^\intercal \bar{e} \geq -k_j$ because the ellipse is centered at origin and symmetric. Therefore, $\bar{e} \in \mathcal{E}(t)$ but $\bar{e} \notin \Omega(t)$, which violates the set-inclusion constraints (6).

Suppose $P(t)$ satisfies the LMI (7), that is, for any $j \in \mathcal{J}(t)$ and for any vector e, $e^\intercal P(t)^{-1} e \geq \frac{\alpha(p)}{k_j^2} e^\intercal h_j h_j^\intercal e$. If $e \in \mathcal{E}(t)$, then $\alpha(p) \geq e^\intercal P(t)^{-1} e$. This implies $k_j^2 \geq e^\intercal h_j h_j^\intercal e$ and thus, $e \in \Omega(t)$ with the same reasoning stated above. Therefore, $e \in \mathcal{E}(t)$ implies $e \in \Omega(t)$, which satisfies the set-inclusion constraints (6). ■

C. Moving Horizon Sensor Selection Problem

Using the Kalman filter theory, we can write the moving horizon sensor selection problem (4) as follows: given $P_{t|t}^{-1}$,

$$\min \quad \sum_{k=0}^{T-1}\sum_{i=1}^{M} \ell_i \mu_{i,t+k} \tag{8a}$$

$$\text{s.t.} \quad P^{-1}_{t+k+1|t+k+1} = P^{-1}_{t+k+1|t+k} + \sum_{i=1}^{M} \mu_{i,t+k} C_i^\intercal V_i^{-1} C_i, \tag{8b}$$

$$P^{-1}_{t+k+1|t+k} = f\left(P^{-1}_{t+k|t+k}\right), \tag{8c}$$

$$P^{-1}_{t+k+1|t+k+1} \succeq \alpha(p)\frac{h_j h_j^\intercal}{k_j^2}, \ \forall j \in \mathcal{J}(t+k+1). \tag{8d}$$

where $f(Q) = A^{-\intercal}QA^{-1} - A^{-\intercal}QA^{-1}(W^{-1} + A^{-\intercal}QA^{-1})^{-1}A^{-\intercal}QA^{-1}$ is the open-loop dynamics of the inverse state covariance $P^{-1}_{t+k|t+k}$ without measurement. If $\mu_{i,t+k} = 1$ for all i, the constraints (8b) and (8c) are the same as the Kalman covariance update. The constraints (8d) guarantee that the chance constraints (5) are satisfied according to Proposition 1.

IV. Problem Solution

Finding the optimal solution of the moving horizon sensor selection problem (8) is computationally demanding since it is non-convex and contains binary variables. Instead, we use lazy and greedy heuristics to find a feasible solution to (8). Simulation results in Section V show that our heuristic, although suboptimal, provides good performances.

A. Lazy Approach

Instead of solving the problem (8) over the horizon T at once, we divide the problem into T subproblems and focus only on one time instance. Each subproblem refers to

$$\min \quad \sum_{i=1}^{M} \ell_i \mu_{i,t} \tag{9a}$$

$$\text{s.t.} \quad P^{-1}_{t+1|t+1} = P^{-1}_{t+1|t} + \sum_{i=1}^{M} \mu_{i,t} C_i^\intercal V_i^{-1} C_i, \tag{9b}$$

$$P^{-1}_{t+1|t} = f\left(P^{-1}_{t|t}\right), \tag{9c}$$

$$P^{-1}_{t+1|t+1} \succeq \alpha(p)\frac{h_j h_j^\intercal}{k_j^2}, \ \forall j \in \mathcal{J}(t+1). \tag{9d}$$

where $\mathcal{J}(t+1)$ indexes the constraints of the polytopic error bounds $\Omega(t+1)$. That is, to satisfy the error bound $e(t+k) \in \Omega(t+k)$, we lazily wait until time $t+k$ before selecting sensors. In this paper, we assume that enough data is available to satisfy constraints (9d) at every stage of problem (8). In other words, $\mu_{i,t} = 1$ for all i is always a feasible solution of the subproblem (9). This assumption ensures the feasibility of all subsequent subproblems, thereby making the solutions to the subproblems (9) a feasible solution to the problem (8). Hence, the rest of this paper focuses on solving the subproblem (9).

B. Greedy Approach

We present a greedy heuristic for solving the single-stage sensor selection problem (9). Using the Schur Complement [11], the matrix positive-semidefinite constraints (9d) are equivalent to the following scalar nonnegative constraints

$$\begin{aligned} &\begin{bmatrix} P^{-1}_{t+1|t+1} & h_j \\ h_j^{\mathsf{T}} & k_j^2/\alpha(p) \end{bmatrix} \succeq 0, \ \forall j \in \mathcal{J}(t+1) \\ \Leftrightarrow\ & \frac{k_j^2}{\alpha(p)} - h_j^{\mathsf{T}} \left(P^{-1}_{t+1|t+1}\right)^{-1} h_j \geq 0, \ \forall j \in \mathcal{J}(t+1), \end{aligned} \tag{10}$$

where $P^{-1}_{t+1|t+1} \succ 0$ is positive definite and $k_j^2/\alpha(p) > 0$ is positive. We define a slack on the j-th constraint as

$$s_{j,t}(Q) := \frac{k_j^2}{\alpha(p)} - h_j^{\mathsf{T}} Q^{-1} h_j.$$

If $s_{j,t}(Q) \geq 0$, the j-th constraint in (9d) is satisfied, that is, $\{e : e^{\mathsf{T}} Q e \leq \alpha(p)\} \subseteq \{e : h_j^{\mathsf{T}} e \leq k_j\}$ according to (10) and Proposition 1.

Using the inequality (10), we define a reward metric $r_{i,t}(Q)$ on the i-th sensor as the maximum difference between the nominal constraint slack $s_{j,t}(Q)$ and the slack after the removal of the i-th sensor $s_{j,t}(Q - C_i^{\mathsf{T}} V_i^{-1} C_i)$, that is,

$$r_{i,t}(Q) := \max_{j \in \mathcal{J}(t+1)} h_j^{\mathsf{T}} (Q - C_i^{\mathsf{T}} V_i^{-1} C_i)^{-1} h_j - h_j^{\mathsf{T}} Q^{-1} h_j.$$

Notice that because the slack is reduced as sensors are taken away, the metric is always nonnegative. The quantity $r_{i,t}(Q)$ measures how the removal of the i-th sensor's data affects the violation of the error constraints $\Omega(t+1)$. The smallest $r_{i,t}(Q)$ means that removing the sensor has least effect on slacks.

Our heuristic greedily removes the sensor that has the minimum reward-relative-to-cost ratio, $r_{i,t}(Q)/\ell_i^2$. The detailed heuristic is provided as follows. At each iteration of the heuristic, I_{used} is a set of sensors that are to be communicated with at time t, and Q is the inverse of *a posteriori* covariance matrix using the sensors in I_{used} (i.e., $Q = f(P^{-1}_{t|t}) + \sum_{i \in I_{\text{used}}} \mu_{i,t} C_i^{\mathsf{T}} V_i^{-1} C_i$).

- **Initialization**: Start with $I_{\text{used}} = \{1, \dots, M\}$. Let $Q = f(P^{-1}_{t|t}) + \sum_{i=1}^{M} C_i^{\mathsf{T}} V_i^{-1} C_i$.
- **Iteration**:
  - *Termination*: If $\min_j s_{j,t}(Q - C_i^{\mathsf{T}} V_i^{-1} C_i) < 0$ for all $i \in I_{\text{used}}$, then terminate the iteration and return I_{used}.
  - *Sensor selection*: Among i in I_{used} satisfying $\min_j s_{j,t}(Q - C_i^{\mathsf{T}} V_i^{-1} C_i) \geq 0$, select the index $i_{\min}$ corresponding to the minimum of $r_{i,t}(Q)/\ell_i^2$.
  - *Update*: Let $I_{\text{used}} = I_{\text{used}} \setminus i_{\min}$ and $Q = Q - C_{i_{\min}}^{\mathsf{T}} V_{i_{\min}}^{-1} C_{i_{\min}}$.

We refer to this heuristic as *greedy subtraction heuristic* because a sensor is subtracted from the set I_{used} at each iteration.

The termination of iterations is determined by the sign of the slack $\min_j s_{j,t}(Q - C_i^{\mathsf{T}} V_i^{-1} C_i)$, which indicates the satisfaction of the estimation error constraints (9d) after the removal of the i-th sensor. For instance, if it is negative, then removing the i-th sensor will cause the estimator error to violate the bounds $\Omega(t+1)$ according to (10). For sensor with a positive slack, the sensor can be removed without violating constraints. The iteration in the heuristic terminates when there are no sensor that can be removed from I_{used} without violating constraints.

Suppose we have $r_{j,t}(Q) > r_{i,t}(Q) > 0$. This tells that removing the i-th sensor has less effect on the constraint slack than the j-th sensor. If the two sensors have the same communication cost, that is, $\ell_i = \ell_j$, then removing the i-th sensor is preferable because it is then more likely that we will be able to remove an additional sensor in the next iteration. Here, we consider ℓ_i^2 rather than ℓ_i to balance the weights of the contribution to the constraints and the associated cost because the slack $s_{j,t}(Q)$ is defined in terms of the square of k_j, which defines half-spaces of the constraints (9d).

We implement the subtraction heuristic in a receding horizon fashion. That is, at time instance t, we use the heuristic to solve the T subproblems (9) and apply the first sensor selection ($\mu_{i,t}$ for all i). At the next time instance $t+1$, we again use the heuristic to solve the subsequent T subproblems.

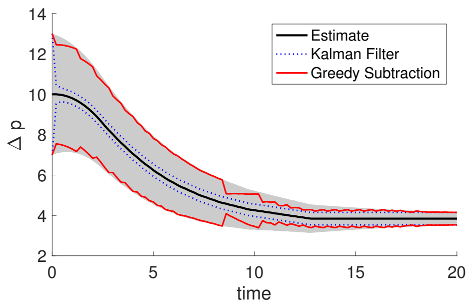
C. Other Heuristics

There are other possible heuristics to solve the subproblem (9). One heuristic is to add sensors that correspond to the maximum value of $r'_{i,t}(Q)/\ell_i^2$ until the constraints are satisfied, where the metric $r'_{i,t}$ is defined as the minimum difference between the slack after adding the i-th sensor $s_{j,t}(Q + C_i^{\mathsf{T}} V_i^{-1} C_i)$ and the nominal slack $s_{j,t}(Q)$. That is,

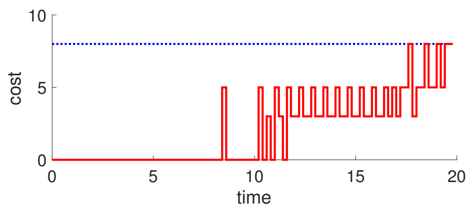
$$r'_{i,t}(Q) := \min_{j \in \mathcal{J}(t+1)} h_j^{\mathsf{T}} Q^{-1} h_j - h_j^{\mathsf{T}} (Q + C_i^{\mathsf{T}} V_i^{-1} C_i)^{-1} h_j.$$

Notice that because the slack increases as sensors are added, the metric is always nonnegative. In the heuristic iterations, Q is initially $f(P^{-1}_{t|t})$ and increases to $Q + C_i^{\mathsf{T}} V_i^{-1} C_i$ as more sensors are added, and we pick the sensor with the maximum value of $r_{i,t}(Q)/\ell_i^2$ because we want to increase the slack as much as possible (to a positive value) with smallest costs by adding sensors. The iteration terminates when $\min_j s_{j,t}(Q)$ becomes positive. We refer to this heuristic as *greedy addition heuristic*.

Another heuristic is to add sensors randomly by choosing one sensor at a time until the constraints are satisfied.



(a) Position difference estimate and estimation errors.



(b) Costs of the Kalman filter and our heuristic.

Fig. 3. Greedy subtraction heuristic yields the estimation error within the given bounds while requiring a significantly lower cost than the conventional Kalman filter.

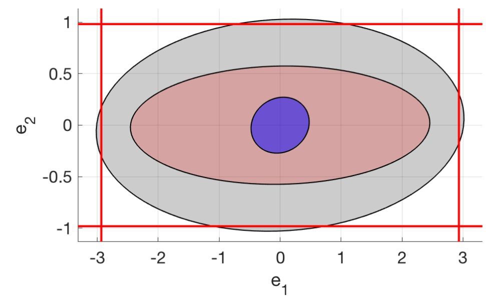


Fig. 4. While the prior distribution (large ellipse) violates the set-inclusion constraints (6) (i.e., being inside the rectangle defined by the four red lines), the posterior distributions resulted from our subtraction heuristic (medium ellipse) and from the Kalman filter (small ellipse) satisfy the constraints.

We refer to this heuristic as *random heuristic*. In the next section, we compare the greedy subtraction heuristic with these heuristics.

V. Simulation Results

We implement the greedy subtraction heuristic in the two motivating examples given in Section II, and compare the results with the state-of-the-art results that are based on the Kalman filter using all available sensors. We also show via computational experiments that the greedy subtraction heuristic exhibits better performances, in the sense that its solutions are close to the optimal solutions in most of cases, than the other two heuristics described in Section IV-C.

A. Rear-end Collision Avoidance

At generic time instant 0, we compute a robust control input that maintains the minimum safety distance, $d_{\min} = 3$, between the ego vehicle and the preceding vehicle. In particular, given estimation error bounds $\Omega(t)$, the control input is robust in the sense that the position between the two vehicles, denoted by Δp, is no smaller than $d_{\min} = 3$ for any bounded errors. In Fig. 3(a), the estimate of Δp and the estimation error bounds $\Omega(t)$ are represented by the black solid line and gray region surrounding the line, respectively. The ego vehicle, which is initially far from the preceding vehicle, is controlled to get closer, but not closer than the minimum safety distance of $d_{\min} = 3$, for any estimation error within the bound with at least probability of $p = 0.95$. The system's performance (safety in this case) is guaranteed with high probability.

In Fig. 3(a), the blue dotted lines around the estimate define a band in which the actual value exists with at least probability of $p = 0.95$ if we update the posterior distribution using the Kalman filter based on all available sensors (three measurements in this example). Similarly, the red solid lines around the estimate represent the error band given by the greedy subtraction heuristic. The Kalman filter error band is contained by that of the greedy subtraction heuristic. Notice that both error bands are inside the given error bounds, thereby satisfying the chance constraints (5), and our approach yields much better communication costs as shown in Fig. 3(b).

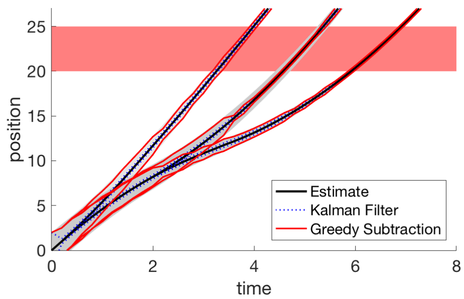
This result is also represented graphically in Fig. 4. The gray large ellipse is $\{e = (e_1, e_2) | e^\intercal P_{1|0}^{-1} e \leq \alpha(0.95)\}$ based on the prior distribution at time 0, and it violates the set-inclusion constraints (6), where $\Omega(1)$ is the rectangle defined by the four red lines. Our heuristic selects a minimal number of sensors to yield the posterior distribution and gives the red medium ellipse, $\{e : e^\intercal P_{1|1}^{-1} e \leq \alpha(0.95)\}$, which satisfies the constraints. The conventional Kalman filter uses all the sensors to yield the posterior distribution corresponding to the blue smallest ellipse. Again, the Kalman filter ellipse is always a subset of the ellipse resulted from our heuristic, and both ellipses satisfy the constraints.

B. Intersection Collision Avoidance

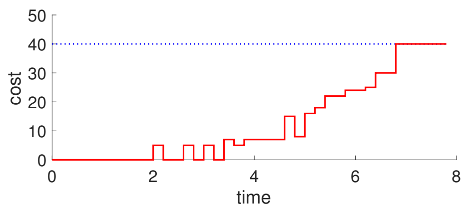
At generic time instant 0, we compute a robust control input that makes the three vehicles in Fig. 1(b) cross the intersection without conflicts for any estimation error within given bounds. Fig. 5(a) shows the position estimate trajectories of the three vehicles (black lines). Any two positions within the error bounds (gray region) are not simultaneously inside the intersection located between 20 and 25. The error bands of the Kalman filter and our greedy subtraction heuristic around the estimate are contained in the error bounds $\Omega(t)$, which confirms that they satisfy the chance constraints (5). Fig. 5(b) shows that our approach significantly reduces the total communication cost; the cost of the Kalman filter is 40 at every time step while the cost of our heuristic has the average of 13.2.

C. Comparison with Other Heuristics and Optimal Solution

We compare the costs of exhaustive search (optimal solutions) and the greedy subtraction, greedy addition, random heuristics described in Section IV on a set of 1,000



(a) Position estimate and estimation errors.



(b) Costs of the Kalman filter and our heuristic.

Fig. 5. Three vehicles in Fig. 1(b) cross the intersection located between 20 and 25 without collisions, and the estimation error of the subtraction heuristic is within the given bounds with significantly reduced communication costs.

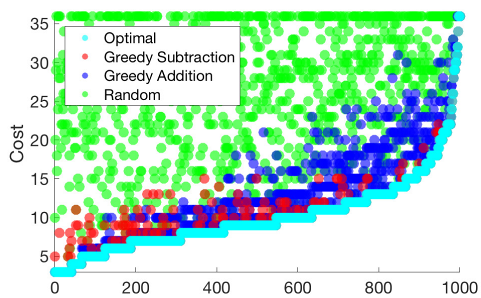


Fig. 6. Costs of different approaches on 1,000 sample data.

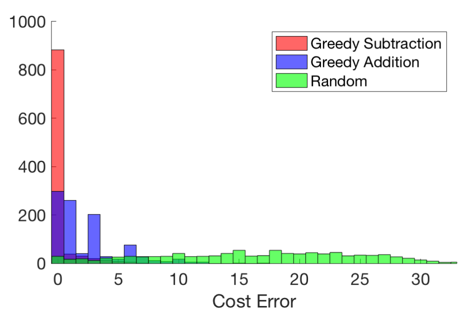


Fig. 7. Histogram of the errors between the optimal costs and the resulting costs.

simulation cases. In each case, the noise covariance V_i for $i \in \{1, 2, \ldots, 8\}$ (8 measurements) and *a priori* covariance matrix $P^{-1}_{t+1|t}$ are randomly generated.

Fig. 6 shows the costs of each approach on 1,000 simulation cases. The optimal solution yields the smallest cost, the greedy subtraction and addition heuristics result in costs close to the optimal costs, and the random heuristic exhibits random performances. Fig. 7 shows the histograms of the cost differences between the optimal and heuristic solutions. The greedy subtraction heuristic yields the optimal costs in 88.3 % of the cases and has a distribution that gives the maximum cost error of 7. The greedy addition heuristic obtains the optimal costs in only 29.8 % of the cases and exhibits a distribution that gives the maximum error of 17. These results show that our greedy subtraction heuristic outperforms the other heuristics, and is able to obtain the optimal solutions in most of cases.

VI. Conclusions

In this paper, we have formulated the moving horizon sensor selection problem for estimating the vehicle states within desired time-varying error bounds in the IoV, to reduce the total cost associated with acquiring and processing data. Rather than exactly solving the problem, which requires significant computation time, we adopt a greedy heuristic approach to approximately solve the problem. Via computer simulations, we have shown that our approach exhibits significant cost reduction compared to the state-of-the-art Kalman filter approach and the other heuristics.

In order to further develop the approach, we plan to exploit the moving horizon structure to update the control input and state estimate at each time instance. Also, quantifying the approximation error of our approach is interesting future work.

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