Fair Blackout Rotation for Distribution Systems under Extreme Weather Events

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***Abstract*—This paper presents an optimization-based method for scheduling fair blackout rotation for distribution systems under extreme-weather induced power shortages. The method first partitions the system into a set of feeder sections according to the locations of switches, then simplifies the system as a connected network by modeling feeder sections as nodes and switches as links. The method simultaneously minimizes the total lost energy, total number of switch operations, maximal section isolation duration, and maximal section isolation frequencies of the system, while minimizing the maximal deviations of isolation durations and frequencies between any two feeder sections. The multiple-objective model is converted into a single objective one through defining each objective's satisfaction degree using a generalized Rectified Linear Unit (ReLU) function. The resulted nonlinear model is further converted into a standard mixed integer linear programming by replacing each nonlinear item including satisfaction function, binary variable operation, and logic operation with an embedded optimization problem with a linear objective function constrained by its feasible region represented using linear equations. The method can be implemented either using a static horizon or rolling horizon strategy. The numerical examples on the modified IEEE 123-node test feeder are given to demonstrate the effectiveness of the proposed method.**

Keywords*—*distribution system, extreme weather, power shortage, rolling blackout, optimization

I. INTRODUCTION

With the increasing frequency, intensity and duration of extreme weathers, the generation deficiency and failures induced by such disastrous events will become inevitable. Therefore, rolling blackouts are expected to play a major role for maintaining the integrity of power system and mitigating the sufferings of power consumers. The latest example of such events includes 2021 Texas power crisis introduced by an extreme cold weather event in the February of 2021. As reported by ERCOT [1], approximately 48.6% of its generation was forced out at the highest point due to the impacts of various extreme weather conditions, and controlled outages were implemented to prevent statewide blackout. Another event occurred in the state of California in 2020. California ISO had triggered two rotating power outages in its footprint to deal with energy supply shortages caused by a historical heat storm in the western US during mid-August of 2020 [2].

Rolling blackouts, or rotating/controlled outages, are systematic, temporary power outages that help bring supply and demand balance of electricity in the power grid. Usually, the operator of power grid will call for rolling blackouts and administer temporary outages to one area at a time to limit the duration of the outages for each area. Rolling blackouts are typically the last step in a series of emergency procedures after a power supply shortage is detected. These temporary outages help prevent everyone in the area from experiencing an even longer blackout. The rolling blackout will conclude when the emergency is ended. The duration of a rolling blackout depends on the severity of the event. The power grid operator will determine which circuits, or sections in their service areas will receive blackouts, and usually attempt to limit the duration of outages in each area. However, it is not easy for power girds to achieve fair blackout rotation particularly for power distribution systems. Apart from lack of measures owing to insufficient investment in power infrastructure, utilities may lack of effective application tools to design blackout rotation scheme as well. The conventional heuristic rules-based approach might not work well when outage scenario is complicated and longer time horizon and larger geographical region coordination are needed.

There are a few works in literature addressing rolling blackout related problems for power distribution systems, such as [3]-[6]. The existing methods can be divided into individual load control based methods, and load group based approaches. [3] investigated load shedding plans to control abnormal situations, which can be implemented at a regional level and in a rotational manner to preserve provision of essential services inside the affected power systems through shedding non-sensitive loads within the neighboring regions. In [4], a strategy was proposed to limit the energy demand in case of power generation shortage in which the consumers who significantly causing the critical peak will be forced to reduce their consumption rather than completely disconnecting electricity. [5] described a method for converting complete blackouts to brownouts which allow selective provisioning of power supply to support essential loads while curtailing supply to less critical loads. [3]-[5] relied on demand responses to rebalance supply with demand, but might fail when the majority portion of load demands are uncontrollable essential loads. [6] offered another type of methods which formulated the rotational load shedding as a bilinear integer programming problem by minimizing the total damage cost while respecting various load shedding preferences among different zones. However, it ignored the connectivity relationship between load zones by assuming each zone can shed independently without affecting the rest of the system, such that the generated plan might be infeasible for practical implementation.

To this end, this paper proposes a new method for implementing a fair blackout rotation for distribution systems under extreme weather events, in which the blackout rotation scheme is determined by scheduling an optimal switch operation and storage dispatch along the evolving of disaster event. The main contributions of this paper include: (1). The distribution system is simplified as a connected network by partitioning the system into a set of feeder sections separated by switchable devices, and modeling each feeder section as a node, and each switchable device as a link. Nodal power balance equations are then used to replace large-scale power flow equations for practical systems, and switch flow limits can be adjusted to clear any violations within feeder sections based on power flow and sensitivity analysis of the feeder section within violations if existing. (2). Multiple optimal objectives have been considered to represent different requirements from the system, the consumers, and the rotation fairness. The system requirements are represented as the minimization of the total lost energy, and total number of switch operations. The consumers' requirements include minimizing the maximal section isolation duration, and frequency among all feeder sections. The rotation fairness is achieved through minimizing the maximal deviations of isolation durations and frequencies between any two feeder sections. (3). The multiple objective mode is converted into a single objective one by defining each objective's satisfaction degree using a generalized Rectified Linear Unit (ReLU) function, and maximizing an overall satisfaction degree over all objectives. The resulted nonlinear model is further converted into a standard mixed integer linear programming by replacing each nonlinear item including satisfaction function, binary variable operation, and logic operation with an embedded optimization problem with a linear objective function constrained by its feasible region represented using linear equations. (4). Considering the evolving of blackouts during disastrous weather event, besides managing the blackout rotation using a static horizon strategy, the blackout rotation is also generated using a rolling horizon strategy to revisit and update disaster situations throughout the evolution of the disaster cycle. This enables the blackout rotation to adapt with situation changes and generate better rotation scheme to take pre-existing conditions into account.

The remainder of this paper is organized as follows. Section II introduces the formulation for fair blackout rotation. Section III discussed the transformation of formulated nonlinear model into a linear one and related soultion technique. Numerical examples are given in Section IV. Section V presents the concluding remarks.

II. FAIR BLACKOUT ROTATION FORMULATION

Assumed a distribution system includes multiple substations fed by the main grid, and each substation includes multiple feeders. Each feeder may connect with a set of renewable generations, battery energy storage systems, and load demands. The feeder can be further divided into a set of feeder sections by opening all switches along the feeder and defining each all-connected area as a feeder section.

When a power shortage occurs at the main grid, the power grid operators have to cut off power supply for some feeder sections, even feeders to reach new balance of power supply and demand through switch operations. The goal of blackout rotation is to determine an optimal switch operation scheme to rebalance power supply and demand dynamically with evolving of power shortages and allocate shortage impacts among consumers in a fair manner.

Due to large space of switch operation combinations and complicated operational coordination along shortage horizon, the blackout rotation problem is usually hard to be modeled and solved as an optimization problem but determined by using heuristic methods instead. The solution optimality is one of major challenges faced by heuristic rules-based methods when dealing with practical system. Another concern is the current practice mainly considering the system needs, such as minimizing energy lost, but ignore the interests of consumers, and concerns for biased blackout rotation.

After partitioning the system into a set of feeder sections, the system can be simplified as a connected network by modeling each section as a node, and each switch as link. Through connectivity analysis of the network, the connectivity relationship between feeder sections, feeders and substations can be established accordingly.

Based on the system configuration determined above, we can formulate the blackout rotation as a multiple-objective optimization problem in terms of switch operation and resource dispatch. The renewable generations and storages are utilized to reduce the amount of energy lost caused by generation deficiency.

Considering the diverse needs from the system, consumers and fairness control, the blackout rotation optimization can be represented using six objectives, including minimizing the total amount of lost energy, $f_c^{out\_mwh}$ (1), minimizing the total number/frequency of switching operations, $f_c^{sw\_fq}$ (2), minimizing the maximum outage hours, $f_c^{out\_hr}$ (3), minimizing the maximum outage frequency/times, $f_c^{out\_fq}$ (4), minimizing the maximum difference of outage hours between two feeder sections, $f_c^{diff\_out\_hr}$ (5), and minimizing the maximum difference of outage frequencies between two feeder sections, $f_c^{diff\_out\_fq}$ (6):

$$f_c^{out\_mwh} = \sum_{s\in SS}\left(f_{s,0,c}^{out\_mwh} + \sum_{t\in T_c} x_{s,t,c}^{isolated}\, P_{s,t,c}^{net}\Delta T_{t,c}\right) \quad (1)$$

$$f_c^{sw\_fq} = f_{0,c}^{sw\_fq} + \sum_{sw\in SW}\sum_{t\in T_c}\left|x_{sw,t,c}^{closed} - x_{sw,t-1,c}^{closed}\right| \quad (2)$$

$$f_c^{out\_hr} = \max_{s\in SS}\left(f_{s,0,c}^{out\_hr} + \sum_{t\in T_c} x_{s,t,c}^{isolated}\Delta T_{t,c}\right) \quad (3)$$

$$f_c^{out\_fq} = \max_{s\in SS}\left(f_{s,0,c}^{out\_fq} + \sum_{t\in T_c} x_{s,t,c}^{out}\right) \quad (4)$$

$$f_c^{diff\_out\_hr} = \max_{\substack{s1,s2\in SS,\\ s1\neq s2}}\left|\begin{matrix} f_{s1,0,c}^{out\_hr} - f_{s2,0,c}^{out\_hr} + \\ \sum_{t\in T_c}(x_{s1,t,c}^{out} - x_{s2,t,c}^{out})\Delta T_{t,c}\end{matrix}\right| \quad (5)$$

$$f_c^{diff\_out\_fq} = \max_{\substack{s1,s2\in SS,\\ s1\neq s2}}\left|f_{s1,0,c}^{out\_fq} - f_{s2,0,c}^{out\_fq} + \sum_{t\in T_c}(x_{s1,t,c}^{out} - x_{s2,t,c}^{out})\right| (6)$$

where c is the index of power shortage scenario, SS is the set of feeder sections within the system. T_c is the set of time periods of shortage scenario c, $\Delta T_{t,c}$ is the duration for period t of scenario c. $f_{s,0,c}^{out\_mwh}$, $f_{0,c}^{sw\_fq}$, $f_{s,0,c}^{out\_hr}$ and $f_{s,0,c}^{out\_fq}$ are the accumulated lost energy for section s, the accumulated number of switch operations for the system, the accumulated outage hours and times for section s at time $t = 0$, under scenario c, respectively. $P_{s,t,c}^{net}$ is the net load for section s at period t of scenario c. $x_{s,t,c}^{isolated}$ is a binary variable indicting the isolated state for feeder section s at period t of scenario c, 1 indicated isolated, and 0 indicated energized. $P_{s,t,c}^{net}$ is the net load for feeder section s at period t of scenario c. SW is the set of switches within the system. $x_{sw,t,c}^{closed}$ is a binary variable indicting the closed state for swicth sw at period t of scenario c, 1 indicated closed, and 0 indicated open. $x_{s,t,c}^{out}$ is a binary variable to indicate if section s enters an outage status at period t of scenario c.

The net load for section, $P_{s,t,c}^{net}$ is calculated as:

$$P_{s,t,c}^{net} = \left(\sum_{ld\in LD_s} P_{ld,t,c}^{D} - \sum_{sg\in SG_s} P_{sg,t,c}^{G} - \sum_{wd\in WG_s} P_{wd,t,c}^{G} - \sum_{es\in ES_s}\left(p_{es,t,c}^{DCH} - p_{es,t,c}^{CH}\right)\right) \quad (7)$$

here LD_s, SG_s, WG_s and ES_s are the sets of loads, solar plants, wind plants and battery energy storages within section s. $P_{ld,t,c}^{D}$, $P_{sg,t,c}^{G}$, $P_{wg,t,c}^{G}$ $p_{es,t,c}^{DCH}$, and $p_{es,t,c}^{CH}$ are load demand, solar power generation, wind power generation, storage discharging and charging powers of load ld, solar plant sg, wind plant wg and stoarge es at period t of scenario c, respectively. The outage status for section s, $x_{s,t,c}^{out}$ is calculated as:

$$x_{s,t,c}^{out} = \max\left\{x_{s,t,c}^{isolated} - x_{s,t-1,c}^{isolated}, 0\right\} \quad (8)$$

It is noted that T_c is the time horizon for the power shortage event if the rotation is implemented without adapting to blackout evolving, i.e. a static rotation scheduling method is adopted, but the rolling scheduling horizon if a rolling rotation scheduling approach is used. For a rolling rotation approach, the pre-existing condition such as previous outage hours and times for each feeder section before the new rolling scheduling horizon is started, are included when evaluating each objective function. The static method will be used if a accurate projection for the power shortage evolving process can be obtained, otherwise the rolling horizon method will be used and the existing conditions will be considerted when triggering a new solution.

The isolated status of the feeder section is determined by the energized statuses for all paths between sources (i.e. feeder heads) and the feeder section. One of paths is energized, then the section is energized. The section is isolated only if all paths are not energized.

$$x_{s,t,c}^{isolated} = 1 - \bigvee_{p \in PT_s^{SRC}} x_{p,t,c}^{energized} \tag{9}$$

where PT_s^{SRC} is the set of paths between section s and all sources (i.e. feeder heads), $x_{p,t,c}^{energized}$ is a binary variable indicating the energized status of path p at period t of scenario c, 1 indicated energized, and 0 not energized.

The energized status of a path between a feeder section and a source depends the closed statuses of all switches resided on the path. Any switch is at open status, the path is not energized. Only all switches are energized, then the path is energized.

$$x_{p,t,c}^{energized} = \bigwedge_{sw \in SW_p^{SRC}} x_{sw,t,c}^{closed} \tag{10}$$

SW_p^{SRC} is the set of swicthes residing on path p.

The blackout rotation is subject to technical constraints required by device characteristics, power flow, system radial operation, and power shortage conditions.

It is assumed that the predictions of load demands, solar and wind generations are available and can be treated as fixed values. Only the discharging and charging powers of battery storages are to be determined. The operational characteristics of the storage $es \in ES$ at time period $t \in T_c$can be modeled using the following equations:

$$SOC_{es,t,c} = (1 - \varepsilon_{es}) SOC_{es,t-1,c} + \eta_{es}^{CH} \frac{p_{es,t,c}^{CH} \Delta T_{t,c}}{\overline{E}_{es}} - \frac{p_{es,t,c}^{DCH} \Delta T_{t,c}}{\eta_{es}^{DCH} \overline{E}_{es}} \tag{11}$$

$$x_{es,t,c}^{CH} + x_{es,t,c}^{DCH} \leq 1 \tag{12}$$

$$0 \leq p_{es,t,c}^{CH} \leq x_{es,t,c}^{CH} \overline{P}_{es}^{CH} \tag{13}$$

$$0 \leq p_{es,t,c}^{DCH} \leq x_{es,t,c}^{DCH} \overline{P}_{es}^{DCH} \tag{14}$$

$$x_{es,t,c}^{CH}, x_{es,t,c}^{DCH} = \{0,1\} \tag{15}$$

where $SOC_{es,t,c}$ is the state of charge for storage es at period t of shortage scenario c. $x_{es,t,c}^{CH}$ and $x_{es,t,c}^{DCH}$ are binary variables to indiccate storage es is charging or discharging at period t of shortage scenario c. ε_{es}, η_{es}^{CH}, η_{es}^{DCH} are the self-consumption coefficiency, charging efficiency and dicharging effiiciency for storage es. $\overline{P}_{es}^{CH}$, $\overline{P}_{es}^{DCH}$ and $\overline{E}_{es}$ are the maximum charging power, maximum dischaging power and maxium energy for storage es. (11) describes the energy balance of the storage.

The power shortage for power grid is described by the profiles for upper and lower generation bounds of each substation under the given scenario along the shortage horizon.

$$\underline{P}_{sub,t,c}^{G} \leq P_{sub,t,c}^{G} \leq \overline{P}_{sub,t,c}^{G} \tag{16}$$

where $P_{sub,t,c}^{G}$ is the power supplied by substaion sub at time period t of scenario c. $\overline{P}_{sub,t,c}^{G}$ and $\underline{P}_{sub,t,c}^{G}$ are upper and lower bounds of substation supplied powers.

The distribution system can only be operated as radial, not meshed or looped. Let FDR be the set of feeders within the distribution system. For any path between any two of feeder heads, $fdr1, fdr2 \in FDR, fdr1 \neq fdr2$, the radial operation contraint must be applied, that is at least one of switches along the path between two feeder heads must be at open state:

$$\sum_{sw \in SW_{fdr1 \leftrightarrow fdr2,p}^{FDR}} x_{sw,t,c}^{closed} \leq \left(\sum_{sw \in SW_{fdr1 \leftrightarrow fdr2,p}^{FDR}} 1 \right) - 1 \tag{17}$$

where $SW_{fdr1 \leftrightarrow fdr2,p}^{FDR}$ is the set of swicthes residing on path p between feeder heads of feeders $fdr1$ anad $fdr2$.

Power balance or power flow constraints are required for both the substations and feeder sections. For any substation, the power supplied from main grid matches the sum of power entered into the distribution system through feeders :

$$P_{sub,t,c}^{G} = \sum_{fdr \in FDR_{sub}} \sum_{sw \in SW_{fdr}^{CB}} \left(f_{sw,t,c}^{+} - f_{sw,t,c}^{-} \right) \tag{18}$$

where FDR_{sub} is the set of feeders connected with substation sub, SW_{fdr}^{CB} is the set of circuit breakers located at feeder head of feeder fdr. $f_{sw,t,c}^{+}$ and $f_{sw,t,c}^{-}$ are bi-directional power flows on switch sw at time period t of scenario c, and postive direction indicates power entering the feeder, and negative indicates power leaving the feeder. For feeder section, the power balance is activated only when the section is not isolated.

$$\left(1 - x_{s,t,c}^{isolated}\right) P_{s,t,c}^{net} = \sum_{sw \in SW_s} \left(f_{sw,t,c}^{+} - f_{sw,t,c}^{-} \right) \tag{19}$$

where the postive-direction power flow $f_{sw,t,c}^{+}$ indicates power entering the section, and negative $f_{sw,t,c}^{-}$ indicates power leaving the feeder.

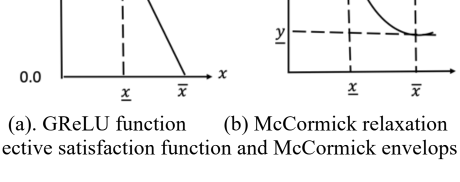
The power flows of the swicthes are aslo limited by their maximum capacities as:

$$f_{sw,t,c}^{+} \leq x_{sw,t,c}^{closed} \overline{F}_{sw,c} \tag{20}$$

$$f_{sw,t,c}^{-} \leq x_{sw,t,c}^{closed} \overline{F}_{sw,c} \tag{21}$$

$\overline{F}_{sw,c}$ is the mximum capacity of switch sw under shortage scenario c.

The multiple objective optimization problem can be combined into a single-objective ones by using a satisfaction function to describe each objective's satisfied degree and defines a new objective to seek the maximum of the weighted sum of satified degrees or minmal satisfied degree, or combined as used in this paper. Usually the satisfacation function is a non-linear function, such as Sigmoid function. For computation simplification, a generalized version of rectified linear activation function (ReLU) [7] is used for such purpose. The ReLU is generalizied as GReLU with output capped at 1 with a configurable non-zero gradient as shown in FIG. 1(a). Fig 1(a) illustrates a GReLU with negative-gradient GReLU, G$ReLU^{-}$ in which the output y is decreased with the increase of input x. $\overline{x}$ and $\underline{x}$ represents its most desirable value and least tolerable value for the input x for a $GReLU^{-}$ function.



(a). GReLU function (b) McCormick relaxation

FIG. 1. Objective satisfaction function and McCormick envelops

The blackout rotation problem can be solved using the following objective and constraints:

$$\text{Max} \sum_{obj\in OBJ} \omega^{obj} GReLU^{-}\left(f_c^{obj}\right) + \lambda \tag{22}$$

$$\text{s.t. } GReLU^{-}\left(f_c^{out\_mwh}\right) \geq \omega^{out\_mgh}\,\lambda \tag{23}$$

$$GReLU^{-}\left(f_c^{sw\_fq}\right) \geq \omega^{sw\_fq}\,\lambda \tag{24}$$

$$GReLU^{-}\left(f_c^{out\_hr}\right) \geq \omega^{out\_hr}\,\lambda \tag{25}$$

$$GReLU^{-}\left(f_c^{out\_fq}\right) \geq \omega^{out\_fq}\,\lambda \tag{26}$$

$$GReLU^{-}\left(f_c^{diff\_out\_hr}\right) \geq \omega^{diff\_out\_hr}\,\lambda \tag{27}$$

$$GReLU^{-}\left(f_c^{diff\_out\_fq}\right) \geq \omega^{diff\_out\_fq}\,\lambda \tag{28}$$

Constraints. (7)-(21)

where OBJ is the set of objectives that modeled using G$ReLU^{-}$ functions, ω^{obj} is the weight factor for objectve obj, λ is the minal satisfied degree among all objectives.

III. Model Conversion and Solution

The above-formulated model is an non-linear optimization problem with mixed binary and continuous variables and logic operations. A directed feasible region based appracoh is proposed to convert the nonlinear optimization model into a standard mixed integer linear programming problem(MILP). This approach replaces the nonlinear operations with an embedded auxiliary optimization problem with a linear objective constarined by a feasible region defined by linear equations.

We first deal with nonlinear function, $GReLU^{-}(f_c^{obj})$ using directed feasible range based approach. Assumed the input variable f_c^{obj} has a tolerable upper limit $\bar{f}_c^{obj}$, and a desirable lower limit $\underline{f}_c^{obj}$, the feasible region for function $GReLU^{-}(f_c^{obj})$ output can be described using three related variables as:

$$GReLU^{-}\left(f_c^{obj}\right) = y_c^{obj} + \Delta y_c^{obj-} - \Delta y_c^{obj+} \tag{29}$$

y_c^{obj} is a continuous variable, Δy_c^{obj-} and Δy_c^{obj+} are two non-negative variables. An auxiliary optimization problem defined as follows can be embedded into the original blackout rotation problem to equivalently represent the function of $GReLU^{-}\left(f_c^{obj}\right)$:

$$maximize\ (y_c^{obj} - \Delta y_c^{obj-} - \Delta y_c^{obj+}) \tag{30}$$

$$\text{Subject to: } y_c^{obj} = \left(\bar{f}_c^{obj} - f_c^{obj}\right) \Big/ \left(\bar{f}_c^{obj} - \underline{f}_c^{obj}\right) \tag{31}$$

$$0 \leq y_c^{obj} + \Delta y_c^{obj-} - \Delta y_c^{obj+} \leq 1 \tag{32}$$

$$-M \leq y_c^{obj} \leq M \tag{33}$$

$$0 \leq \Delta y_c^{obj-}, \Delta y_c^{obj+} \leq M \tag{34}$$

M is a positive number that greater than one. The auxiliary objective (30) is embedded into the original problem's objective function to direct the optimization of variable f_c^{obj} within the feasible region represented by (31)-(34). In summary, using the approach, $GReLU^{-}\left(f_c^{obj}\right)$ in (22) and (23)-(28) are replaced with $(y_c^{obj} - \Delta y_c^{obj-} - \Delta y_c^{obj+})$ and $(y_c^{obj} + \Delta y_c^{obj-} - \Delta y_c^{obj+})$, and (31)-(34) are added into the existing constraints of the original problem.

The second type of issues to be dealt with directed feasible region approach are the absolute and max operations of binary or continuous variables, such as used in (2), and (3). The absolute function is defined as:

$$y = |x| \tag{35}$$

Its directed feasible range can be defined by using (36) as the objective, and (37)-(38) as constraints.

$$\max -Ny \tag{36}$$

$$\text{s.t.} \quad y \geq x \tag{37}$$

$$y \geq -x \tag{38}$$

where N is a small number and much smaller than any value for other variable defined in the objection function (22), such as 0.001. The max function is defined as:

$$y = \max_{1\leq i\leq n} x_i \tag{39}$$

Its directed feasible range can be defined as constraint (40) with optimization directed by (36):

$$y \geq x_i, \quad \forall\ 1 \leq i \leq n \tag{40}$$

The third type of issues to be dealt with by the directed feasible region approach are the logic operations of binary variable used by the feeder sections and switches, such as in (9) and (10). Assumed

$$y = \wedge_{i=1}^{n} x_i \tag{41}$$

where x_i is a binary variable. The consecutive "and" operations can also be handled by a directed feasible region by taken (36) as objective and (42) and (43) as constraints.

$$y \leq x_i, \forall i = 1,2,\dots,n. \tag{42}$$

$$y \geq \sum_{i=1}^{n} x_i - n + 1, \forall i = 1,2,\dots,n. \tag{43}$$

Similarly, for a series of binary variables, x_i , its consecutive "or" operations can be replaced as a directed feasible region as well. Assumed

$$y = \vee_{i=1}^{n} x_i \tag{44}$$

Its directed feasible range can be defined using (36) as auxiliary objective, and (45)-(46) as constraints.

$$y \geq x_i, \forall i = 1,2,\dots,n. \tag{45}$$

$$y \leq \sum_{i=1}^{n} x_i \tag{46}$$

In addition to above-described functions and operations, we also have non-linear items introduced by multiplying one binary variable with one continuous variable in the formulated model, such as $x_{s,t,c}^{isolated} p_{s,t,c}^{net}$ in (1). Those operations can be handled by using McCormick relaxation method [8] to represent as linear function and constraints. Fig. 1(b) gives a graphical representation of McCormick relaxation method. Assumed there are one binary variable, x and one continuous variable, y. Their product, $w = xy$ has a feasible region defined using as:

$$x\underline{y} \leq w \leq x\bar{y} \tag{47}$$

$$y + (x-1)\bar{y} \leq w \leq y + (x-1)\underline{y} \tag{48}$$

where $\bar{y}$ and $\underline{y}$ are the upper and lower bounds of y. We can simply replace the product using a single variable and add those constraints into the optimization problem.

By using above mentioned method, the blackout rotation problem can be converted into a MILP problem, and solved using commercial out-of-shelf software.

IV. Numerical Examples

The proposed method has been tested using a sample system as shown in Fig. 2. This system is modified from the IEEE 123 node test feeder [9] which operates at a nominal voltage of 4.16 kV.

The modified system includes two substations, and six feeders. Substation SUB-I connects with feeders FDR-I(node 150), FDR-II(node 195) and FDR-III(node 251), and substation SUB-II connects with feeders FDR-IV(node 350), FDR-V(node 451) and FDR-VI(node 610). The system includes three solar plants at nodes 13,76,101 with capacities 20 kW each per phase, two wind plants at nodes 47 and 57 with capacities of 20 kW each per phase, and five battery energy storage systems at nodes 7,36,58,105,72 with capacities 500 kWh/50kW each per phase. The efficiency of charging/discharging for each battery is 95%, and its self-consumption factor is 0.1%. The initial setting and lower/upper bounds for state of charge for each battery are 0.5 and 0.4/0.8, respectively. The system includes seven normally-open switches (13-152, 18-135, 23-235, 158-105, 160-60, 72-672, and 94-54), and eleven normally-closed switches (150-149, 195-95, 251-250, 350-300, 451-450, 610-61, 7-178, 447-44, 97-197, 867-86, and 151-300).

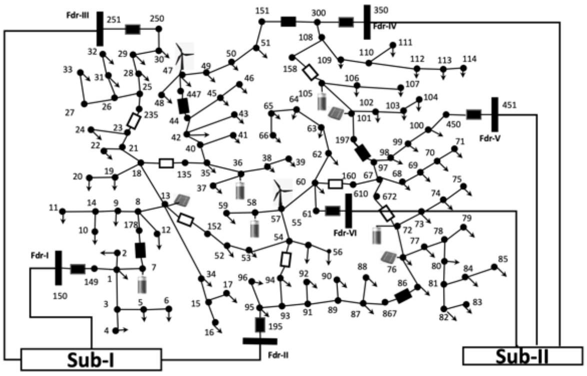


Fig. 2. The modified IEEE 123 node test feeder

As shown in FIG. 3, according to switch locations within the distribution system, the system can be partitioned into 11 feeder sections.

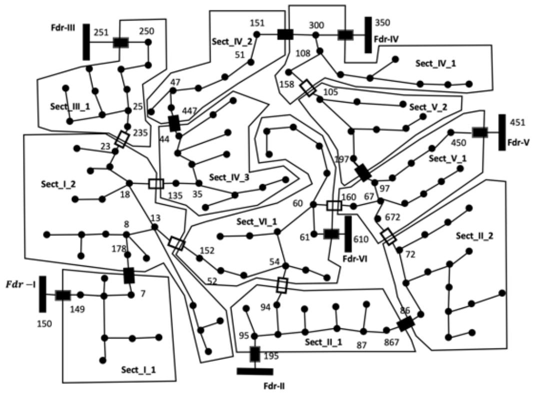


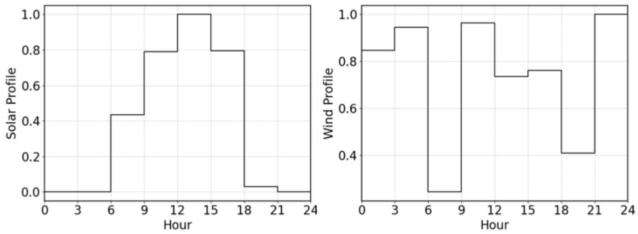
Fig. 3. The partitioned feeder sections for the sample system

Assumed that extreme weather caused 50% generation capacity of the main grid lost for 24 hours. The generation profiles (in per unit) for solar plants and wind plants during the power shortage are given in Fig. 4(a) and Fig. 4(b). The projected hourly load profile during power shortage, and substation normal power supply profile are given in Fig. 5(a), and Fig. 5(b). All profiles are given by 24 hours with 8 time periods.

Three blackout rotation schemes have been generated under different optimization and resource conditions. Scheme I is generated by using total lost energy minimization as optimization objective and ignoring the battery storages for energy balancing. Same as scheme I, scheme II also ignores the storages, but uses multiple objectives including rotation fairness when generating the blackout rotation scheme. Scheme III is generated by considering both multiple optimization objectives and storage contribution for energy balancing.

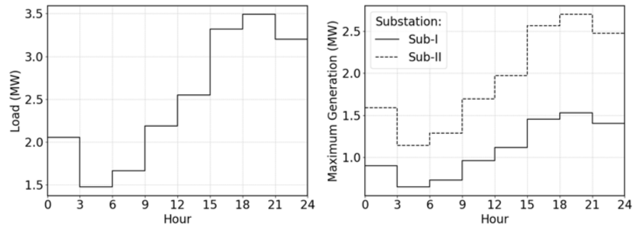
The summarized attributes for 3 different schemes are given in Table I. The final statuses for switch operations at the beginning of each time period within power shortage horizon are given in Table II.

The rolling blackout schedules (in terms of blackout starting hours and blackout ending hours) for each scheme are shown in Fig. 6, Fig. 7 and Fig. 8, respectively.



(a). Solar Plant (b). Wind Plant

FIG. 4. Hourly Generation profiles



(a).Load (b). Substation

FIG. 5. Hourly load and substation generation profile

TABLE I. EVALUATION ATTRIBUTES FOR ROTATION SCHEMES

Attribute	**Scheme**			**Expected Ranges**
	I	***II***	***III***	
Lost Energy (MWh)	22.64	25.95	19.97	[0, 60]
Switching times	2	26	11	[0, 100]
Maximum blackout hours	24	12	15	[0, 24]
Maximum blackout times	1	2	1	[0, 10]
Maximum blackout-hour difference	24	3	6	[0, 24]
Maximum blackout-times difference	1	0	0	[0, 10]

TABLE II. SWITCH OPERATIONS FOR ROTATION SCHEMES

Blackout Period	**Scheme**		
	I	***II***	***III***
0:00-3:00	Open: 195-95,610-61	Closed: 72-672;Open: 150-149,195-95,251-250,451-450	Open: 150-149, 251-250,610-61
6:00-9:00		Closed: 195-95,451-450; Open: 350-300, 610-61, 867-86	Open:350-300
9:00-12:00		Closed: 150-149, 251-250,867-86; Open: 195-95	Closed: 150-149, 251-250,610-61; Open: 195-95
12:00-15:00		Closed: 18-135,158-105;Open: 72-672	Open: 450-451
15:00-18:00			Closed: 350-300
18:00-21:00		Closed: 195-95,610-61,13-152,23-235; Open: 150-149,451-450,7-178,18-135	
21:00-24:00		Closed: 7-178	Closed: 158-105

As shown in the tables, using Scheme I, the system will lose energy 22.64 MWh, needs 2 switch operations, and causes blackout lasting maximally 24 hours and occurring once for the event period. As shown in Fig. 6, this is a totally biased rotation scheme. The consumers on feeders FDR-II and FDR-VI are expected to experience a blackout for 24 hours, and consumers at other feeders do not have any blackouts at all. This scheme is generated by taking minimizing total lost energy as objective which are commonly used by utilities. Although it has minimized the lost energy and switching operations to maximum extent, it does not do any reasonable blackout rotation among consumers at all.

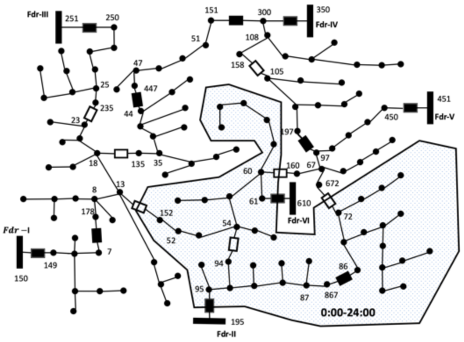


FIG. 6. Blackout rotation scheme for Scheme I.

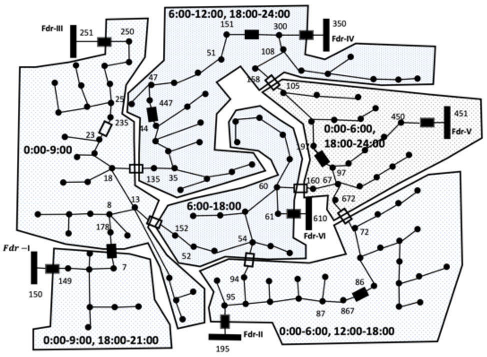


FIG. 7. Blackout rotation scheme for Scheme II.

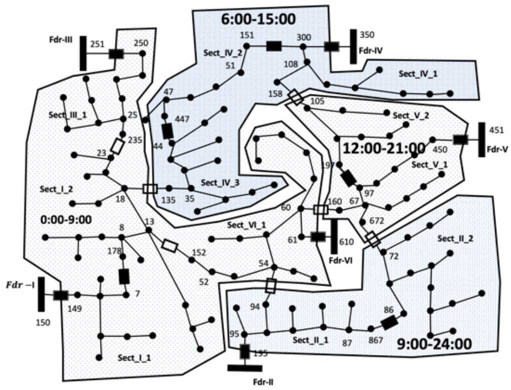


FIG. 8. Blackout rotation scheme for Scheme III.

In comparison, Scheme II is generated using all objectives representing different interests from consumers, system and fairness consideration. As indicated in Tables I-II, its lost energy is 25.95 MWh, switching times is 26, and maximum outage duration and times are 12 and 2, respectively. Compared with Scheme I, the lost energy is slightly increased 14.62%, and the number switching operations has also increased to an amount within its tolerable bounds (i.e. [0,100]). However, the consumers will experience shorter blackout and fair blackout rotation. Using scheme II, the maximum blackout hours will be reduced 50.0%, from 24 hours to 12 hours, and the maximum blackout hour difference will be 3 hours instead of 24 hours for scheme I, decreased 75%. The maximum deviations of blackout duration and times among different sections are 3 hours and 0, respectively. As shown in Fig. 7, the blackout will be rotated among 6 different regions, and only 3-hour difference will be observed for blackouts among different regions of the system. Each region may include multiple feeder sections.

Using Scheme III, the system will lose energy 19.97 MWh and require 11 switch operations. It will result in outage lasting longest 15 hours and occurring once during the power shortage period. This scheme rotates blackout among 4 different regions, and corresponding maximum deviations for outage duration and frequency are 6 hours and 0, respectively. Comparing schemes III with II, it can be observed that using storages during blackout rotation, it can help reducing the total lost energy and the required switch operations while maintaining reasonable deviations on outage duration and frequency among consumers at different locations.

V. CONCLUSION

This paper has presented a multiple-objective optimization model for scheduling a fair blackout rotation for distribution systems under power shortage events. The model considers the system requirements on lost energy and switch operation, the consumers' requirements on outage duration and frequency, and fairness requirements on inter-regional deviations of outage duration and frequency simultaneously. The formulated multi-objective nonlinear optimization problem is converted into a standard single-objective mixed integer linear programming one by defining each objective's satisfaction degree using a Generalized Rectified Linear Units (GReLUs), and replacing each non-linear items (including GReLUs, binary variable operation and logic operation) using an auxiliary optimization problem with a linear objective and feasible regions bounded by linear equations. The test results on the modified IEEE-123 node test feeder have preliminarily demonstrated the effectiveness of the proposed method.

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