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TR2019-076 July 25, 2019

Abstract

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IEEE International Conference on Industrial Informatics (INDIN)

A Data-Driven Method for Predicting Capacity Degradation of Rechargeable Batteries

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***Abstract*—Rechargeable batteries supply numerous devices with electric power and are critical part in a variety of applications. An accurate monitoring and prediction of capacity degradation is directly related to making timely decision as to when a battery should be replaced, so that power disruption of the system it supplies power to is avoided. We propose a methodology for predicting capacity of a battery over future time horizon. The proposed method is based on training data consisting of occasional measurements, taken under the same conditions, of capacity and charge/discharge voltage/current of a certain number of batteries sharing the same chemistry and manufacturer, that otherwise undergo different usage patterns. In the operational/online stage, capacity degradation over future time horizon of a test battery cell of unknown state of health and previous usage pattern is predicted based on its capacity and voltage/current measurements over one charge/discharge cycle and the training dataset. The experimental validation reveals that the proposed method predicts capacity of a test battery cell over prediction time horizon of few hundred days of battery's operation with relative prediction error below 1%.**

***Index Terms*—battery prognosis, degradation prediction, data-driven method, rechargeable battery, state of health**

I. Introduction

Rechargeable batteries provide power supply to numerous systems and applications, most prominent examples of which include electric vehicles (EV), consumer electronic devices, uninterrupted power supply (UPS) systems and photo-voltaic (PV) cells. Battery management system (BMS) monitors and manages operation of the associated battery based on measurements from a variety of sensors it employs. The monitoring part is conventionally comprised of estimation of the battery's state of charge (SoC) [1]–[3], state of health (SoH) [4] and state of power (SoP) [5], which have been in the research focus over the past years. More recently, battery state of degradation (SoD) and the related remaining useful life (RUL), regarded as more useful monitoring metrics, have started to gain research interest [6].

The problem we address in this paper is related to predicting how battery's capacity will degrade over future time horizon. The capacity of a battery is measured by the amount of charge (expressed in Ah) it can store and deliver to its load. For example, a capacity 10 Ah means that the battery can deliver to its load a constant current 10 A during 1 h time period. The capacity of a battery degrades over time due to aging, where the degradation pattern depends on its usage, chemistry and manufacturer. Once the capacity degrades to a certain level, the battery is declared dead and needs to be replaced. Therefore, it is important to monitor battery's health and predict how its capacity evolves over time so as to replace it when needed and thus avoid power disruption.

One approach for predicting capacity degradation is to model capacity evolution over time using an empirical model, such as the one based on decaying exponential [7]–[11]. The model parameters are tracked by using an adaptive filter, such as particle filer, based on capacity measurements over discharge cycles. The capacity evolution over future time horizon is predicted by extrapolating empirical model using the most recent values of the model parameters. The main limitation of this approach is that the battery needs to be fully charged and fully discharged over its operation so that its capacity can be measured in each cycle. In addition, this approach does not take into account voltage/current measurements during charge/discharge cycles which are readily available and indicative of battery's health.

A data-driven approach [12], [13] is based on discharge voltage measurements taken over lifetimes of one or more batteries sharing the same chemistry and manufacturer, and undergoing a fixed, pre-defined usage pattern. The remaining useful life of a battery under test in the operational stage is predicted based on the training data and measurements of its voltage during the most recent discharge cycle. In comparison to model-based approaches, this approach does not necessarily rely on capacity measurements, meaning that the battery does not need to be fully charged and discharged. However, the main limitation is that the tested battery has to be of the same manufacturer and chemistry, as well as to undergo the same usage pattern as the batteries from the training data. In other words, a separate training dataset is needed for each relevant usage pattern, manufacturer and chemistry, making this approach less practical.

We propose in this paper a data-driven method for predicting capacity evolution of a test battery cell over future time horizon. The training dataset comprises of capacity, voltage and current measurements taken under the same conditions of a certain number of batteries sharing the same chemistry and manufacturer that otherwise undergo different usage patterns. In other words, the training batteries are used in different manners and the measurements that populate training dataset are occasionally taken under the same conditions that force the batteries to get fully charged and discharged so as to obtain their capacity measurements. In the operational stage, capacity and other measurements of a test battery (of the same chemistry and manufacturer as the batteries used for collecting training dataset) of unknown previous usage patterns are taken under the same conditions (that are used for collecting the training dataset) and leveraged to predict capacity degradation of the test battery cell over future charge-discharge cycles. To the best of our knowledge, our approach constitutes the most practical setup for predicting capacity degradation of batteries, compared to the existing literature. As such, in comparison to [12] and [13], the train and test battery cells in our setup are not required to undergo the same usage pattern. On the other hand, unlike in [7]–[11], the batteries in our framework are fully charged and discharged under the same conditions only occasionally, however they need not be fully charged/discharged during their regular course of operation.

The rest of the paper is organized as follows. Section II introduces battery experimental dataset we use to aid the development of the degradation model and validate the proposed approach. Section III presents main modelling principles behind the proposed algorithm. Section IV describes the proposed degradation prediction method. Section V discusses possible extensions of the proposed methodology. Section VI validates the proposed algorithm. Finally, Section VII concludes the paper.

II. Experimental Dataset

We present an experimental dataset used to conceive the main idea behind the proposed methodology. This dataset is also used to validate the proposed algorithm.

Thirteen battery cells (labeled Cell 6, Cell 7, . . ., Cell 18) of the same manufacturer and chemistry undergo different usage patterns over a time period of few years. The cells are tested on up to 43 random occasions during that time period under the same conditions. In each test, a cell is charged with constant current (CC) 1 A until the voltage on its terminals reaches 4.2 V. This voltage is kept constant (i.e., this is a floating voltage of the cell) until the current falls to 0.05 A, which concludes the charging cycle. The cell is then discharged with constant current (CC) -1 A until the voltage on its terminals falls to 3 V. We emphasize that at a given test occasion, a subset of cells is tested and their charge-discharge current, voltage and temperature variations are measured. The recorded current and voltage waveforms of a cell on one such occasion are shown in Fig. 1. We note that behavior of the voltage and current just before the voltage starts floating during the charge cycle (at ~ 7.3 h in the plots) is due to a glitch in the battery measurement system.

The amount of charge a cell stores (delivers) during a charge (discharge) cycle is computed as the time integral of the charge (discharge) current over the charge (discharge) cycle, where the constant of proportionality depends on the cell's chemistry. The capacity or state of health (SoH) of a cell, $C(t)$, is defined

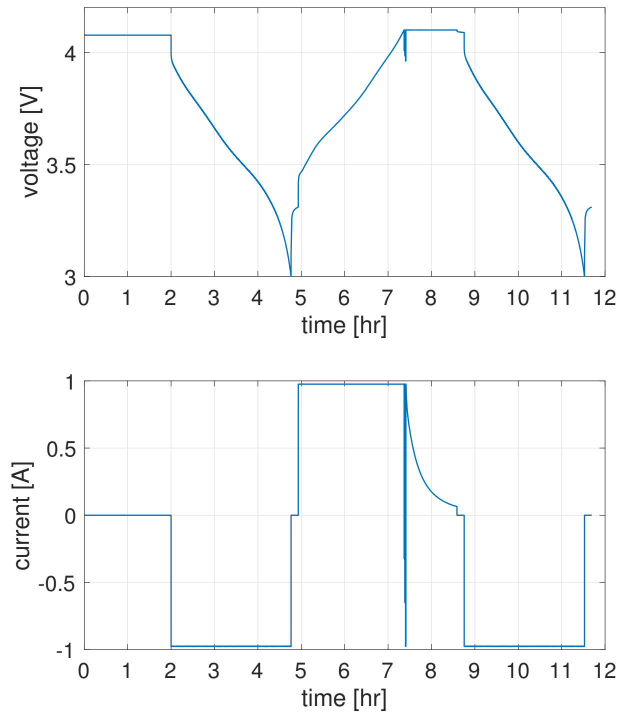

Fig. 1. Recorded voltage and current of a battery cell during one test.

as the maximum amount of charge it can deliver to its load during a discharge cycle ending at time t, after it has been fully charged in the preceding charge cycle. Given that all cells in our dataset are tested under the same charge/discharge conditions that aim to (almost) fully charge/discharge them, the measured charge each cell delivers during its test discharge cycle is essentially its capacity associated with the time instant at which that cycle ends. Time evolutions of capacities of all 13 cells from our dataset are shown in Fig. 2.

As shown in Fig. 2, the capacities of the examined cells exhibit quite different behavior over time. While some cells are fairly heathy and last over a large number of charge-discharge cycles, other cells relatively quickly degrade. This implies that despite sharing the same chemistry and manufacturer, the cells exhibit different degradation patterns due to distinct usage patterns. This can be further evidenced from the voltage waveforms measured during discharge cycles, referred to as tail voltages. For example, the tail voltages of Cell 6 and Cell 13, respectively shown in Figures 3 and 4, present distinct features and thus indicate that those two cells are impacted by quite different degradation mechanisms. We note that the same effect can be observed from the voltage waveforms measured during test charge cycles.

III. Degradation Modeling

Building upon the insights from the experimental dataset, we formulate capacity and tail voltage models in this part

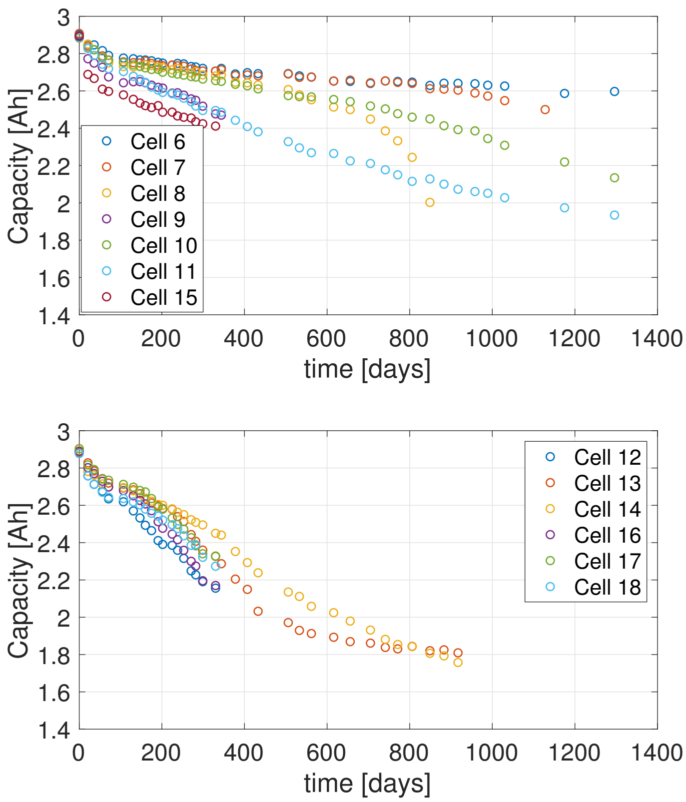

Fig. 2. Measured capacity traces of battery cells in our dataset.

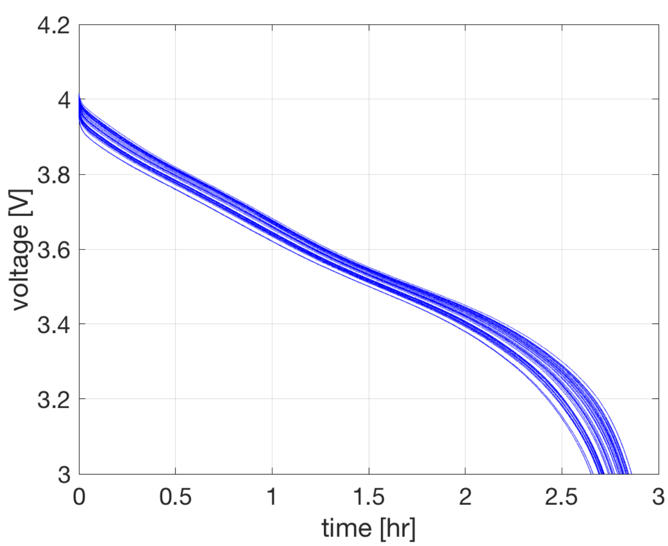

Fig. 3. Tail voltages of Cell 6.

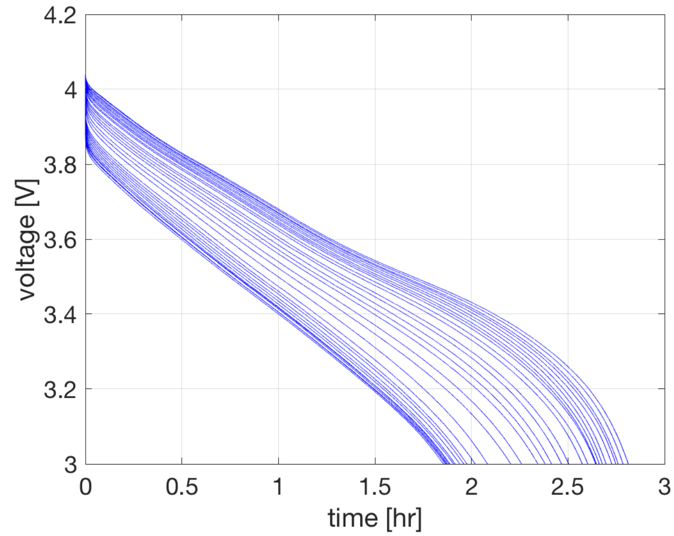

Fig. 4. Tail voltages of Cell 13.

and use them in Section IV to develop the proposed capacity prediction method.

As seen in Fig. 2, the examined battery cells experience fairly different capacity degradation patterns over time. In general, a battery cell is a complex physical and chemical system which is susceptible to a variety of degradation patterns. Different degradation patterns of battery cells of the same chemistry and manufacturer are mostly due to different manners the cells are used and handled. While, in principle, one may try to discern all possible degradation patterns along with their causes, this would be an exceedingly challenging task. Instead, possible degradation patterns can be gleaned from measured capacity traces of a number of cells over longer time period. Each distinct degradation pattern effectively defines one possible degradation class a battery cell can undergo. In other words, each battery cell n is associated with a specific degradation class d such that its capacity at time t is modelled as

$$C_n(t) = C_n(t; d) \tag{1}$$

Additional information about degradation of a battery cell is obtained from its measured tail voltage. As indicated in Figures 3 and 4, distinct degradation classes result in different tail voltage curves. This means that the tail voltages of two cells, associated with different degradation classes, differ even when they are recorded during discharge cycles at which they have the same capacity. For example, Fig. 5 shows tail voltages of Cell 7 and Cell 10 recorded when they both have the same capacity 2.55 Ah. As a side remark, we note that discharge cycles of both cells are of equal duration because they correspond to same capacity value and the cells are discharged with CC -1 A. Overall, a measured tail voltage of a cell n depends on its degradation class d and capacity (i.e., state of health) at the corresponding discharge cycle,

$$V_n(\tau) = V_n(\tau; d, C_n), \tag{2}$$

where τ is a "local" time of a discharge cycle. In comparison, t represents "global" time and can also be thought of as a discharge cycle index. Although the dependance of tail voltage on degradation class is implicit from (1), we still make it explicit in (2).

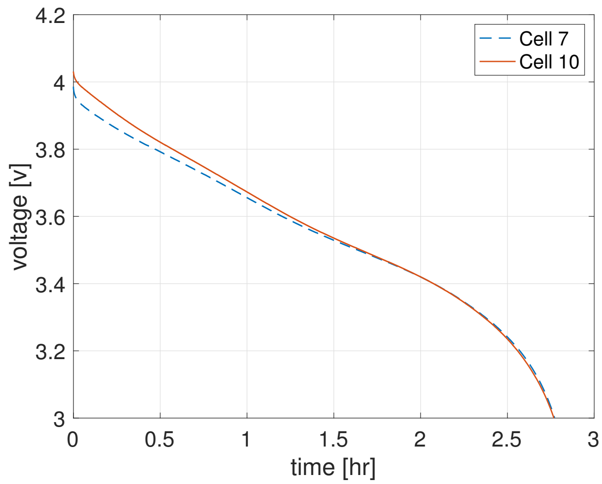

Fig. 5. Tail voltages of Cell 7 and Cell 10 corresponding to capacity 2.55 Ah.

IV. Capacity Degradation Prediction

We describe in this part the proposed methodology for predicting capacity degradation of a battery cell over future time horizon based on measurements of capacity and tail voltage during its most recent discharge cycle. As already indicated, our data-driven methodology relies on measurements related to battery cells sharing the same chemistry and manufacturer, and undergoing different usage patterns. Those measurements are recorded over a relatively small number of charge-discharge cycles, driven under the same conditions. The proposed methodology comprises of model learning stage and online prediction stage.

A. Model Learning Stage

Assuming N battery cells undergo a sequence of test charge-discharge cycles under the same conditions, the training data comprises of capacity $C_n(t)$ and tail voltage measurements $V_n(\tau, t)$, where $n = 1, \ldots, N$ is battery cell index, t is discharge cycle index, and τ is time axis of a discharge cycle. In general, the number of possible degradation mechanisms is finite. Thus, if N is large enough, some of the capacity traces $C_n(t)$ exhibit similar patterns. Therefore, to extract a representative set of possible capacity degradation patterns, the collected capacity traces may be clustered into a certain number of clusters, i.e., degradation classes K. As a result of clustering, each cluster is represented with its own capacity model $\tilde{C}_d(t)$, where $d = 1, \ldots, K$ is a degradation class index. In case the number of cells N is relatively small such that the capacity traces $C_n(t)$ are fairly distinct, each capacity trace may represent one possible degradation model.

As indicated in (2), the tail voltage depends on degradation class d and capacity C. Therefore, a model for tail voltage $\tilde{V}_d(\tau, C)$ is learned for each degradation class d and possible capacity value C using the training dataset. A simple approach to learn tail voltage models is to average measured tail voltages of the battery cells associated with the same degradation class d and recorded when the corresponding capacities are within small ϵ value of C. That is, assuming $\mathcal{V}(d, C) = \{V_n(\tau; d, c)\}_{c \in (C-\epsilon, C+\epsilon)}$ is a collection of all recorded tail voltages corresponding to the same degradation class d and ϵ neighborhood of C,

$$\tilde{V}_d(\tau, C) = \frac{1}{|\mathcal{V}(d, C)|} \sum_{V_n(\tau) \in \mathcal{V}(d,C)} V_n(\tau), \tag{3}$$

where $|\mathcal{V}|$ denotes the cardinality of the set $\mathcal{V}$. In the case where a single capacity trace represents a separate capacity degradation model, each measured tail voltage associated with that capacity trace is one tail voltage model, parameterized with the corresponding capacity value C.

Overall, the model learning part yields K degradation classes with capacity models $\tilde{C}_d(t)$ and tail voltage models $\tilde{V}_d(\tau, C)$, where C takes values from the quantized range of possible capacity values with the quantization step size ϵ.

B. Online Prediction Stage

The goal of the online stage is to predict capacity evolution of a tested battery cell over future time horizon based on measurements of its tail voltage $V(\tau)$ and capacity C, taken under the same test conditions used to record the training data. As a side remark, the capacity C can be directly measured or computed based on discharge current. The discharge current is, in turn, either measured or known in advance as part of a discharge protocol. Essentially, the online prediction stage determines degradation class of the test battery cell. Once the degradation class is detected, the capacity trace is predicted as the capacity degradation model associated with the detected degradation class.

The first step in the online prediction stage comprises of determining a similarity between the measured tail voltage $V(\tau)$ and tail voltage models of all degradation classes that correspond to the measured capacity C of the test battery cell. More formally,

$$S_d = \mathcal{S}\left(V(\tau), \tilde{V}_d(\tau, C)\right) \tag{4}$$

where $d = 1, \ldots, K$, while $\mathcal{S}$ is a similarity operator. An example of the similarity metric we use in the experimental validation is

$$S_d \propto \exp\{-\|V(\tau) - \tilde{V}_d(\tau, C)\|_2^2\}, \tag{5}$$

where, with a slight abuse of notation, $V(\tau)$ and $\tilde{V}_d(\tau, C)$ are, respectively, vectors of discretized tail voltage measurement and tail voltage model, while $\|\|_2$ denotes the L2 norm of a vector.

Upon normalization so that $\sum_{d=1}^{K} S_d = 1$, the resulting similarity S_d can be viewed as a likelihood that the capacity of the test battery cell degrades according to degradation class d. Hence, a hard decision on degradation class the test battery cell belongs to is obtained as

$$\hat{d} = \arg\max_{d} S_d \tag{6}$$

Finally, the predicted capacity trace of the test battery cell, $C_h(t)$, is the capacity model corresponding to the detected degradation class $\hat{d}$,

$$C_h(t) = \tilde{C}_{\hat{d}}(t) \tag{7}$$

The capacity prediction (7) implicitly assumes that all possible degradation classes are learned from the training dataset in the training stage. However, this may not be the case, especially when the training dataset contains measurements from a relatively small number of battery cells N. Alternatively, a soft prediction of capacity degradation trace, $C_s(t)$, of the test battery cell is obtained as a weighted combination of all capacity degradation models $\tilde{C}_d(t)$, with the weights given by the similarity scores S_d, such that

$$C_s(t) = \sum_{d=1}^{K} S_d \tilde{C}_d(t) \tag{8}$$

V. Discussion

We discuss in this part possible extensions and variations of the proposed method.

A. Generalizations of Model Learning

The proposed prediction methodology involves clustering capacity traces recorded over test charge-discharge cycles of a number of battery cells as part of the model learning in the training stage. As already pointed out and used in the experimental validation, when the number of battery cells in the training data is relatively small, each recorded capacity trace is associated with one possible degradation class and used as the corresponding capacity degradation model. In general, we do not restrict our methodology to any particular clustering method. Since the number of possible degradation classes is not known in advance, Dirichlet Process Mixture Model (DPMM) [14], [15] is a suitable clustering approach as it has a built-in mechanism to automatically detect the number of clusters.

As already elaborated, the tail voltage model for a given degradation class and capacity value is obtained by averaging the measured tail voltages associated with the same class and capacity value. In general, other approaches for modelling tail voltages may be used. As such, the tail voltage model for each degradation class and capacity value can be obtained by empirical curve fitting using the set of corresponding tail voltage measurements. More generally, a capacity trace of a battery cell, along with its recorded tail voltages, can be treated as a data point in the clustering stage. In such a case, the clustering procedure automatically yields degradation classes, each one associated with models for capacity traces and tail voltages. The DPMM [15] algorithm is a possible approach to achieve that.

Finally, other approaches for computing similarity scores between measured tail voltage of a test battery cell and tail voltage models can be used. Essentially, online prediction stage boils down to clustering a newly obtained data point, comprised of the measured tail voltage and capacity, into one of the clusters previously determined in the training stage.

B. Types of Measurements

Although we use capacity and tail voltage measurements to learn degradation models and predict capacity evolution of a test battery cell, measurements of other quantities can also be utilized. For example, voltage measurements during test charge cycles also contain battery degradation information. Consequently, they can be used instead of tail voltage measurements. More generally, a voltage waveform recorded during test charge and the following test discharge cycle is another type of measurement that, along with the battery cell's capacity, can be used for capacity prediction. We emphasize that the proposed prediction methodology does not conceptually change irrespective of the type of utilized voltage measurements.

In addition to charging and discharging a battery cell with constant current, as is the case in our dataset, a variety of other test charge and discharge protocols can be used. In general, all quantities (charge and discharge voltage and current) that do not follow a pre-determined pattern should be measured and utilized for the prediction task. This is because the variations of all those quantities over test charge-discharge cycles depend on degradation pattern and can be leveraged to accurately predict capacity evolution of a test battery cell.

Finally, the measurements of current and/or voltage during more than one test charge and/or discharge cycle can be taken and used to aid capacity prediction, provided that the capacity does not considerably change over those cycles. As in the previous case, the proposed algorithm is amenable to be generalized to accommodate such measurements.

VI. Experimental Validation

We validate the proposed algorithm using the experimental battery dataset described in Section II. Since Cell 7 is among battery cells with longer measurement log, we assume it is a test battery cell. The measurements corresponding to other battery cells comprise the training dataset, with the exception of Cell 8 which is excluded from the dataset due to its quite atypical and unhealthy capacity evolution.

For the experimental evaluation, we assume that capacity and tail voltage of Cell 7 corresponding to a certain discharge cycle t are available, predict using the proposed method how its capacity will evolve in future discharge cycles, and compare the predicted capacity with the measured capacity degradation, considered as the ground truth. We measure the performance of the proposed algorithm with the relative prediction error over prediction time horizon for various discharge cycle indices t. Note that the length of the prediction time horizon depends on the discharge cycle index t. To the best of our knowledge, the framework considered in this work is the first of its kind and, consequently, we are not aware of other methods that can be used to benchmark the proposed algorithm to.

The measurement log of Cell 7 contains measurements from 39 test charge-discharge cycles. For an illustration of the experimental evaluation, we predict capacity evolution of that battery cell using capacity and tail voltage measurements taken during the 20th test discharge cycle, which occurred on the 344th day of the cell's operation. As previously elaborated, each capacity trace from the training dataset represents one possible degradation class. Similarly, each measured tail voltage from the training dataset is the tail voltage model for the corresponding degradation class and capacity value. The measured capacity of Cell 7 on the 20th test discharge cycle is 2.7135 Ah. The similarity score between Cell 7's measured tail voltage during that discharge cycle and measured tail voltages from the training dataset associated with capacities from the range of width 0.1 Ah around 2.7135 Ah, are computed using (5). Their plot in Fig. 6 suggests that the tail voltage of the test battery cell exhibits similar behavior to tail voltage corresponding to degradation classed associated with Cell 6 and, to some extent, Cell 17. Consequently, as suggested in (8), the capacity trace of the test battery cell is predicted as the weighted combination of the capacity traces of the battery cells from the training dataset, where the weights are the similarity scores. Since not all cells from the training dataset underwent all test charge/discharge measurements, the similarity scores corresponding to cells with missing capacity measurements on a given test discharge cycle are assumed zero and the remaining similarity scores are thus re-normalized.

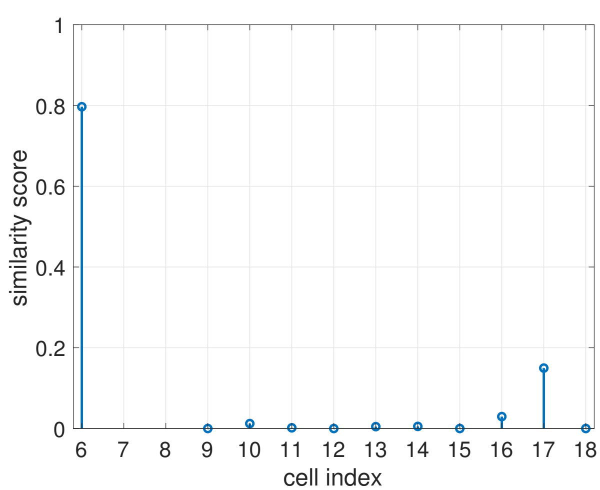

Fig. 6. Similarity score for Cell 7 on the 20th test discharge cycle (344th day of its operation).

The comparison between the predicted and measured (i.e., ground truth) capacity evolution of the test battery cell is shown in Fig. 7. The horizontal axis represents the prediction time horizon with respect to the reference day on which the test discharge cycle measurements that are used for capacity prediction are recorded. As can be seen, the predicted capacity is fairly close to the ground truth even more than 500 days after the reference day. More specifically, the relative prediction error, shown in Fig. 8, does not exceed 3% over the prediction time horizon, and is below 1% within the first 600 days after the reference day.

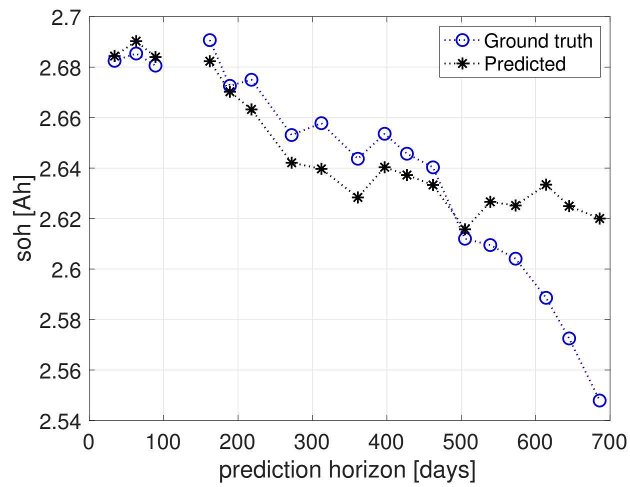

Fig. 7. Comparison between the predicted and measured capacity evolution of Cell 7 over the prediction time horizon.

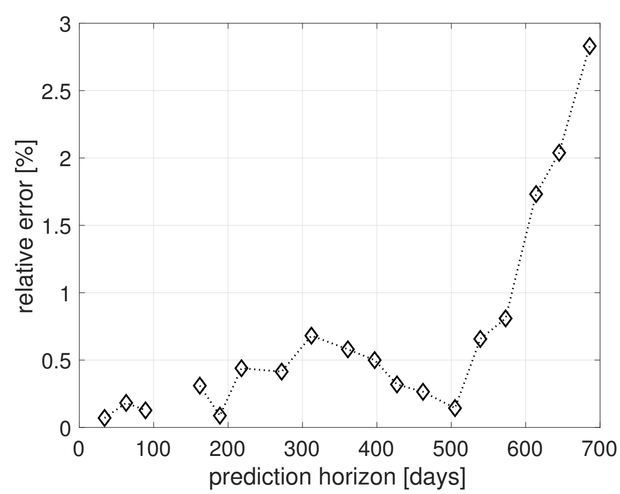

Fig. 8. Relative prediction error of the capacity degradation of Cell 7 over the prediction time horizon.

Finally, we predict capacity degradation of the test battery cell using capacity and tail voltage measurements taken on different test discharge cycles. The maximum relative prediction error over the corresponding prediction time horizon for each considered test discharge cycle is shown in Fig. 9. We note that as the index of the measurement log (i.e., test discharge cycle) increases, the test discharge cycle occurs later in the battery cell's life and the prediction time horizon is shorter. As such, the prediction time horizons corresponding to the maximum relative errors reported in Fig. 9 range from 959 to 597 days. Notably, the maximum relative prediction error, which usually occurs towards the end of the corresponding prediction time horizon, is below 3% for all but two measurement logs.

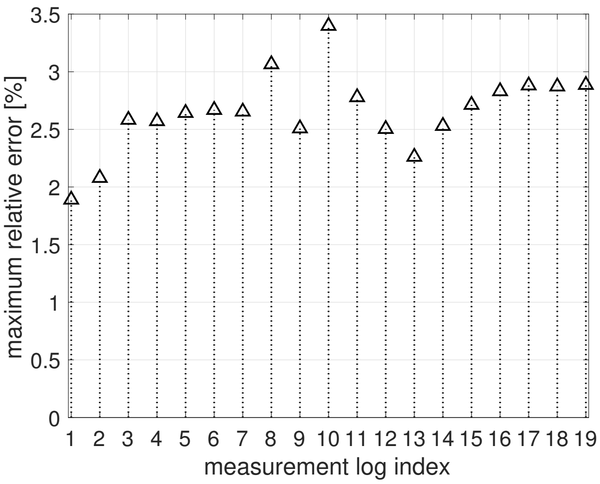

Fig. 9. Maximum relative prediction error for test battery Cell 7 for different test discharge cycle indices.

VII. Conclusions

We propose in this paper a data-driven method for predicting how capacity of a rechargeable battery will evolve over future time horizon. Given a wide application area of rechargeable batteries such as in electric vehicles, consumer electronic devices, uninterrupted power supply systems, photovoltaic cells, predicting capacity degradation is one of the key prerequisites needed for making timely decisions as to when a battery should be replaced so as to avoid power disruption of a system it supplies. The proposed algorithm predicts capacity of a battery over future charge-discharge cycles using measurements of the discharge voltage waveform and capacity over the most recent discharge cycle. The training data contains such measurements, taken under the same conditions, of a number of batteries sharing the same chemistry and manufacturer but undergoing otherwise different usage patterns. In comparison to other approaches, the batteries in the training dataset do not exhibit the same degradation pattern and the algorithm learns possible capacity degradation traces from the training data in the model learning stage. The proposed algorithm classifies a test battery cell into one of possible degradation classes based on voltage and capacity measurements during the most recent test discharge cycle and predicts its capacity evolution over future time horizon. The experimental validation of the proposed methodology reveals that the relative error in predicting capacity of the test battery cell is below 1% over a prediction time horizon of few hundred days of battery's operation. The relative error increases over longer prediction time horizons, however still remains within reasonable range. As such, it is below 3% over several hundred days of battery's operation.

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