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Distributed Average Consensus Algorithm for Damage Assessment of Power Distribution system

Shiva Poudel[1], Hongbo Sun[2], Daniel Nikovski[2], and Jinyun Zhang[2]
[1]Washington State University, Pullman, WA 99164
[2]Mitsubishi Electric Research Laboratories, Cambridge, MA 02139

***Abstract*—In this paper, we propose a novel method to obtain the damage model (connectivity) of a power distribution system (PDS based on distributed consensus algorithm. The measurement and sensing units in the distribution network are modeled as an agent with limited communication capability that exchanges the information (switch status) to reach an agreement in a consensus algorithm. Besides, a communication graph is designed for agents to run the consensus algorithm which is efficient and robust during the disaster event. Agents can dynamically communicate with the other agent based on available links that are established and solve the distributed consensus algorithm quickly to come up with the correct topology of PDS. Numerical simulations are performed to demonstrate the effectiveness of the proposed approach with the help of an IEEE 123-node test case with 3 different sub-graphs.**

***Index Terms*—Power distribution system, outage, topology, agent, distributed consensus algorithm**

I. INTRODUCTION

The electric power grid is one of the most critical infrastructures of a nation; virtually every aspect of modern society (transportation, water supply, school, city halls, airports and so on) relies on the supply of electricity. Unfortunately, the increased frequency, duration, and intensity of extreme weather events pose severe threats to the power grid causing wide-area power outages primarily affecting in low- and mid-voltage power distribution grid that contributes to an estimated of 90% of the outages [1]. After a disaster occurs, the topology connectivity identification is the first but crucial step for service restoration. Commonly, the communication system to the central controller is failed, and the power grid has to rely on the distributed metering point's local measurement or status of switches to identify its connectivity.

Several articles in the literature have sought to identify the topology of the power transmission and distribution system after an outage. Thanks to phasor measurement units, which makes data available in near real-time and can provide unique insights into the global operation of the grid. With this consideration, extensive research has been already done in utilizing PMU data for situational awareness including state estimation, visualization and outage detection in the transmission network. Unlike transmission networks, distribution networks are distinguished by radial configuration (tree-topology) and because of a one-way flow of power, they have less monitoring, observability and state estimation as compared to the transmission system. Traditionally, outage handling methods use customer trouble calls based on artificial intelligence method [2], [3]. Several works have been done based on data-driven methods for topology identification in the distribution network [4], [5]. Authors in [6] proposed a topology identification approach with real-time and pseudo-measurements using maximum likelihood (ML). Outage detection using load and line flow measurements is proposed by [7], [8].

The aforementioned literature, however, has one or more limitations. Traditional methods based on trouble calls depends on the availability of trouble call from the customer side and absence of customer might prolong the outage detection and as a result, the overall restoration action will be prolonged. A majority of literature is focused on identifying the correct topology of PDS during normal conditions and such methods are inapplicable for disastrous events [6], [9]. Few are based on the assumption that a sensor or a smart meter has the communication capability with utility center outage is detected based on the collected information [7], [8]. However, these methods are prone to single-point failure and rely on communication over a large range. One of the visions of the future electric power grid is to have the cost-effective communication system and it is envisioned that there will be ubiquitous sensing of grid components and end-user devices with an interoperable machine to machine communication [10]. It is followed form the investment made under the Smart Grid Investment Grant (SGIG) program managed by U.S. Department of Energy [11], which is focused on distribution automation via automated feeder switching (smart switches and automatic reclosers).

With these considerations, we propose a distributed average consensus algorithm for obtaining the topology connectivity of the damaged distribution system. We assume that measurement and sensing units are modeled as agent and are distributed in a distribution network. These agents could be a line-switch or sensors with a limited range for communication. An efficient and robust communication graph is modeled for each area. Local agents in each area utilize the communication graph for running the consensus algorithm. The proposed approach is then tested on an IEEE 123-node distribution feeder.

II. PROBLEM FORMULATION

A. Average Consensus Algorithm

Consensus and cooperation problems are in the domain of computer science from the early years and they form the foundation of the field of distributed computing. In a network of several agents, "consensus" refers to reach an agreement regarding a certain quantity of interest that depends on the state of all agents. For our problem, to identify the healthy portion of the damaged network, the quantity of interest could be switch status or line flow measurements.

A physical distribution network can be modeled as a graph $\mathcal{G}_p(\mathcal{V}, \mathcal{E}_{ph})$ with set of buses or nodes $\mathcal{V}=\{1, 2, 3, ..., n\}$ and set of edges $\mathcal{E}_{ph} \subseteq \mathcal{V} \times \mathcal{V}$. Similarly the communication network among agents can be modeled as a graph $\mathcal{G}_c(\mathcal{V}, \mathcal{E}_{cm})$ with same set of nodes as in physical system and set of communication links $\mathcal{E}_{cm} \subseteq \mathcal{V} \times \mathcal{V}$. $\mathcal{E}_{ph}$ and $\mathcal{E}_{cm}$ can be same or $\mathcal{E}_{cm}$ can have more links than distribution lines to increase the system redundancy and for faster convergence in the consensus algorithm. Being a radial network and having relatively less degree of a node, $|\mathcal{E}_{ph}| \lll |\mathcal{V} \times \mathcal{V}|$. Let, N_i be the neighbors of agent i and are given as, $N_i = \{j|(ij) \in \mathcal{E}_{ph}\}$ The topology of the graph is characterized by the adjacency matrix $\mathbf{A} = \{a_{ij}\}$ where $a_{ij} = 1$ if $(ij) \in \mathcal{E}_{ph}$ and $a_{ij} = 0$ otherwise. Suppose a degree matrix, $\mathbf{D}$ is defined as, $\mathbf{D} = \text{diag}[\text{deg}_1, \text{deg}_2, ..., \text{deg}_n]$ where the diagonal elements represents the degree of a particular node given by, $\text{deg}_i = \sum_{i \neq j} a_{ij}$. At this point it is noteworthy to define the graph Laplacian matrix $\mathbf{L}$ with eigenvalues λ_1, λ_2, λ_3,..., λ_n which is defined as $\mathbf{L} = \mathbf{D} - \mathbf{A}$. According to the definition of graph laplacian, all row-sums of $\mathbf{L}$ are zero because of $\sum_j l_{ij} = 0$. Therefore, $\mathbf{L}$ always has a zero eigenvalue $\lambda_1 = 0$ and this zero eigenvalue corresponds to the eigenvector $\mathbf{1} = (1, ..., 1)^T$. From the spectral graph theory, it is known that second smallest eigenvalue of the Laplacian matrix $\mathbf{L}$ can tell much information about the graph and also the behavior of the average consensus algorithm. The performance of consensus algorithms often depends on $\lambda_2(\mathbf{L})$, which is also known as algebraic connectivity. A linear iterative form of the consensus algorithm for any state x_i (can be switch status or line flow) can be formulated as in (1), where each node updates its information state [12].

$$x_i(k+1) = w_{ii}\ x_i(k) + \sum_{j \in N_i} w_{ij}\ x_j(k) \tag{1}$$

where, N_i is the set of nodes that can transmit information to node i directly. The selection of weights (w_{ij}) determines the convergence rate of the algorithm and hence it should be chosen intelligently. With the exception of diagonal entries, setting $w_{ij} = 0$ for $j \notin N_i$, above equation (1) can be re-written as,

$$\begin{aligned} x(k+1) &= \mathbf{W}x(k) \\ x(k+1) &= x(k) - \epsilon \mathbf{L} x(k) \end{aligned} \tag{2}$$

where, $\mathbf{W} = \mathbf{I} - \epsilon \mathbf{L}$. With the proper value of step size and required tolerance, the value of $x(k)$ will converge to the average of their initialized values.

$$\lim_{k \to \infty} x(k) = \lim_{k \to \infty} \mathbf{W}x(0) = \frac{1}{\mathcal{N}} \mathbf{1}\mathbf{1}^T x(0) \tag{3}$$

where, $\mathbf{1}$ denotes vector of all ones. Since the algorithm converges to the average of the initialized values (3), proper initialization needs to be done as shown below,

$$x_i(0) = \begin{cases} \mathcal{N} s_i, & \text{if } i \in \mathcal{V} \\ 0, & \text{otherwise} \end{cases}$$

where $\mathcal{N}$ is the number of agents in a network and s_i is the information each agent is carrying.

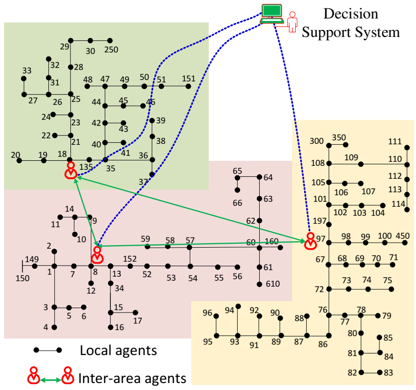

Fig. 1. A framework for distributed consensus algorithm.

B. Distributed Average Consensus Algorithm

Each communication graph is divided into sub-graphs with each graph (area) having one inter-area agent as shown in Fig. 1. It is assumed that each inter-area agent has a relatively strong communication capability in comparison to a local agent. The number of inter-area agents or the number of sub-graph can be determined based on the size of the network, infrastructure cost, and the required overall convergence rate. Suppose, a large distribution network with $\mathcal{N}$ number of agents is divided into several areas $\mathcal{A} = \alpha_1, \alpha_2, \alpha_3, ..., \alpha_n$. At each area of the distribution network, the consensus update can be concurrently solved for each of them and local consensus agreement can be achieved. If the number of areas is large, then several areas are again grouped such that reasonable time for convergence is achieved. The problem forms layers or stages of sub-graphs where local consensus algorithm is solved at each stage. Following equation (2), we can write (4) for each area on a particular stage,

$$\begin{aligned} x_{\mathcal{L}_1}^{\alpha_i}(k+1) &= \mathbf{W}_{\mathcal{L}_1}^{\alpha_i}\ x_{\mathcal{L}_1}^{\alpha_i}(k) \\ x_{\mathcal{L}_2}^{\beta_i}(k+1) &= \mathbf{W}_{\mathcal{L}_2}^{\beta_i}\ x_{\mathcal{L}_2}^{\beta_i}(k) \quad ... \\ x_{\mathcal{L}_n}^{\gamma_i}(k+1) &= \mathbf{W}_{\mathcal{L}_n}^{\gamma_i}\ x_{\mathcal{L}_n}^{\gamma_i}(k) \end{aligned} \tag{4}$$

The inter-area agents can run average consensus among themselves. If the number of inter-area agents is large (i.,e., a large number of sub-graphs), they can again be divided into several groups to get rid of the computational burden. Thus, different layers of the consensus algorithm can be formulated for better convergence. They reach a consensus and converge upon the damaged system model for the entire feeder and communicate it to the corresponding decision-support systems. The overall convergence time for global consensus is then equal to the sum of the maximum time taken by each area to reach consensus for different layers of sub-graph ($\mathcal{L}$) which can be expressed as,

$$\begin{aligned} \tau_{total} =& \max_{\mathcal{L}_1}(\mathcal{T}_{\alpha_1}, \mathcal{T}_{\alpha_2}, ..., \mathcal{T}_{\alpha_n}) + \max_{\mathcal{L}_2}(\mathcal{T}_{\beta_1}, \mathcal{T}_{\beta_2}, ..., \mathcal{T}_{\beta_n}) \\ &+ ..... + \max_{\mathcal{L}_n}(\mathcal{T}_{\gamma_1}, \mathcal{T}_{\gamma_2}, ..., \mathcal{T}_{\gamma_n}) \end{aligned} \tag{5}$$

III. COMMUNICATION NETWORK OF POWER DISTRIBUTION SYSTEM

The power grid physical layer $\mathcal{G}_{ph}$ depends on its communication network or a cyber layer $\mathcal{G}_{cm}$ (See Fig. 2) such that any device in $\mathcal{G}_{ph}$ (e.g., a generator, load, or a switch) is operable

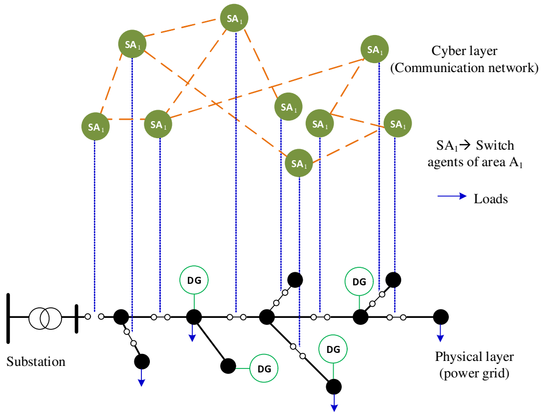

Fig. 2. A cyber-physical system for power distribution system.

only when its associated control point can receive a signal from some control center in $\mathcal{G}_{cm}$ [13]. The cyber layer with communication among the smart switches is exploited in this work for obtaining the damaged model of a faulted power distribution network. In this section, the optimal design of cyber layer (communication graph) for a power distribution network and its robustness to link failure is discussed.

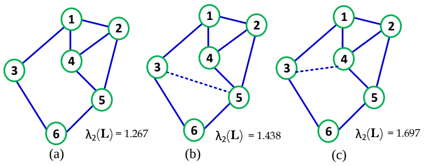

Fig. 3. Effect of adding an edge on eigenvalues of graph Laplacian.

A. Communication graph for agents

The communication among agents is crucial for convergence of the average consensus algorithm. Moreover, the local agents have constraints of communication capability limited by infrastructure cost, the range of communication and energy source to support its performance. It is well understood that the connectivity of the communication graph determines the convergence of consensus algorithm. It is always desired to have a well-connected graph where each agent has communication with another agent. However, this is practically impossible for a power distribution network at this point in time because of cost and resource constraints. In such circumstance, we propose an algorithm for obtaining the optimal communication topology of a sub-graph $\mathcal{G}_{cm}(\mathcal{V},\mathcal{E}_{cm})$ which helps to increase the convergence rate and robustness for disaster condition. The proposed approach is summarized in Algorithm 1.

Algorithm 1: Design of communication topology

```
1  Given: $\mathcal{G}_{ph}(\mathcal{V},\mathcal{E}_{ph})$, $d^{ij}_{range}$, $C^{\$}_{ij}$
2  Initialize $\mathcal{E}_{cm}$=$\mathcal{E}_{ph}$
3  Form a connected fully graph $\mathcal{G}_{cn}(\mathcal{V},\mathcal{E}_{cn})$. where,
     $\mathcal{E}_{cn}=\{\mathcal{V}\times\mathcal{V}\}$
4  Find a potential graph $\mathcal{G}_{pos}(\mathcal{V},\mathcal{E}_{pos})$. where,
     $\mathcal{E}_{pos}=\{\mathcal{V}\times\mathcal{V}\}-\mathcal{E}_{ph}$
5  for each edge in $\mathcal{E}_{pos}$ do
6      if $d(ij)\geq d^{ij}_{range}$, then
7          $\mathcal{E}_{pos}\leftarrow\mathcal{E}_{pos}-(ij)$;
8      if $(i,j)\in\mathcal{V}_n$, then
9          $\mathcal{E}_{pos}\leftarrow\mathcal{E}_{pos}-(ij)$;
10 for each edge in $\mathcal{E}_{pos}$ do
11     // Find the importance of each link
       $\mathcal{M}=\mathrm{F}\left[\deg_i,\deg_j,\deg_{N_i},\deg_{N_j},n(\mathcal{C})\right]$
12 ind← Sort $\mathcal{M}$
13 Initialize $\mathcal{A}_{cost}$ = 0
14 for $k\leftarrow 1$ to $\mathcal{E}_{pos}$ do
15     $\mathcal{E}_{cm}\leftarrow\mathcal{E}_{cm}+\mathcal{E}_{pos}(\mathrm{ind}(k))$
16     $\mathcal{A}_{cost}\leftarrow\mathcal{A}_{cost}+C^{\$}_{ij}$
17     if $\mathcal{A}_{cost}\geq Budget$, then
18         break;
19 Output $\mathcal{G}_{cm}(\mathcal{V},\mathcal{E}_{cm})$
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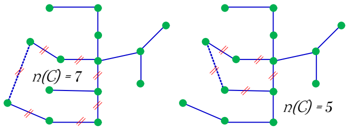

Fig. 4. Loop formation on a graph after addition of a new link and its robustness to link failures.

Let us assume, the communication among agent is constrained by the range (i.e., geographical distance) and available infrastructures for enhancing the agent performance. Suppose, d^{ij}_{range} be the maximum range a local agent i can reach to another local agent j and $C^{\$}_{ij}$ be the infrastructure required for establishing new links by upgrading the agent so that it can handle additional communication burden (Step 1). As the agents are located on the node or edges (switches) in a distribution network, it is desired to have a base topology as a radial network similar to the physical network (Step 2). The fully connected graph of the agents is represented by $\mathcal{G}_{cn}(\mathcal{V},\{\mathcal{V}\times\mathcal{V}\})$, where each agent can communicate with another and possible edges (links) ($\mathcal{E}_{pos}$) are identified for adding to the communication network (Steps 3 and 4). For each possible link, the range of communication and agent capability is checked. It is well understood that the communication bandwidth is often a scarce resource in decentralized settings. If communication cannot be established between any edge ij or agents i,j belong to a set which is not available for establishing of further links ($\mathcal{V}_n$), then it is removed from the set possible links (Steps 5-9). For the updated set of links, a function is defined based on the degree of an agent, the degree of their immediate neighbors and the size of cycle formed after the link is established (Steps 10 and 11). It is known that the second eigenvalue is dependent on the degree of a node and also the connectivity of neighbors (See Fig. 3). Similarly, the robustness of the graph is realized based on the size of a loop formed in the network after a new link is established, $n(\mathcal{C})$ and is graphically illustrated based on Fig. 4. It can be observed that a large size loop is more robust for a link failure than small size loop. For instance, consider a case of a single link failure in the given graph. The bigger loop is robust to failure of one of the 7 links in the network while smaller loop is robust to failure of one of the 5 links only. By robustness, we mean the graph still remains connected after a failure. The importance of communication link defined by $\mathcal{M}$ is sorted and new links

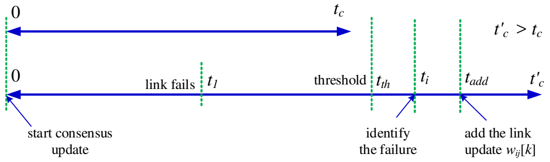

Fig. 5. General time-line for dynamic topology update.

with higher importance are established in the communication graph until the available infrastructures are exhausted (Steps 12-18).

B. Modeling Switching (agents or links failure)

Sometimes during disaster conditions, challenges may occur in the convergence of consensus algorithm for doing a damage assessment because of link failures due to the agent not succeeding in establishing the communication or node failures (e.g., due to draining of batteries supporting an agent). Thus, in such a case the communication graph will be time-varying (i.e., the weight matrix will be denoted by $\mathbf{W}(k)$. To capture dynamically changing topologies it is assumed that the set of agents is fixed, $\mathcal{V} = 1, 2, ..., n$ but the set of links among them might change at various time steps during the consensus update. For such a highly volatile system, an approach based on dynamic topology change is used to ensure the fault-tolerant information dissemination among the distributed agents. We make the following assumptions for handling the changing interconnections in a disaster-impacted communication graph:

- At any instant 'k', a transmitting node requires to know the number of neighboring nodes receiving its information and this requirement is not difficult in any undirected graph.
- To keep things simple, it is assumed that there exist no delays in any communication links.
- Communication links can be established or terminated throughout the iterative algorithm by any nodes.

Note that specific weights in $\mathbf{W}[k]$ are set to zero which corresponds to a pair of nodes that are not connected at a particular time step 'k'. Mathematically,

$$w_{ij}[k] = 0, \ \forall (i,j) \in \mathcal{E}^f, \ i \notin j \tag{6}$$

where, $\mathcal{E}^f$ is the set of failed links and w_{ij} is one of the elements of matrix $\mathbf{W}$ representing a weight. Now, equation (2) can be rewritten as,

$$x(k+1) = \mathbf{W}(k)x(k) \tag{7}$$

This means, at any instant, a node i has a set of neighboring nodes $N_i[k]$ and degree $\deg_i[k]$. Specifically, we assign the weight on an edge (link) based on the larger degree of its two incident nodes in real time.

$$w_{ij}[k] = \frac{1}{\max(\deg_i[k], \deg_j[k])}, \ \forall (ij) \in \mathcal{E}_{cm} \tag{8}$$

Fig. 5 shows the convergence timeline for a consensus algorithm. During normal communication among agents, the algorithm converges in time t_c. Suppose, a communication link fails at time t_1. Let us assume that failure makes the communication graph disconnected. It is obvious that the algorithm fails to converge within time t_c. A threshold time ($t_{th} > t_c$) is defined to ensure the graph became disconnected because of link failure and algorithm ceases to converge. This is critical because it is likely that the algorithm can converge for $t_c \leq t \leq t_{th}$ because not all failure makes a graph disconnected but will result in reduced $\lambda_2(\mathbf{L})$. A failure is identified by observing the real-time error of the consensus update at time t_i. Once it is identified, new links are established to ensure the graph is connected. To establish a new link, it is required to remove some redundant links because an agent has a limited communication capability and cannot handle any additional new links. This is following the fact that agent capability and resources are fully utilized for designing the communication graph in Algorithm 1. With the removal or addition of links in the graph, the weight $w_{ij}[k]$ is also updated. Now the algorithm finally converges at time $t_c^{'}$ (See Fig. 5).

IV. SIMULATION RESULTS

The proposed algorithm is tested on an IEEE-123 node test feeder which has experienced severe damage after a natural disaster. Each line has a smart switch which acts as an agent and can communicate with its neighboring agent. After several iterations, all agents come to an agreement and an overall estimate of the PDS topology can be made. However, for this relatively small feeder, the number of iteration was observed to be 3253 and simulation time for convergence was found to be around 27.36 seconds with an error tolerance of 10^{-1}. In real, practical distribution feeder has thousands of nodes and hence it is not efficient to run consensus algorithm for a large scale network. So, we divide the IEEE-123 node feeder into three sub-graphs each having an inter-area agent (See Fig. 1) and solve the distributed consensus algorithm concurrently.

TABLE I
DISTRIBUTED CONSENSUS ALGORITHM FOR EACH AREA

S.No	Area 1	Area 2	Area 3
Number of agents	35	38	52
Simulation time (s)	1.9	2.65	5.86
No. of iterations	582	747	971

For each sub-graph, initially, the communication links are established based on the algorithm proposed in Section III A. The simulation result is shown in Fig. 6. First, a fully connected graph is formed based on agent geographical location, Fig. 6a. Next, a base case topology is identified which is the same as the physical topology of the network, Fig. 6b. Finally, Algorithm 1 is implemented and several links are then established to get final communication graph Fig. 6c. The local agents in a sub-graph now run their own consensus algorithm. Note that, a similar process is repeated for each area. After the consensus algorithm converges in each area, the inter-area agent then communicates among themselves for obtaining the overall damage model of the distribution feeder. Table I summarizes the performance of consensus algorithm for each sub-graph. It is observed that the number of iterations and simulation time reduces drastically by splitting the large network into several sub-graphs. This approach thus helps to achieve the damage model of PDS quickly after a disaster event.

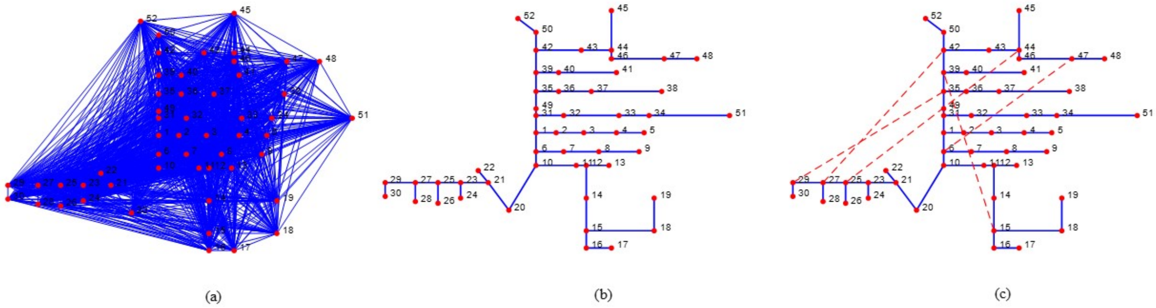

Fig. 6. Design of a communication topology for a sub-graph $\mathcal{G}_{cm}(\mathcal{V}, \mathcal{E}_{cm})$. (a) Fully connected sub-graph, (b) Base radial topology of the communication graph. (c) New added links to obtain an optimal graph for consensus algorithm.

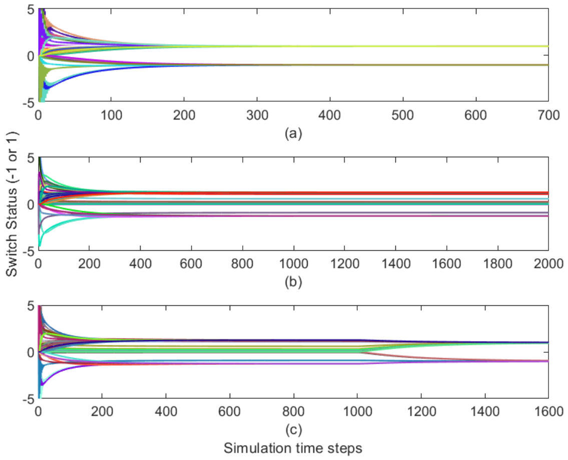

Fig. 7. Convergence of average consensus algorithm for random link failure.

To validate the robustness of the proposed approach for link failures, a random failure is introduced into the system. Area 3 of IEEE-123 node (Fig. 6c) is taken as a test case for simulating the failure. Fig. 7a shows, in absence of failure, the consensus algorithm converges and provide corresponding switch status. At particular time step k, a random link failure is introduced. With this failure, the algorithm doesn't converge for several rounds of iterations (See Fig. 7b). At this point, a failure link has to be identified and a new link has to be established. Based on this, a disconnected portion of the graph is identified (32-33-34-51). Now, a new link is established after a few time steps between nodes 32 and 47. To do this, link 6-47 is taken down because agent 47 cannot handle new additional link as it is not coped with enough capability for additional communication burden. The weight matrix is then updated dynamically following equation (8). Finally, the convergence of consensus update is guaranteed as seen in Fig. 7c.

V. CONCLUSION

In this paper, a distributed average consensus method for quick damage assessment of a PDS is proposed. An effective communication topology suitable for a PDS is designed for the exchange of information among agents. A fast outage identification of a PDS is achieved by distributing the algorithm into sub-graphs and hence the damage system model is obtained at a relatively small amount of time with the less computational burden. The measurement and sensing units in the distribution network are modeled as an agent with limited communication capability that exchanges the information (switch status) to reach an agreement in a consensus algorithm. The proposed algorithm is thus suitable for practical implementations during a disaster as it relies on local measurements and local communication. In addition, the proposed framework is robust to the random link failures and hence the convergence is guaranteed during a disaster condition. An efficient restoration algorithm can be called after the topology is identified for restoring the critical loads.

REFERENCES

[1] H. Farhangi, “The path of the smart grid,” *IEEE power and energy magazine*, vol. 8, no. 1, 2010.

[2] Z. Sumic and R. Vidyanand, “Fuzzy set theory based outage determination,” in *Proc. of 1996 International Conference on Intelligent Systems Applications to Power Systems*, pp. 204–208, 1996.

[3] J. Aguero and A. Vargas, “Inference of operative configuration of distribution networks using fuzzy logic techniques-part i: real-time model,” *IEEE Transactions on Power Systems*, vol. 20, no. 3, pp. 1551–1561, 2005.

[4] G. Cavraro, R. Arghandeh, K. Poolla, and A. Von Meier, “Data-driven approach for distribution network topology detection,” in *Power & Energy Society General Meeting, 2015 IEEE*, pp. 1–5, IEEE, 2015.

[5] K. Soumalas, G. Messinis, and N. Hatziargyriou, “A data driven approach to distribution network topology identification,” in *PowerTech, 2017 IEEE Manchester*, pp. 1–6, IEEE, 2017.

[6] Y. Sharon, A. M. Annaswamy, A. L. Motto, and A. Chakraborty, “Topology identification in distribution network with limited measurements,” in *Innovative Smart Grid Technologies (ISGT), 2012 IEEE PES*, pp. 1–6, IEEE, 2012.

[7] Y. Zhao, R. Sevlian, R. Rajagopal, A. Goldsmith, and H. V. Poor, “Outage detection in power distribution networks with optimally-deployed power flow sensors,” in *Power and Energy Society General Meeting (PES), 2013 IEEE*, pp. 1–5, IEEE, 2013.

[8] R. A. Sevlian, Y. Zhao, R. Rajagopal, A. Goldsmith, and H. V. Poor, “Outage detection using load and line flow measurements in power distribution systems,” *IEEE Transactions on Power Systems*, vol. 33, no. 2, pp. 2053–2069, 2018.

[9] R. Arghandeh, M. Gahr, A. von Meier, G. Cavraro, M. Ruh, and G. Andersson, “Topology detection in microgrids with micro-synchrophasors,” in *Power & Energy Society General Meeting, 2015 IEEE*, pp. 1–5, IEEE, 2015.

[10] G. Alliance, “The future of the grid: Evolving to meet americaâĂŹs needs,” *US Department of Energy*, 2014.

[11] U.S. Department of Energy, "Smart Grids Investments Improve Grid Reliability, Resilience, and Storm responses", Nov 2014. [Online]. Available: https://www.smartgrid.gov/files/B2-Master-File-with-edits_120114.pdf.

[12] R. Olfati-Saber, J. A. Fax, and R. M. Murray, “Consensus and cooperation in networked multi-agent systems,” *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[13] G. Byeon, P. Van Hentenryck, R. Bent, and H. Nagarajan, “Communication-constrained expansion planning for resilient distribution systems,” *arXiv preprint arXiv:1801.03520*, 2018.