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TR2019-079 July 30, 2019

Abstract

Image registration is of crucial importance in image fusion such as pan-sharpening. Mutual information (MI)-based methods have been widely used and demonstrated effectiveness in registering multi-spectral or multi-modal images. However, MI-based methods may fail to converge in searching registration parameters, resulting mis-registration. In this paper, we propose an outlier robust method to improve the robustness of MI-based registration for multiple rigid transformed images. In particular, we first generate registration parameter matrices using a MI-based approach, then we decompose each parameter matrix into a low-rank matrix of inlier registration parameters and a sparse matrix corresponding to outlier parameter errors. Results of registering multi-spectral images with random rigid transformations show significant improvement and robustness of our method.

IEEE International Geoscience and Remote Sensing Symposium (IGARSS)

ROBUST MUTUAL INFORMATION-BASED MULTI-IMAGE REGISTRATION

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ABSTRACT

Image registration is of crucial importance in image fusion such as pan-sharpening. Mutual information (MI)-based methods have been widely used and demonstrated effectiveness in registering multi-spectral or multi-modal images. However, MI-based methods may fail to converge in searching registration parameters, resulting mis-registration. In this paper, we propose an outlier robust method to improve the robustness of MI-based registration for multiple rigid transformed images. In particular, we first generate registration parameter matrices using a MI-based approach, then we decompose each parameter matrix into a low-rank matrix of inlier registration parameters and a sparse matrix corresponding to outlier parameter errors. Results of registering multi-spectral images with random rigid transformations show significant improvement and robustness of our method.

***Index Terms*—** Image registration, multi-spectral image, mutual information, sparsity constraint

1. INTRODUCTION

Image registration is of crucial importance in integrating information from images of the same area of interest that are collected from different measurements, either at different time, space, or using a different modality. For example, in remote sensing, pan-sharpening is a technique to fuse high spatial-resolution panchromatic image with low spatial resolution RGB or multi-spectral images. In order to achieve high spatial and spectral resolution images, an accurate registration between the Pan and the multi-spectral images is required. A single pixel or even sub-pixel error in the registration may lead to significant color or spectral distortion. Therefore, image registration has been an interesting research topic in image processing and has attracted a lot of attention.

There are many different approaches for image registration. One approach is to measure the similarity of intensity patterns in two images or patches by computing cross correlation [1] or mutual information [2]. The similarity can be computed in either the image domain or the frequency domain. These similarity-based methods are typical valid for registering images with rigid transformations, but less effective for non-rigid transformations. Another approach is to model the transformation between images to be registered. To determine the parameters of transformation model, correspondence points in different images are explored using features such as SIFT [3], the transformation parameters are then determined by fitting the correspondence points using algorithms such as RANSAC [4]. It is clear that model-based registration methods are more flexible in registering both rigid and non-rigid transformed images.

In remote sensing, image fusion is generally executed between images of different spectra such as Pan and multi-spectral or hyper-spectral images, of different modalities such as radar and optical [5], or of different view-angles [6, 7]. Since the registration problem is a non-convex problem, there is no guarantee that a single image registration method will always succeed in searching optimal registration parameters, especially for multi-spectral or multi-modal images. For example, for multi-spectral or multi-modal images, mutual information(MI)-based methods have been demonstrated to be effective in most situations [2]. However, we observe that the MI-based method may fail to register some multi-spectral images viewed from different angles. Further investigation shows that this mis-registration is due to the convergence characteristic of the Powell algorithm, which is widely used in the MI-based method to search for the optimal registration parameter.

To improve the robustness of multiple image registration, one feasible idea is to take each one of the images as a reference and try to register the others. A robust registration plan is then suggested by combining all the registration information. Following this idea and motivated by the work of robust principle component analysis (RPCA) [8], in this paper we propose a sparsity-driven method which is capable of extracting parameters for robust multi-image registration. In particular, we first generate multiple matrices of registration parameters, where each matrix corresponds to a parameter of registration, such as rotation angle, horizontal shift, and vertical shift, *etc.*. Each column of the parameter matrix corresponds to a reference image and each row corresponds to a floating image to be registered. Therefore, each entry of the matrix corresponds to the registration parameter between the floating-reference image pair. We then decompose each parameter matrix into a low-rank matrix of robust registration parameters and a sparse matrix corresponding to parameter errors. We verify our method by registering multi-spectral images and the panchromatic image with random rigid transformations. Experiments demonstrate that our proposed method significantly improves the performance for registering multi-spectral/modality images viewed from different viewing-angles.

2. MUTUAL INFORMATION BASED REGISTRATION

When we register two images, one image is treated as the reference and the other one as the floating image. Pixel samples of the floating image are then transformed to the reference image such that both images are in the same coordinate system. Let $f(\mathbf{s})$ denote the image intensity in the floating image at position $\mathbf{s}$ and $r(\mathbf{T}_\alpha \mathbf{s})$ the intensity at the transformed position $\mathbf{T}_\alpha \mathbf{s}$ in the reference image, where $\mathbf{T}_\alpha$ represents the transformation matrix with parameter α. The mutual information based registration process determines α through the following processes[2].

The joint image intensity histogram $h_\alpha(f,r)$ of the overlapping volume of both images at position α is computed by binning the image intensity pairs $(f(\mathbf{s}), r(\mathbf{T}_\alpha \mathbf{s}))$ for all $\mathbf{s} \in \mathbf{S}_\alpha$, where $\mathbf{S}_\alpha$ is the set of grid pixels for which $\mathbf{T}_\alpha \mathbf{s}$ falls inside the domain of the reference image. The joint marginal and joint image intensity distributions are obtained by normalization of $h_\alpha(f,r)$:

$$p_{FR,\alpha}(f,r) = \frac{h_\alpha(f,r)}{\sum_{f,r} h_\alpha(f,r)}, \tag{1}$$

$$p_{F,\alpha}(f) = \sum_r p_{FR,\alpha(f,r)}, \tag{2}$$

$$p_{R,\alpha}(r) = \sum_f p_{FR,\alpha(f,r)}. \tag{3}$$

The Powell algorithm [9] is typically utilized to search the optimal registration parameter α^* which maximizes the mutual information between $f(\mathbf{s})$ and $r(\mathbf{T}_\alpha \mathbf{s})$, *i.e.*,

$$\alpha^* = \text{argmax}_\alpha \sum_{f,r} p_{FR,\alpha}(f,r) \log_2 \frac{p_{FR,\alpha}(f,r)}{p_{F,\alpha}(f) p_{R,\alpha}(r)}. \tag{4}$$

For rigid transformations of 2D images, we have three-degrees of freedom

$$\alpha^* = \{\varphi^*, x^*, y^*\}, \tag{5}$$

where φ^*, x^*, and y^* represents rotation angle, horizontal shift, and vertical shift respectively. Once the parameters are determined, image registration can be executed with image transformation.

3. ROBUST REGISTRATION BY MATRIX ANALYSIS

3.1. Problem formulation

Assume we have N images including a panchromatic (Pan) image and $(N-1)$ multi-spectral (MS) images with objective to register all the MS images with the Pan. To improve the robustness of registration, we consider all possible pairs of N images for registration and jointly analyze the registration parameters. Let $\alpha_{i,j} = \{\varphi_{i,j}, x_{i,j}, y_{i,j}\}$ be the true registration parameter corresponding to the i^{th} floating image with the j^{th} reference image. For all possible image pairs, a set of matrices of true registration parameters can be formed as follows

$$\mathbf{A} = \{\mathbf{\Phi} = [\varphi_{i,j}], \mathbf{X} = [x_{i,j}], \mathbf{Y} = [y_{i,j}]\}. \tag{6}$$

In particular, if we take the Pan image as the reference, *i.e.*, $j = 1$, then $\alpha_{i,1} = \{\varphi_{i,1}, x_{i,1}, y_{i,1}\}$ is the parameter of the transform matrix of the $i^{th}(i = 1, 2, ..., N)$ MS floating image. We define $\phi = [\varphi_{1,1}, ..., \varphi_{n,1}] \in \mathcal{R}^N$, $\mathbf{x} = [x_{1,1}, ..., x_{n,1}] \in \mathcal{R}^N$, and $\mathbf{y} = [y_{1,1}, ..., y_{n,1}] \in \mathcal{R}^N$. For rigid transformations, we have $\varphi_{i,j} = \varphi_{i,1} - \varphi_{j,1}$. It is straightforward to verify that

$$\mathbf{\Phi} = \phi \mathbf{1}^T - \mathbf{1} \phi^T, \tag{7}$$

where $\mathbf{1}$ is a N-dimensional vector with all entries being 1. Similarly, we have $\mathbf{X} = \mathbf{x}\mathbf{1}^T - \mathbf{1}\mathbf{x}^T$ and $\mathbf{Y} = \mathbf{y}\mathbf{1}^T - \mathbf{1}\mathbf{y}^T$ for rigid image transformations. Note that (7) shows that the true registration matrices have rank$(\mathbf{\Phi}) \leq$ rank$(\phi \mathbf{1}^T)$ + rank$(\mathbf{1}\phi^T) = 1 + 1 = 2$, meaning $\mathbf{\Phi}$ is a low-rank matrix of rank not greater than 2. We will rely on this property to denoise the registration matrices when registration errors occur.

In practice, the registration parameter matrices $\mathbf{A}^* = \{\mathbf{\Phi}^*, \mathbf{X}^*, \mathbf{Y}^*\}$ acquired by MI-based methods are generally noisy. To achieve robust image registration, one approach is to solve a least-squares problem with an explicit rank-2 constraint to extract the registration parameter. For example, for the rotational angle, we solve

$$\hat{\phi} = \text{argmin}_\varphi \|\mathbf{\Phi}^* - \mathbf{L}(\varphi)\|_F^2, \tag{8}$$

where

$$\mathbf{L}(\varphi) = \varphi \mathbf{1}^T - \mathbf{1}\varphi^T. \tag{9}$$

The underlying assumption of the least-squares method is that the parameter error is random Gaussian noise, which is however not true in our problem.

Inspired by the robust principle component analysis (RPCA)[8], alternative to the least-squares method we propose a sparsity-driven method to achieve robust registration parameters. It is realized by solving the following problem:

$$\min_{\varphi, \mathbf{S}} \frac{\beta}{2} \|\mathbf{\Phi}^* - \mathbf{L}(\varphi) - \mathbf{S}\|_F^2 + \|\text{vec}\{\mathbf{S}\}\|_1, \tag{10}$$

where $\mathbf{L}$ represents a low-rank matrix and $\mathbf{S}$ denotes a sparse outlier matrix. The difference between (10) and RPCA is that here we impose a strict rank ≤ 2 structure on $\mathbf{L}$ whereas RPCA looks for a general low rank matrix. The low-rankness is satisfied automatically by its definition in (9). Other registration parameters such as the horizontal shift and the vertical shift can be achieved in a similar way.

3.2. Algorithm

To solve (10), we use an alternating minimization method. We first initialize $\mathbf{S}$ as $\mathbf{S}^0 = \mathbf{0}$, then update $\mathbf{S}$ and φ sequentially as follows.

For $k = 1, ..., K$

$$\varphi^k = \text{argmin}_{\varphi} \frac{\beta}{2} \|\mathbf{\Phi}^* - \mathbf{L}(\varphi) - \mathbf{S}^{k-1}\|_F^2, \quad (11)$$

$$\mathbf{S}^k = \text{argmin}_{\mathbf{S}} \frac{\beta}{2} \|\mathbf{\Phi}^* - \mathbf{L}(\varphi^k) - \mathbf{S}\|_F^2 + |\text{vec}\{\mathbf{S}\}|_1. \quad (12)$$

The first updating process in (11) is a standard least-squares problem which can be solve in a straightforward way using the psudo-inverse of the project matrix of φ. The second updating process in (12) is a simplified LASSO problem [10] for which the solution is given by

$$\mathbf{S}^k = (\mathbf{\Phi}^* - \mathbf{L}(\varphi^k)) \circ \max(0, 1 - \frac{1}{\beta|\mathbf{\Phi}^* - \mathbf{L}(\varphi^k)|}), \quad (13)$$

where $\circ$ represents the element-wised product.

The iterative algorithm is terminated until a convergence criterion meets such as

$$\frac{|\varphi^{k+1} - \varphi^k|_2}{|\varphi^{k+1}|_2} < \epsilon, \quad (14)$$

where $\epsilon \ll 1$ is a preset small positive number.

4. SIMULATION RESULTS

To validate our method, we examine the problem of registering MS images with the corresponding Pan image using mutual information based method. The high resolution Pan image is shown in Fig. 1. A total of 16 multi-spectral images (including RGB, infra-red, near-infra-red, and short wave infrared, *etc.*) are considered to register with the Pan image for further fusion process. To simulate un-registered images, we perform rigid transformations on a well-registered image set, each band image with a random transformation of parameters $\tilde{\alpha} = \{\tilde{\phi}, \tilde{x}, \tilde{y}\}$ i.i.d drawn from uniform distributions($\tilde{\phi} \in [-3, 3]$ in degree, $\tilde{x} \in [-50, 50]$ and $\tilde{y} \in [-50, 50]$ in pixel). In Fig. 2 we show in the first row three examples of unregistered MS images covering three different spectral bands respectively. The second row shows the registered images respectively using the MI method, as indicated in (4). We can observe that the middle column image is not well registered in both rotation and translation. While if we combine all registration parameters of the MI method using the least-squares method, as indicated in (8), the results are shown in the third row. We notice that the middle column image registration is getting better, but still with a small rotation angle error; and that the first and the third column images are slightly worse registered than the previous ones due to the least-squares data fitting. With our proposed method, we set $\beta = 100$, $K = 2000$, and $\epsilon = 1 \times 10^{-6}$. The registered images are shown in the last row of Fig. 2. It is clear that all the MS images are very well registered with the Pan image visually.

To further examine the registration performance, we compare the registration parameters of different methods. We take the rotation angle as an example. In Fig. 3 (a) we present the parameter matrix of rotation angle using MI method. With our proposed method, the low-rank parameter matrix is recovered as shown in Fig. 3 (b), and sparse error matrix in Fig. 3 (c). The errors between the registration parameter and the true image transformation parameter are compared in Fig. 3 (d). We observe that for the MI based method, some spectral images are not well registered with relative large rotation angle errors. By combining all parameters using the least-squares analysis, the errors are significantly reduced, but still lie in the range of $[0, 4]$ degrees. While if we use our proposed robust method, the rotation angle errors are reduced significantly to almost zero (with a maximum absolute error not greater than 0.022 degree) for all 16 multi-spectral images. Similarly, the corresponding translational shift errors are also reduced to the sub-pixel level. We omit the detailed results to save the space of this paper. Consequently, these registration parameters estimated by our proposed method lead to accurate and robust image registration.

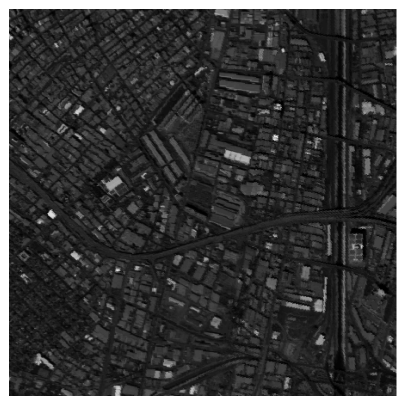


Fig. 1. High resolution panchromatic image as reference.

5. CONCLUSION

We proposed a robust sparsity-driven image registration method for multiple image registration to solve the divergence problem of mutual information based image registration. We examine our method on registering high resolution panchromatic image and low-resolution multi-spectral images under rigid transformations with random parameters. Results show that our method significantly improves the accuracy and robustness for multiple image registration when the mutual information based registration fails to register some images correctly.

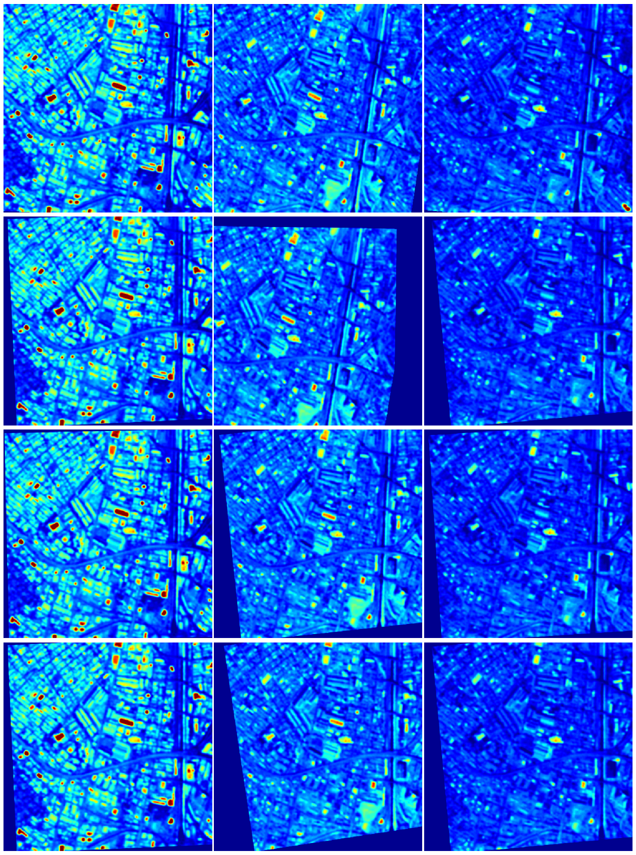
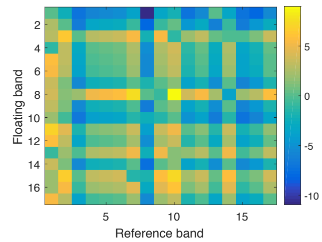
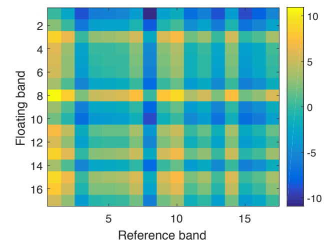


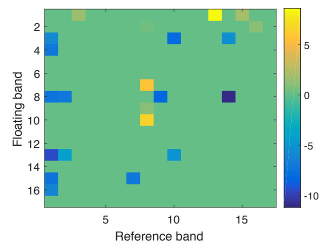
Fig. 2. Each column corresponds to a spectral band. From top to bottom, each row includes three example spectral bands of (a) Unregistered MS images; (b) Registered images using MI; (c) Registered images using MI and least squares; (d) Registered images using our proposed method.



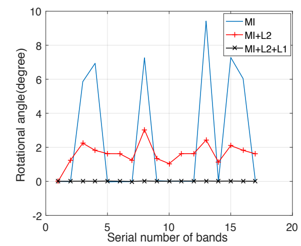
(a) Rotation angle using MI,



(b) Recovered rotation angle,



(c) Sparse rotation angle error,



(d) Rotation angle errors.

Fig. 3. Rotation angle analysis for image registration.

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