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Learning Heuristic Functions for Mobile Robot Path Planning Using Deep Neural Networks

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Abstract

Resorting to certain heuristic functions to guide the search, the computation efficiency of prevailing path planning algorithms such as A*, D* and their variants is solely determined by how good the heuristic function approximates the true path cost. In this study, we propose a novel approach to learn heuristic functions using a deep neural network (DNN) to improve the computation efficiency. Even though DNNs have been widely used for object segmentation, natural language processing, and perception, their role in helping to solve path planning has not been well investigated. This work shows how DNNs can be applied to path planning problems and what kind of loss functions is suitable for learning such a heuristic. Our preliminary results show that an appropriately designed and trained DNN can learn a heuristic which effectively guides conventional path planning algorithms and speeds up the path generation.

Introduction

Efficient path planning is required for many applications such as self-driving cars and home service robots. A variety of path planning algorithms have been proposed, yet heuristic learning using deep neural networks (DNNs) have not been studied well. Deep learning has been successful in a variety of applications, to name a few: object recognition (Krizhevsky, Sutskever, and Hinton 2012), video games (Mnih et al. 2015) and image generations (Goodfellow et al. 2014). Inspired by the object segmentation and the image generation algorithms such as U-Net (Ronneberger, Fischer, and Brox 2015) and Generative Adversarial Network (GAN) (Goodfellow et al. 2014), we investigate how to apply these techniques to path planning. We focus on learning a heuristic function instead of taking end-to-end approaches that map sensory inputs to actions directly so that we can still use the current well developed search-based path planning algorithms. This paper shows a potential direction of heuristic learning for path planning. Our contributions are as follows.

- We applied a neural network architecture commonly used in semantic segmentation in order to learn heuristic functions for path planning.
- We investigated loss functions that are suitable for learning heuristic functions.
- We showed that the learned heuristic functions can reduce the search cost in many scenarios in two domains: 2D grid world and continuous domain.

The overall framework is outlined in Figure 1 and is explained in the section of our proposed method.

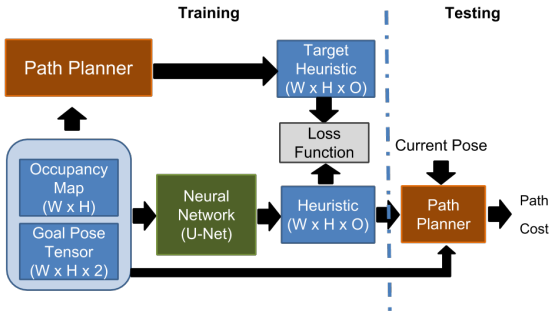

Figure 1: Learning heuristic functions for path planning.

Related Work

Path Planners

A variety of search algorithms have been proposed such as A* (Hart, Nilsson, and Raphael 1968), Anytime Repairing A* (ARA*)(Likhachev, Gordon, and Thrun 2004), Rapidly-exploring Random Tree (RRT) (LaValle 1998), Hybrid-A* (Dolgov et al. 2008), local-minima-free potential field (Connolly and Grupen 1993), and two-stage RRT (Wang, Jha, and Akemi 2017). Heuristic-based methods often use admissible hand-crafted heuristic functions such as Manhattan distance, Euclidean distance, and length of Reed-Shepp paths. Cost functions of the states can be also created from the topology of the state (Mahadevan and Maggioni 2007) (Connolly and Grupen 1993) and can be learned from demonstration (Ratliff, Bagnell, and Zinkevich 2006). Although optimality can be achieved with admissible heuristics, the complexity can be beyond control with complex environments. Anytime Repairing A* (ARA*) (Likhachev, Gordon, and Thrun

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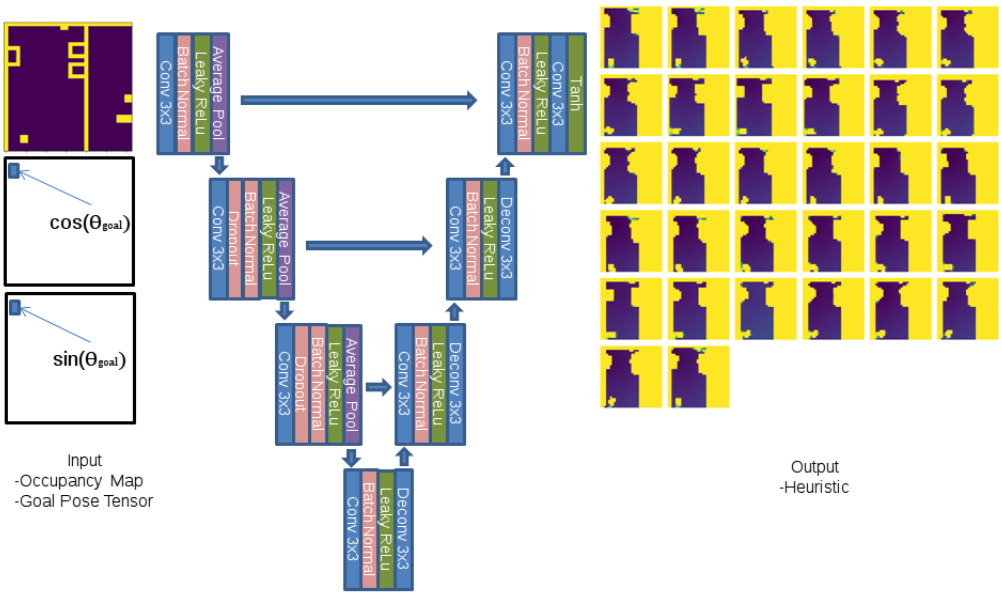


Figure 2: Network architecture used in our experiments (continuous domain)

2004) uses a scaling factor to reduce the complexity by adjusting the upper bound of the cost of a path. Hybrid-A* uses the max of two different admissible heuristics to guide grid cell expansion albeit it does not preserve completeness and optimality. Two-stage RRT (Wang, Jha, and Akemi 2017) uses two RRTs. The upper stage uses a low-cost RRT without the robot kinematics to produce waypoints, and the waypoints guide the other RRT that considers the robot kinematics.

Navigation with Neural Networks

The recent advancement of deep neural networks enables learning navigations directly from high dimensional sensory inputs. Banino *et al.* show that neural networks for Simultaneous Localization and Mapping (SLAM) can learn grid-like representations that appear in animal brains (Banino et al. 2018). Pan *et al.* use imitation learning for a real autonomous car to drive off-road at high speed using raw RGB images with wheel speeds. Model Predictive Control (MPC) experts were used to provide training data.

Deep reinforcement learning has been used to learn a function that maps raw sensory inputs (images) to actions. However, state-of-the-art algorithms have only been tested in simple domains with a little variation of the environment (Tamar et al. 2016) (Khan et al. 2017) (Panov, Yakovlev, and Suvorov 2018). This is mainly due to the necessity of very long training time. Tamar *et al.* propose Value Iteration Networks (VIN) that computes value iteration by applying convolution kernels that represent learned control dynamics. The results show that their network is better than a reactive policy that is learned by standard convolutional neural networks (Tamar et al. 2016). Khan *et al.* combine Value Iteration Network with Differential Neural computing to learn navigation (Khan et al. 2017). Panov *et al.* apply DQN (Deep Q-Network) to solve 2D grid navigation using a reward that is a function of optimal paths (Panov, Yakovlev, and Suvorov 2018). Zhang *et al.* use neural networks with external memory for SLAM (Zhang et al. 2017). They showed that their neural SLAM helps a reinforcement learning algorithm to learn navigation tasks. Imitation learning with neural networks is used to reduce the search cost (Bhardwaj, Choudhury, and Scherer 2017). They learn a heuristic policy for each domain separately. The algorithm was tested on simple domains but relatively high dimensional states with the fixed start and the goal. Once start positions, goal positions and environments are randomized, local features may not be able to guide the planner to a goal efficiently as discussed in (Dhiman et al. 2018). On the contrary, we learn a heuristic function defined over the entire environment given the goal configuration and the map so that we can avoid the misguiding problem. In addition, we need only one feed-forward computation of a neural network to obtain the heuristic function. In our toy domain, we learn a heuristic function that can be applied to six different environments rather a specific environment. We also consider environments where a path does not exist while the work on navigation with deep learning cited above often consider only environments where a path always exists. This is important to investigate behaviors of planners in terms of the completeness.

Proposed Methods

Task

We work on a problem where a robot finds a path $\pi : [0, 1] \mapsto C_{free}$ from a start configuration $\pi(0) := s_0 \in C_{free}$ to a goal configuration $\pi(1) := s_{goal} \in C_{free}$ given a map of an environment m and a heuristic function h where C_{free} is free space. Our goal is to learn a heuristic function h using DNNs to reduce the search cost. Our overall approach

is shown in Figure 1. We first introduce the network architecture we adopted, and then we explain loss functions and training procedures.

Network Architecture

We adopt U-Net (Ronneberger, Fischer, and Brox 2015) that was initially proposed for object segmentation problems. The architecture has recently been used as a generator in GAN architecture to solve image-to-image translation (Isola et al. 2017). We use the U-Net architecture to generate heuristic values. U-Net achieves the precise image generation by using the information from early convolutional layers directly to the latter layers. However, our approach does not limit to U-Net, and other image generation networks can be used. Figure 2 shows the architecture of the network. The input is an occupancy map and a goal tensor G. A goal tensor consists of two matrices that represent $s_{goal} = (x_{goal}, y_{goal}, \theta_{goal})$ where (x_{goal}, y_{goal}) is a goal position and θ_{goal} is a goal orientation of a robot. Let (i_{goal}, j_{goal}) be a position in the occupancy map that contains (x_{goal}, y_{goal}). We define the goal tensor G as follows.

$$G(0,i,j) = \begin{cases} cos(\theta_{goal}), & \text{if } i = i_{goal} \text{ and } j = j_{goal} \\ 0, & \text{otherwise} \end{cases}$$

$$G(1,i,j) = \begin{cases} sin(\theta_{goal}), & \text{if } i = i_{goal} \text{ and } j = j_{goal} \\ 0, & \text{otherwise} \end{cases}$$

It is possible that we also discretize θ, but this increases the input dimensions. We use $cosine$ and $sine$ instead of the raw value in radian to compute the proper distance between two orientations (for example, the distance between 2π and 0 should be 0 instead of 2π). The output is a discretized heuristic values that is a tensor of $W \times H \times O$ where W and H is the number of columns and rows in the occupancy map respectively, and O is the number of orientations.

Loss Functions

The objective of the path planning problem is to find the shortest path with a constraint on search cost. The path cost can be defined as a function of path length or the number of actions. The search cost can be defined as a function of the number of expanded nodes during the search process. In this paper, we work on a research question of whether we can learn a heuristic function that is better than hand-crafted heuristics regarding the search cost.

We first explain why learning a heuristic function is not a straight-forward supervised learning problem, and then explain why the choice of the loss function is important.

Let L be the cost of a path generated by a planner and T be the search cost of the planner given m, s_0, s_{goal}, and h. The objective function for learning a heuristic function, $J(h)$, is a reward function of a path cost and/or a search cost. $J(h)$ satisfies the following condition.

$$\text{If } L_1 \geq L_2 \text{ and } T_1 \geq T_2 \text{ , then } J(h_1) \leq J(h_2) \quad (1)$$

Equation 1 means that h_2 has a better reward if both of the path cost and the planning cost are lower than those of h_1. This reward can be computed by solving path planning problems with a path planner and h. Thus J is in general not differentiable. In order to compute J appropriately, we need to solve a path planning problem using a planner with h. Hence, evaluating J is computationally expensive. Non-differentiability and expensive computation make it challenging to learn heuristics in an end-to-end learning manner.

Let $d(\cdot)$ be a loss function, h_1 and h_2 be learned heuristics and h^* be an optimal heuristic. We can reduce the problem above to a supervised learning problem whose objective is to minimize errors between learned heuristics and optimal heuristics if the following condition is satisfied (Barto and Dietterich 2004).

$$\text{If } d(h_1, h^*) \leq d(h_2, h^*), \text{then } J(h_1) \geq J(h_2) \quad (2)$$

However, standard loss functions such as mean squared error (MSE) or mean absolute error (MAE) do not satisfy this condition although lower loss usually indicates higher J. Figure 3 shows one simple example in which Equation 2 does not hold. In this example, both a heuristic function h_1 and a heuristic function h_2 can find an optimal path, but the search cost is different. h_1 has better MSE and MAE, but it needs to expand more nodes. Therefore, it is essential to understand what loss functions are suitable so that learning a heuristic can be treated by solving the supervised learning problem (minimizing $d(\cdot)$) rather than solving the original computationally expensive optimization problem (maximizing $J(\cdot)$). We first introduce the standard loss functions and then propose new loss functions for this heuristic learning problem.

MSE between two tensors h and h^* is defined as follows

$$MSE(h, h^*) = \frac{1}{N} \sum_{i=0}^{N-1} \left(h(i) - h^*(i)\right)^2 \quad (3)$$

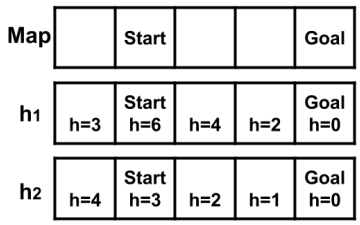
where h is a learned heuristic, h^* is an optimal heuristic and N is the number of elements in the tensor.

MAE is computed as follows

$$MAE(h, h^*) = \frac{1}{N} \sum_{i=0}^{N-1} |h(i) - h^*(i)| \quad (4)$$

With MAE, we equally punish the errors in all directions. We, however, want to generate heuristics which are between known lower bound of the cost and the optimal cost. Thus, we can penalize more if the learned heuristic is out of the range between these bounds. Especially if the learned heuristic is larger than the optimal cost, it does not guarantee the optimality anymore. Hence, we propose the following piecewise MAE.

$$\begin{aligned} Loss_{piece}(h, h^*) = \frac{1}{N} \Bigg(& \\ & \alpha_1 \sum_{i=0}^{N-1} |h(i) - h^*(i)| \left[h(i) < h_{min}(i)\right] \\ & + \sum_{i=0}^{N-1} |h(i) - h^*(i)| \left[h_{min}(i) \leq h(i) \leq h^*\right] \\ & + \alpha_2 \sum_{i=0}^{N-1} |h(i) - h^*(i)| \left[h^*(i) < h(i)\right] \Bigg) \end{aligned}$$



	MSE	MAE	Search Cost
h_1	5.0	1.0	5
h_2	5.8	1.4	4

Figure 3: Equation 2 does not satisfy in this example. We have a 1D map that contains five nodes. A robot can move either left or right. A cost of an action is 2. h_1 and h_2 represent some learned heuristics. h in each node represents a value of the heuristic function. Mean squared error (Equation 3) and mean absolute error (Equation 4) between the heuristic and the optimal heuristic are shown in the table on the right. The search cost is the number of expanded nodes when we apply A^* algorithm. h_2 has more errors than h_1, but h_2 has less search cost.

where h_{min} is a lower bound of the cost, $[\cdot]$ is an indicator function, $\alpha_1 \geq 1$ and $\alpha_2 \geq 1$ are positive constants. The lower bound can be usually computed by considering the environment without obstacles. The first term computes the sum of absolute difference between the learned heuristic and the optimal heuristic if the learned heuristic is lower than the minimum cost at state. Similarly, the second term computes the sum of the absolute difference if the heuristic is between the minimum cost and the optimal cost. The third term is computed if the heuristic is more than the optimal cost. If $\alpha_1 = 1$ and $\alpha_2 = 1$, this loss function is reduced to MAE.

It is in general hard to learn the optimal heuristic for every task; thus the learned heuristic does not necessarily preserve gradients of the heuristic with respect to actions as you can see in Figure 3. We use the following loss function to capture errors on the gradients.

$$Loss_{grad}(h, h^*) = \sum_{a \in A} MAE(K_a * h, K_a * h^*) \tag{5}$$

where $*$ operator denotes a discrete convolution operation. $K_a * h$ approximates the gradient of h with respect to $a \in A$ where A is an action set. This operation is similar to Sobel-Feldman operator (Sobel 1968) that captures gradients of an image. For example , K_a for "*north*" action in a 2D grid world is as follows

$$K_{north} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We use the weighted sum of these loss functions for our neural network.

$$Loss = Loss_1 + \alpha Loss_{grad} \tag{6}$$

where α is a constant that controls the importance of $Loss_{grad}$. We use MSE, MAE or $Loss_{piece}$ as $Loss_1$. Both $Loss_1$ and $Loss_{grad}$ are differentiable, and we can apply backpropagation to update the weights of the network.

Instead of using some handcrafted properties like the gradient described above, it may be possible to use another DNN to learn properties associated with optimal heuristics automatically. Thus, we investigate the use of GAN for this purpose. We introduce another DNN to discriminate the learned heuristic with the optimal heuristic. We train a generator network (U-Net) that generates heuristic values which minimize $Loss$ and fools a discriminator network f_D. We use a loss from a discriminator of Wasserstein GAN (WGAN)(Arjovsky, Chintala, and Bottou 2017). In our case, we use the goal pose tensor and a heuristic function as an input to the discriminator. Then, a loss function for the discriminator tensor $Loss_{des}$ is computed as follows.

$$Loss_{des} = f_D(h, G) - f_D(h^*, G) \tag{7}$$

where h is the learned heuristic (output of the generator network) and G is a goal tensor. The learned heuristics are fake examples and the optimal heuristics are the real examples. We train the discriminator network using $loss_{des}$, and train the generator network (U-Net) using the loss function of WGAN $Loss_{WGAN}$, $Loss_1$ and $Loss_{grad}$.

$$Loss_{WGAN} = -f_D(h, G) \tag{8}$$

The loss function from WGAN computes the loss regarding the properties of optimal heuristic. We use a weighted sum of loss functions to train the generator network.

$$Loss = Loss_1 + \alpha Loss_{grad} + \beta Loss_{WGAN} \tag{9}$$

where β is a constant. The discriminator network architecture we use in our experiments is as follows: Input - Convolution - Convolution - Average Pooling - Dropout - Dense - ReLU - Dense - ReLU - Dense - ReLU - Dense - Output. One of the major drawbacks of WGAN is that it needs twice as much time and memory as other methods in the training stage.

Training

We make environments with randomized start and goal poses. Then, we create optimal heuristic h^* using Dijkstra algorithm starting from s_{goal} assuming that each action has an inverse action. Then, we train the network with h^*. We use Adam optimization to train the network (Kingma and Ba 2014). To evaluate the network, we generate h using the network, and pass it to a path planner (either A^* or hybrid A^*), and measure the search cost. As we explained earlier, the lower loss does not necessarily mean the better performance, and we need to use path planners with the learned heuristic to evaluate the network.

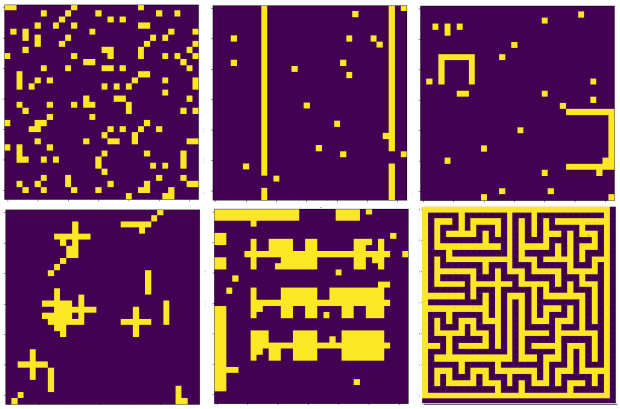


Figure 4: Examples of environments used in toy domain.

Experiments

We consider two simulated domains: toy 2D domain and continuous 2D domain.

Toy domain

Toy domain is a simple 2D grid world. A robot can move either north, west, east or south. The robot volume is equivalent to one cell. There is no orientation. Thus, we use a one-hot matrix to represent the goal position instead of using the goal tensor explained earlier for the input. We generated 180,000 training examples from six different environments (Figure 4). We used different rules to create such environments with random parameters. We also generated 10,000 test examples. We used Manhattan distance and scaled Manhattan distance (inflation factor is 1.5) as baselines. We introduce *difficulty* of a task as follows to evaluate performance in toy domains:

$$\textit{difficulty} = \frac{h_{Opt}(s_0)}{h_{Base}(s_0)} \tag{10}$$

where h_{Base} is a commonly used simple heuristic function that is admissible and consistent such as Manhattan distance or Euclidean distance in our task. We use this criterion for a fair comparison against baselines because baselines work well in simple environments. *difficulty* measures how the optimal cost at s_0 deviates from the estimated cost (heuristic). If this value is large, the estimated cost is much smaller than the optimal cost. Hence, a simple heuristic may not be able to guide the planner appropriately. We use MAE, MSE, $Loss_{grad}$, $Loss_{piece}$ and $Loss_{WGAN}$ for loss functions. There will be a number of combinations of these, and we investigate some of the combinations. We use $\alpha_1 = 1.0$, $\alpha_2 = 2.0$, $\alpha = 1.0$ (when we use $Loss_{grad}$) and $\beta = 1.0$ (when we use $Loss_{WGAN}$). We selected these hyper parameters after we tried several values.

Continuous domain

We consider a four-wheeled robot. The environment is a $2.318m \times 2.318m$ parking lot. We randomize occupied parking spaces and added small obstacles randomly. The robot size is $0.25m \times 0.20m$. We use a bicycle model (Rajamani 2011) to describe the robot's kinematics. The robot uses five steering angles for both forward and backward direction with the fixed speed. There are ten actions in total. Collision checkings are performed in continuous space. To plan a path, we use a hybrid A^*. This planner does not hold optimality anymore, but it can still produce sub-optimal drivable paths. We discretize states (x, y, θ) into $32 \times 32 \times 32$ states for the hybrid A^*. We generate 3040 training samples and 130 test samples. We also randomize s_0 and s_{goal} in continuous space. The robot reaches s_{goal} when the robot reaches a voxel containing s_{goal}. Hence, the robot's orientation must be close to the goal orientation. We use $\alpha_1 = 1.0$ and $\alpha_2 = 2.0$ for $Loss_{piece}$ and $\alpha = 0.01$ for $Loss_{grad}$. We do not use the *difficulty* measure because hybrid A^* is not guaranteed to produce an optimal solution or to find a path even if a path exists. We do not investigate WGAN loss in this domain because it does not bring many benefits in toy domain (we describe it in the next section) despite the fact that it takes very long time to train.

Results and Discussion

Toy domain

Figure 5 shows successful cases and Table 1 summarizes results. As in Figure 5, a planner with the learned heuristic function does not need to expand many nodes (green stars) if the solution exists. If there is no path, learned heuristic produces high values for unreachable states from the goal. Note that we used a single learned network to generate heuristic values in all testing environments. Hence the results also show the generalizability of our proposed algorithm. In Table 1, the bold numbers show that the learned heuristic function outperforms baselines regarding the search cost. The gradient loss helps MSE and MAE to reduce the search cost. Use of GAN also contributes to reducing the search cost of MSE and MAE. Piecewise loss works better especially when tasks are difficult. Although WGAN improves the performance a little, it needs twice as much training time and memory as other methods. We do not provide the statistical testing because the number of testing examples is too large for the meaningful statistical testing for our experiments and a small difference can easily lead to the statistical significance (Lin, Lucas Jr, and Shmueli 2013). Such statistical significance does not indicate meaningful results. Thus, we show the mean of the ratio of the number of expanded nodes only. When the tasks are easy (*difficulty*$= [1.0, 1.2]$), scaled Manhattan distance is better than learned heuristics.

Contrary to computer vision tasks such as object segmentation and object recognition, it is possible that a change of one cell can change the optimal heuristic drastically. Hence, learning heuristic functions using DNN can have a difficulty that does not appear in other DNN applications. As the results show, even we randomize goal positions, the neural network can still learn heuristic that is better than baselines. Thus, our results indicate that DNN is capable of dealing with such difficulty. There are some cases that the learned heuristic does not produce good results as shown in Figure 6. The number of unsuccessful examples can be reduced by

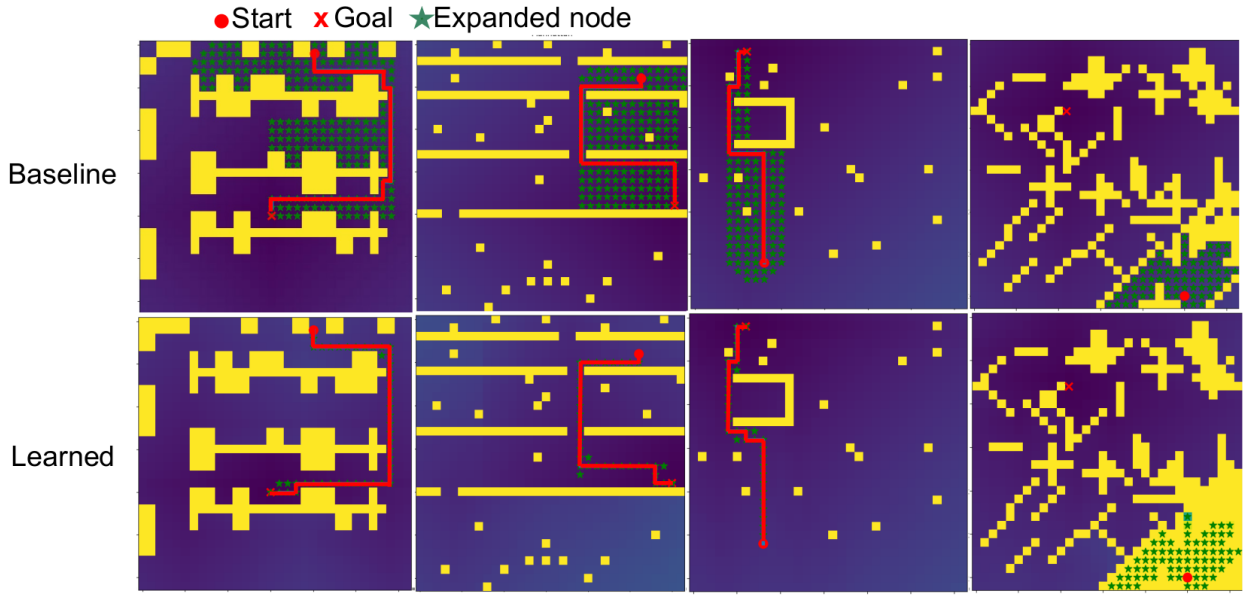


Figure 5: Successful results in toy domains. The top row shows results of baselines that use Manhattan distance, and the bottom row shows results of the learned heuristic that used our proposed piecewise loss function. The color of each cell represents the value of heuristic. Dark blue represents small heuristic. Yellow indicates the highest heuristic, and it usually means the obstacles or unreachable region from a goal. Red circles are s_0, red crosses are s_{goal} and green stars are expanded nodes in A^*. The rightmost column shows an environment where a path does not exist.

Loss	Complexity Evaluation Ratio of the number of expanded nodes on different task difficulties									
	1.0-1.2	1.2-1.4	1.4-1.6	1.6-1.8	1.8-2.0	2.0-2.2	2.2-2.4	2.4-2.6	2.6-2.8	≥ 2.8
MSE	0.64	0.57	**0.56**	**0.57**	**0.56**	**0.56**	**0.60**	**0.64**	**0.65**	**0.71**
MSE, $loss_{grad}$	0.50	**0.42**	**0.41**	**0.40**	**0.46**	**0.46**	**0.48**	**0.53**	**0.53**	**0.62**
MAE	0.57	**0.46**	**0.45**	**0.42**	**0.48**	**0.51**	**0.58**	**0.60**	**0.59**	**0.62**
MAE, $loss_{grad}$	0.60	**0.45**	**0.45**	**0.46**	**0.47**	**0.51**	**0.51**	**0.60**	**0.55**	**0.65**
MSE, $loss_{WGAN}$	0.58	0.53	**0.52**	**0.49**	**0.47**	**0.50**	**0.51**	**0.53**	**0.63**	**0.71**
MAE, $loss_{grad}$, $loss_{WGAN}$	0.54	**0.45**	**0.42**	**0.41**	**0.43**	**0.45**	**0.47**	**0.53**	**0.56**	**0.61**
Piecewise	0.57	**0.46**	**0.44**	**0.42**	**0.44**	**0.47**	**0.46**	**0.54**	**0.52**	**0.58**
Piecewise, $loss_{grad}$	0.54	**0.45**	**0.45**	**0.43**	**0.46**	**0.51**	**0.48**	**0.58**	**0.51**	**0.58**
Baseline (Manhattan)	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Baseline (Scaled Manhattan)	0.41	0.49	0.63	0.69	0.72	0.75	0.78	0.81	0.82	0.84

Table 1: Results of experiments in the toy domain. The number shows the mean of the ratio of the number of expanded nodes where ratio$= \frac{N}{N_b}$, N is the number of expanded nodes using the learned heuristic, and N_b is the number of expanded nodes using Manhattan distance. Task difficulty is computed by Equation 10. If the task difficulty is $[1.0, 1.2]$, this indicates the task is "easy". The bold numbers indicate that the leaned heuristic is better than the baselines.

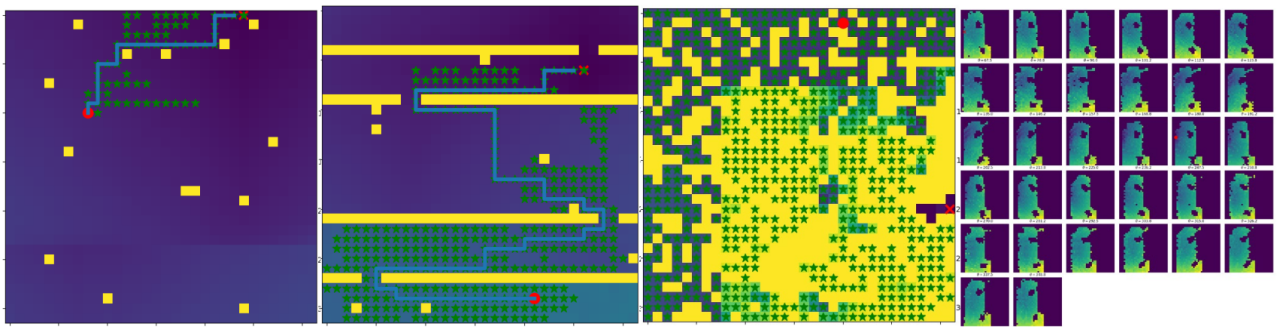


Figure 6: First three figures show unsuccessful results in the toy domain and the rightmost figure shows an unsuccessful result in the continuous domain. Learned heuristic expanded many unnecessary nodes in the first, second, and the fourth figures. The third figure from the left shows the environment where the path does not exist. In the third figure, the learned heuristic function generated small heuristic values in many unreachable states from the goal. However, all cells except several cells around the goal should be yellow (high heuristic values) because there are no paths from these cells. Note that the learned heuristic function generates these images (except for green star, path, red circle, and red cross). Thus, free space can be yellow if the space is unreachable from the goal. You see some expanded nodes in yellow cells in the third figure because of this.

increasing training samples.

Continuous domain

Figure 7 shows a successful example in the continuous domain where the hybrid A^* with the learned heuristic does not need to expand many nodes. With only 130 testing examples, such as χ^2 test can be applied to produce statistically meaningful results. Table 2 summarizes the results of statistical testing. The results are similar to those of toy 2D domain. MAE is better than MSE. The use of gradient loss improves the results for MSE and MAE; however, there is no statistical significance when we compare them against scaled Euclidean heuristic. Piecewise loss is statistically better than the scaled Euclidean. The loss of gradient degrades the results of piecewise loss. We also show the ratio of the number of expanded nodes in Table 2. If we use the piecewise loss, the ratio of expanded nodes is less than 0.081 in half tasks, 0.248 in 75% of tasks, and 0.77 in 90% of tasks. These results indicate that it is possible that DNNs can learn a heuristic function that is better than simple baselines. It also shows that the proposed piecewise loss function captures errors that simple loss functions such as MSE and MAE cannot. There are several cases where the learned heuristic function does not work well. One of the examples is shown in Figure 6. In this case, hybrid $A*$ expands almost all of the reachable nodes from s_0. Coarse discretization of orientations is one of the issues that a Hybrid A^* needs to expand many nodes to find a path. Increasing the number of orientations can avoid such cases. When we create h^* during training, we need to apply many collision checkings which are computationally expensive. However, once it learns h, we do not need to apply collision checkings to generate h during the test time.

Conclusion and Future work

In this paper, we applied the neural network architecture commonly used in object segmentation and creative image generation appropriately to learn heuristic functions for path planning. Our goal of this paper is to show preliminary results on this kind of approach, and we showed that the learned heuristic function can reduce the search cost in many scenarios in both the toy domain and the continuous domain. We also investigated loss functions and showed that the proposed piecewise loss helps DNNs to learn better heuristics. The learned DNN does not produce high-quality heuristics in some cases, but the path planner can still find a path even with those low-quality heuristics. This is one of the benefits of combining existing path planning algorithms with DNNs. Loss of gradient with respect to actions also helps other loss functions such as MSE and MAE to learn heuristic functions. Use of GAN automatically captures the properties of optimal heuristic and produces good heuristic, but it needs long training time compared to other approaches.

As future work, we want to investigate network architectures including the representation of the goal tensor to produce better heuristic values. We also would like to examine the scalability of the proposed approach using large maps. We, however, consider that our proposed method is suitable for parking scenarios because occupancy maps can easily contain both start and goal poses. Although we worked on the fully observable environments in this study, the proposed method can easily create new heuristic values given the new information because feed-forward computation of the neural network is fast thanks to high-performance GPUs. Thus, it should be able to deal with partially observable environments by replanning. We can also easily change the inputs to multiple frames of occupancy maps to deal with dynamic objects. Therefore, we would like to work on modifying the proposed approach for partially observable environments and dynamic environments as future work.

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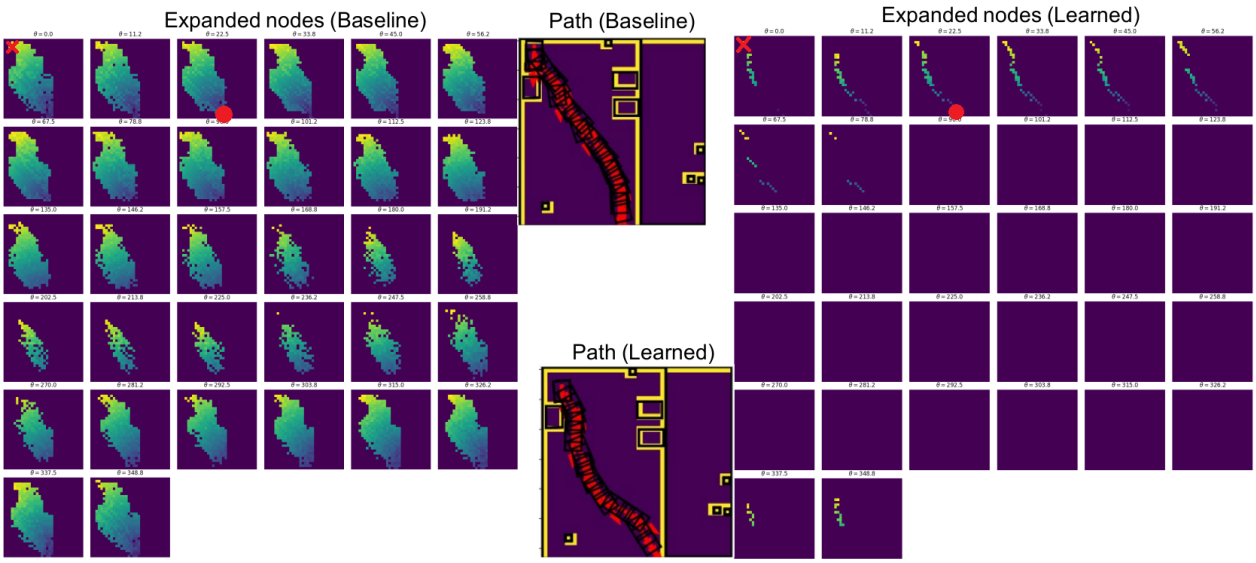

Figure 7: A successful example in the continuous domain. Left figures show results of Euclidean distance and the right figures show results of piecewise loss. We use 32 orientations in the hybrid A^*, and each figure corresponds to each orientation. Start orientation is about $\theta = 22.5°$ and the goal orientation is about $\theta = 0°$. The red circle is the start position (first row, third column) and the red cross is the goal position (first row, first column). Each figure represents the expanded nodes. Cells with dark blue are non expanded nodes. Cells with other colors represent the expanded nodes. The learned heuristic does not need to expand many nodes. The figures titled with path represents an path generated with the heuristics. Red lines indicate paths and black lines indicate either contours of a robot and obstacles. Yellow cells indicate obstacles in an occupancy map.

Loss functions	Complexity Evaluation								
	Comparison against baselines 130 test tasks N: The number of expanded Nodes N_b:Baseline, N: Learned				Ratio of the number of expanded nodes q th percentile Ratio= $\frac{N}{N_{Euclidean}}$				
	Euclidean		Scaled Euclidean		25th	Median	75th	90th	95th
	$N_b < N$	$N_b > N$	$N_b < N$	$N_b > N$					
MSE	76	44	102	15	0.650	1.939	4.348	8.256	12.380
MSE, $loss_{grad}$	60	62	90	32	0.423	1.014	2.397	6.314	9.527
MAE	34	**87**	69	51	0.145	0.383	1.095	2.351	5.272
MAE, $loss_{grad}$	21	**99**	53	67	0.084	0.179	0.859	1.456	4.292
Piecewise	13	**107**	36	**83**	0.035	0.081	0.248	0.770	1.082
Piecewise, $loss_{grad}$	16	**105**	37	**83**	0.046	0.109	0.307	0.908	1.656
Baseline (Euclidean)	NA	NA	87	42	1.00	1.00	1.00	1.00	1.00
Baseline (Scaled Euclidean)	42	87	NA	NA	0.122	0.265	0.641	0.888	1.500

Table 2: Results of experiments in the continuous domain. The numbers in the comparison against baselines show the number of tasks that are better than the other method. For example, $N_{base} < N$ shows the number of tasks where the baseline has less number of expanded nodes than that of the learned heuristic. The bold numbers indicate that the leaned heuristic is statistically better than the baseline (χ^2 test, $p < 0.05$). Outliers dominate the mean and the variance. Instead, we show q th percentile.

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