

Flying Start of Sensorless Synchronous Machines with Reactive Power Injection

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Abstract

In this work, a flying start of sensorless synchronous machines based on the reactive power injection is proposed to identify the initial rotor position and angular speed under non-stationary conditions. This is realized with a novel control structure designed in the stator current reference frame such that it produces negligible braking torque and has minimal disturbances on the load. The proposed method benefits from a simple structure, low reliance on the motor model and is applicable to the family of synchronous machines. Moreover, the paper provides guidelines for gain selection based on the nominal nameplate parameters and discusses sensitivity analysis to errors in stator resistance. The proposed method is validated in simulation on a 5.5 kW permanent magnet-assisted synchronous reluctance (PM-SyR) machine.

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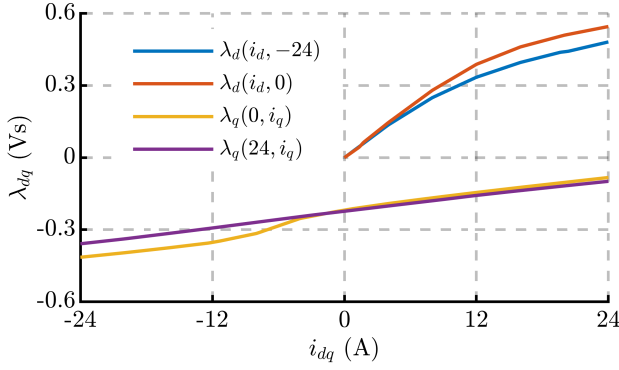


Fig. 2. Flux map of the 75.5 kW PM-SyR motor under test, experimentally identified with the constant speed test [12].

Real space vectors will be used; for example, the stator current is $\mathbf{i}_{dq} = [i_d, i_q]^T$ where i_d and i_q are the vector components in rotor reference frame. Space vectors in the stationary reference frame are denoted by subscript $\alpha\beta$. Note that the convention of a SyR machine is followed, i.e., d -axis is defined along the maximum inductance path. Magnets, if any, are along the negative q -axis.

A. Mathematical Model in dq Reference Frame

The voltage equation of a synchronous machine in rotor reference frame is expressed as

$$s \boldsymbol{\lambda}_{dq} = \mathbf{v}_{dq} - R_s \mathbf{i}_{dq} - \omega \mathbf{J} \boldsymbol{\lambda}_{dq} \quad (1)$$

where R_s is the stator resistance and $\boldsymbol{\lambda}_{dq}$ is the stator flux linkage. The incremental inductance is defined as

$$\mathbf{L}_\partial = \frac{\partial \boldsymbol{\lambda}_{dq}}{\partial \mathbf{i}_{dq}} = \begin{bmatrix} l_d & l_{dq} \\ l_{dq} & l_q \end{bmatrix} \quad (2)$$

where l_d, l_q represents the incremental inductance along direct d and quadrature q -axis, respectively, while l_{dq} is the cross-saturation term. All quantities are functions of \mathbf{i}_{dq} . The stator flux linkage is expressed as

$$\boldsymbol{\lambda}_{dq} = \mathbf{L}(\mathbf{i}_{dq}) \cdot \mathbf{i}_{dq} + \boldsymbol{\lambda}_m = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \cdot \mathbf{i}_{dq} + \begin{bmatrix} 0 \\ -\lambda_m \end{bmatrix} \quad (3)$$

where \mathbf{L} is the apparent inductance matrix and $\boldsymbol{\lambda}_m$ is the open circuit permanent magnet flux linkage; $\lambda_m = 0$ corresponds to the SyR machine. The saturation and cross-saturation properties of the PM-SyR machine under test is illustrated in the flux-map in Fig.2 that is experimentally identified with constant speed test reported in [12].

The average electromagnetic torque of a synchronous machine is given by

$$T = \frac{3p}{2} \mathbf{i}_{dq}^T \mathbf{J} \boldsymbol{\lambda}_{dq} \quad (4)$$

where p is the number of pole pairs.

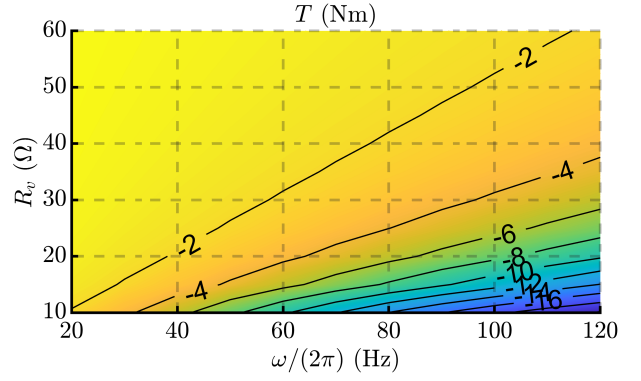


Fig. 3. Analytical evaluation of the steady-state braking torque for the virtual resistance flying start method; contours are plotted as functions of the rotor speed (0.2 - 2 p.u.) and the virtual resistance.

B. Flying Start with Virtual Resistance Method

The virtual resistance based flying start [10] is among the state-of-art method to estimate the initial mechanical states. The inverter output voltage and the maximum virtual resistance condition are given by

$$\mathbf{v}_{\alpha\beta} = -R_v \mathbf{i}_{\alpha\beta} \quad 0 < R_s + R_v < \frac{L_q}{T_s} \quad (5)$$

where R_v is the virtual resistance and T_s is the sampling time. For the voltage (5), the dynamics of the stator current derived from (1) and (3) are governed by

$$\mathbf{i}_{dq} = \frac{-\omega \mathbf{J} \boldsymbol{\lambda}_m}{(R_v + R_s) \mathbf{I} + \omega \mathbf{J} \mathbf{L} + s \mathbf{L}_\partial} \quad (6)$$

and the dq steady-state currents are given by

$$\mathbf{i}_{dq} = -\omega \left((R_v + R_s) \mathbf{I} + \omega \mathbf{J} \mathbf{L} \right)^{-1} \mathbf{J} \boldsymbol{\lambda}_m. \quad (7)$$

Using (7) in (4), the resulting steady-state braking torque is computed and shown in Fig. 3 as functions of the angular rotor speed and the virtual resistance for the machine under test. The braking torque is observed to increase with increasing rotor speed and decreasing virtual resistance; it is, however, always non-zero and the machine is susceptible to disturbances. Having $R_v = -R_s$ in (7) results in zero torque ($i_d = 0$) but the poles in (6) become purely imaginary that is practically unstable.

The rotor position estimation uses a phase-locked-loop (PLL) to track the stator current, i.e.,

$$\hat{\theta} = \angle \mathbf{i}_{\alpha\beta} \quad (8)$$

where $\hat{\theta}$ is the estimated rotor position. The stator current (7) converges to the second quadrant for positive motor speed (and first quadrant for negative motor speed), leading to a steady-state position error given by $\hat{\theta} = \theta - \hat{\theta} = \angle \mathbf{i}_{dq}$. Fig. 4 shows the position error contour where it is observed to increases with the virtual resistance. Thus, additional compensation that is equal to the current angle (7) based on the machine parameters is necessary for the position error correction.

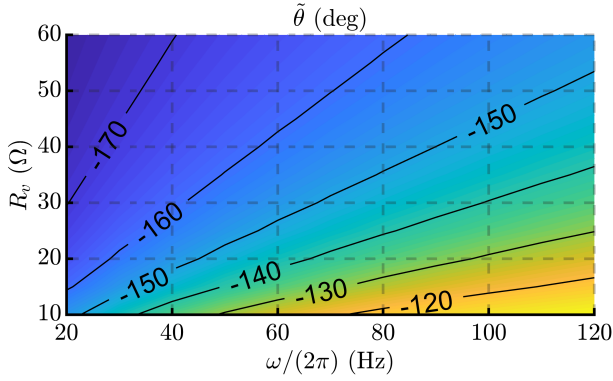


Fig. 4. Analytical evaluation of the position error for the virtual resistance flying start method; contours are plotted as functions of the rotor speed (0.2 - 2 p.u.) and the virtual resistance.

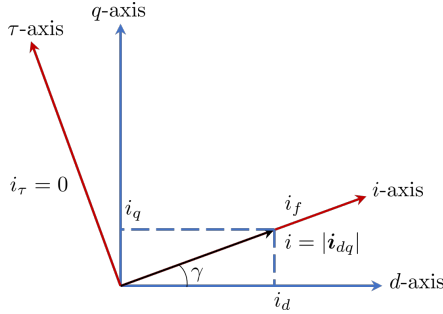


Fig. 5. Illustration of the $i\tau$ reference frame where the i -axis is oriented along the stator current vector.

Alternatively, a virtual impedance has been proposed [11] with resistive (R_v) and inductive (L_v) components as

$$\mathbf{v}_{\alpha\beta} = -(R_v + sL_v) \mathbf{i}_{\alpha\beta} \quad (9)$$

where the steady-state stator current expression becomes

$$\mathbf{i}_{dq} = -\omega \left((R_v + R_s) \mathbf{I} + \omega \mathbf{J} (L + L_v \mathbf{I}) \right)^{-1} \mathbf{J} \boldsymbol{\lambda}_m. \quad (10)$$

Using $L_v \mathbf{I} = L$ mitigates the position error and the stator current converges to the d -axis ($i_q = 0$). However, this is only valid for non-salient surface PMSM machines ($L_d = L_q$). Moreover, the braking torque due to the magnetic component is non-zero.

III. PROPOSED REACTIVE POWER INJECTION METHOD

The block diagram of the proposed flying start method is shown in Fig.1 and is implemented in the $i\tau$ stator current reference frame.

A. Mathematical Model in $i\tau$ Reference Frame

The stator current reference frame, shown in Fig.5, is denoted by the subscript $i\tau$ where the i -axis is aligned along the stator current vector and τ -axis is the quadrature axis, i.e.,

$$\mathbf{i}_{i\tau} = \begin{bmatrix} i_i \\ 0 \end{bmatrix} = e^{-\mathbf{J}\gamma} \cdot \mathbf{i}_{dq} \quad (11)$$

where $i_i = |\mathbf{i}_{dq}|$ is the current magnitude and $\gamma = \angle \mathbf{i}_{dq}$ is the current angle. The voltage equation (1) transforms to

$$e^{\mathbf{J}\gamma} \mathbf{L}_{\partial} e^{-\mathbf{J}\gamma} \begin{bmatrix} s i_i \\ i_i s \gamma \end{bmatrix} = \mathbf{v}_{i\tau} - R_s \begin{bmatrix} i_i \\ 0 \end{bmatrix} - \omega \mathbf{J} \boldsymbol{\lambda}_{i\tau} \quad (12)$$

where the state variables are i_i and γ . The torque equation (4) simplifies to

$$T = \frac{3p}{2} \lambda_{\tau} i_i. \quad (13)$$

Thus, it follows that the voltage equation (12) can be rewritten in terms of active and reactive power as

$$e^{\mathbf{J}\gamma} \mathbf{L}_{\partial} e^{-\mathbf{J}\gamma} \begin{bmatrix} s i_i \\ i_i s \gamma \end{bmatrix} = \mathbf{v}_{i\tau} - R_s \begin{bmatrix} i_i \\ 0 \end{bmatrix} - \frac{2}{3 i_i} \begin{bmatrix} P_{mech} \\ Q \end{bmatrix} \quad (14)$$

where Q is the reactive power and P_{mech} is the mechanical active power output. For simplicity in analysis, the cross-saturation inductance is neglected, $l_{dq} = 0$, i.e.,

$$e^{\mathbf{J}\gamma} \mathbf{L}_{\partial} e^{-\mathbf{J}\gamma} = \begin{bmatrix} l_{\Sigma} + l_{\Delta} \cos(2\gamma) & l_{\Delta} \sin(2\gamma) \\ l_{\Delta} \sin(2\gamma) & l_{\Sigma} - l_{\Delta} \cos(2\gamma) \end{bmatrix} \quad (15)$$

where $l_{\Sigma} = (l_d + l_q)/2$ and $l_{\Delta} = (l_d - l_q)/2$.

B. Control System for Reactive Power Injection

The proposed control scheme has two objectives: (i) inject a constant stator current of magnitude i_i^* and (ii) ensure the active power is driven to zero, $P_{mech} = 0$, to mitigate the braking torque disturbances. To this end, a proportional-integral (PI) controller in the i -axis is designed to establish a constant current $i_i = i_i^*$ as

$$v_i^* = \left(k_{pi} + \frac{k_{ii}}{s} \right) (i_i^* - i_i) + R_s i_i \quad (16)$$

where k_{pi} and k_{ii} are the proportional and integral gains, respectively. An orthogonal τ -axis controller is used to regulate the active power to zero, P_{mech} . Assuming $i_i^* \approx i_i$, the active power estimate \hat{P}_{mech} can be expressed as

$$\hat{P}_{mech} = \frac{3 i_i}{2} (v_i^* - R_s i_i) \quad (17)$$

Thus, the τ -axis PI controller is designed as the following:

$$v_{\tau}^* = \left(k_{p\tau} + \frac{k_{i\tau}}{s} \right) \hat{P}_{mech} \quad (18)$$

where and $k_{p\tau}$ and $k_{i\tau}$ are the corresponding gains.

C. Tuning Guidelines

The controller gains are tuned by representing the control variables (i_i and P_{mech}) in terms of the state variables (i_i and γ). The i -axis gains for the critically damped poles at $s = -\Omega_i$ are given by

$$k_{pi} = \mu_i 2 \Omega_i \quad k_{ii} = \mu_i \Omega_i^2 \quad (19)$$

where the scaling factor μ_i follows from (14) and (15) as

$$\mu_i = l_{\Sigma} + l_{\Delta} \cos(2\gamma). \quad (20)$$

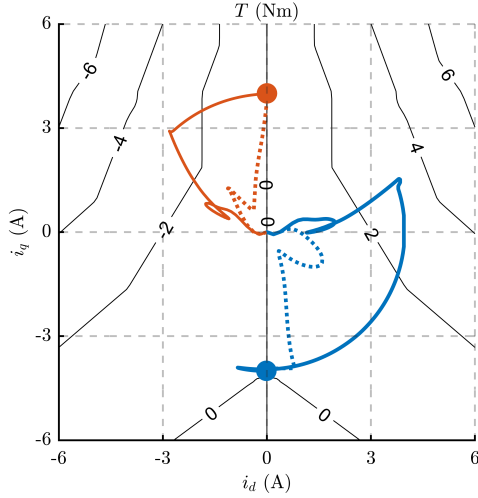


Fig. 6. Simulated current trajectories of the proposed technique shown against the torque contour in black. Positive rated speed $\omega = 2\pi \cdot 60$ rad/s (1800 rpm) is shown in red and negative rated speed $\omega = -2\pi \cdot 60$ rad/s (-1800 rpm) in blue. The solid lines have slow τ -axis dynamics $\Omega_\tau = 2\pi \cdot 10$ rad/s and the dotted lines have faster dynamics $\Omega_\tau = 2\pi \cdot 50$ rad/s. Marker \circ denotes the steady-state settling point. The current reference is $i_i^* = 4$ A, satisfies (24).

Likewise, the τ -axis gains are tuned for critically damped poles at $s = -\Omega_\tau$ as

$$k_{p\tau} = \mu_\tau 2\Omega_\tau \quad k_{i\tau} = \mu_\tau \Omega_\tau^2. \quad (21)$$

The scaling factor μ_τ is computed from the linearized model as

$$\mu_\tau = \frac{i \left(l_\Sigma - l_\Delta \cos(2\gamma) \right)}{\partial P_{mech} / \partial \gamma} \quad (22)$$

where the term $\partial P_{mech} / \partial \gamma$ can be expressed as

$$\frac{\partial P_{mech}}{\partial \gamma} = \frac{3}{2} \omega \left(l_\Delta i^2 2 \cos(2\gamma) - \lambda_m i \sin(\gamma) \right) \quad (23)$$

It is apparent that the scaling factors are operating point dependent. Hence, an approximate gains are computed using the nominal parameters at rated speed and considering the initial condition $\gamma = 0$.

D. Convergence Discussion

For positive angular speeds, the open-circuit back-emf due to the magnets in the negative q -axis is directed along the negative d -axis. Hence, the initial dq stator current evolves in the braking region in second/third quadrant. The negative mechanical active power (P_{mech}) is input to the τ -axis controller that drives the state variable γ in the clockwise direction. This leads to the convergence of the stator currents to the zero-torque locus in the positive q -axis.

Fig. 6 illustrates the evolution of the dq current trajectories juxtaposed against the torque contour. The results for the flying start at positive rated speed (shown in red) appear in the negative torque region and converge to the positive q -axis. The

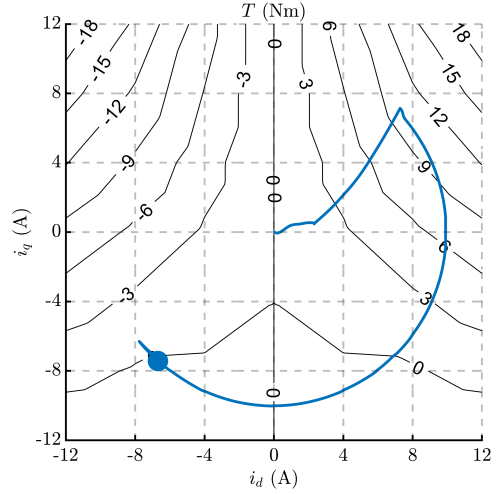


Fig. 7. Simulated current trajectories of the proposed technique at negative rated speed $\omega = -2\pi \cdot 60$ rad/s (-1800 rpm) and $\Omega_\tau = 2\pi \cdot 10$ rad/s. Marker \circ denotes the steady-state settling point. The current reference is $i_i^* = 10$ A, violates (24) leading to position estimation error.

solid curves correspond to the test with slow τ -axis dynamics to help illustrate the convergence where the stator current is observed to move along the constant current locus, $i_i^* = i_i = 4$ A, towards the q -axis.

Similar discussion follows for the flying start at negative angular speed where the currents evolve in the positive torque region in first/fourth quadrant. However, to ensure that the stator currents settle in the zero-torque locus in the negative q -axis, the maximum i_i should be limited to

$$i_i < \frac{\lambda_m}{L_q}. \quad (24)$$

Fig. 6 is set to the maximum value $i_i = 4$ A. For higher values of i_i , the negative q -axis is no longer a stable equilibrium and the control settles on the zero-torque locus in the third quadrant as shown in Fig. 7, leading to inaccurate initial position estimation. No such limitation on i_i exist for synchronous reluctance machines without magnets.

E. Position Observer

A conventional phase-locked-loop (PLL) with a proportional-integral (PI) controller is employed to drive the error signal ϵ to zero as in (25)

$$\hat{\omega} = k_p \epsilon + \int k_i \epsilon dt \quad \hat{\theta} = \int \hat{\omega} dt \quad (25)$$

where k_p and k_i are the respective gains; the gains are tuned for critically damped poles at $s = -\Omega_\omega$. The position error signal to track the stator current is given by

$$\epsilon = \angle \hat{i}_{\alpha\beta} - \hat{\theta} \quad (26)$$

where $\hat{\theta}$ is the estimated position. Note that the PLL converges to the q -axis and a correction of $\pm\pi/2$ based on the direction of angular speed is necessary to orient along the d -axis.

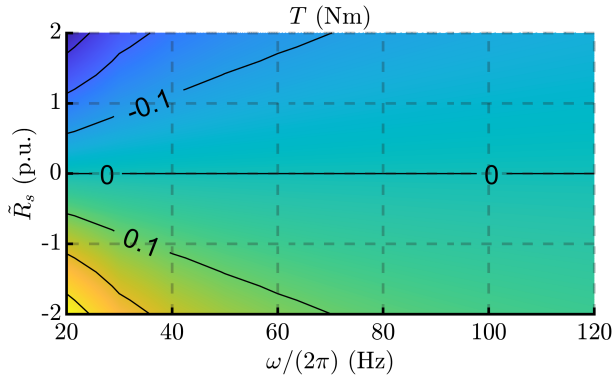


Fig. 8. Sensitivity analysis to stator resistance: contour of the steady-state torque plotted as functions of the rotor speed (0.2 - 2 p.u.) and the resistance error ($\hat{R}_s = \pm 2$ p.u. $\Rightarrow \tilde{R}_s = \pm 3 R_s$).

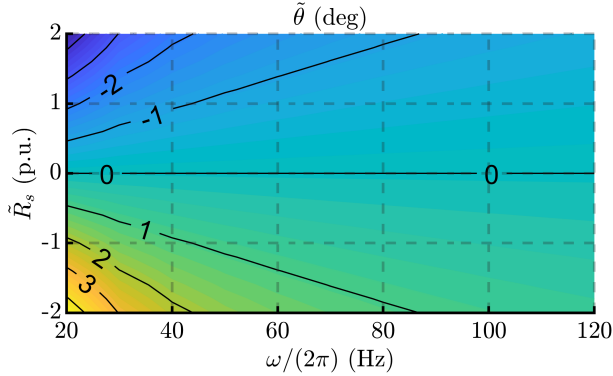


Fig. 9. Sensitivity analysis to stator resistance: contour of the steady-state position error plotted as functions of the positive rotor speed and the resistance error ($\hat{R}_s = \pm 2$ p.u. $\Rightarrow \tilde{R}_s = \pm 3 R_s$).

F. Sensitivity to Stator Resistance

Let $\tilde{R}_s = R_s - \hat{R}_s$ denote the resistance error where \hat{R}_s is the estimated resistance. Then, it follows from (1) and (17) that in steady-state,

$$\frac{3i_i}{2} (v_i^* - \hat{R}_s i_i) = 0 \quad \Rightarrow \quad \lambda_\tau = -\frac{\tilde{R}_s i_i}{\omega}. \quad (27)$$

Thus, the residual torque and active power are given by

$$T = -\frac{3p}{2} \frac{\tilde{R}_s i_i^2}{\omega} \quad \Rightarrow \quad P_{mech} = -\frac{3}{2} \tilde{R}_s i_i^2 \quad (28)$$

The contour of the torque (28) as functions of rotor speed and resistance error is shown in Fig. 8 and is observed to be insignificant, particularly at medium/high speeds. The steady-state position error for positive and negative speeds under resistance error is shown in Figs. 9 and 10, respectively. The negative low speeds region is more sensitive, yet the position error is $|\hat{\theta}| < 10^\circ$ for resistance error $|\tilde{R}_s| < 1$ p.u. ($|\hat{R}_s| < 2 R_s$).

IV. SIMULATION

The proposed flying start method is validated in the Matlab Simulink environment. The parameter of the PM-SyR machine

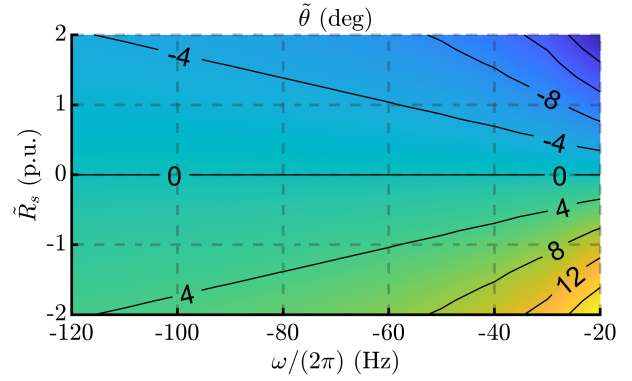


Fig. 10. Sensitivity analysis to stator resistance: contour of the steady-state position error plotted as functions of the negative rotor speed and the resistance error ($\hat{R}_s = \pm 2$ p.u. $\Rightarrow \tilde{R}_s = \pm 3 R_s$).

TABLE I
PM-SyR MOTOR PARAMETERS

Parameters	Symbol	Values	Units
Rated power	P_n	5.52	kW
Rated voltage (line rms)	V_n	260	V
Rated speed	ω_n	1800	rpm
Rated current (rms)	I_n	16.3	A
Rated torque	T_n	29.8	Nm
Pole pairs	p	2	-
Stator resistance	R_s	0.46	Ω
PM flux magnitude	λ_m	0.22	Vs
d -axis inductance	L_d	24	mH
q -axis inductance	L_q	7	mH

under test is tabulated in Table. I. The flying start operation is considered at no-load with a shaft inertia $J = 0.02$ kg m². The switching frequency of the system is 10 kHz.

The PLL poles are placed at $\Omega_\omega = 2\pi \cdot 60$ rad/s. The i -axis current controller is tuned for $\Omega_i = 2\pi \cdot 150$ rad/s and τ -axis controlled for $\Omega_\tau = 2\pi \cdot 50$ rad/s.

A. Virtual Resistance Based Flying Start

The performance of the virtual resistance based flying start method at rated speed $\omega = 1800$ rpm (60 Hz electrical) is shown in Fig. 11 where the drive is powered-up at time $t = 0$ s. A constant virtual resistance $R_v = -20 \Omega$ is considered. For $t > 0$ s, the braking torque is $T = -3.5$ Nm and the steady-state position error $\hat{\theta} = -145^\circ$, in agreement with Figs. 4 and 3, respectively. The rotor speed (magnified shown in red dotted line) decelerates by 160 rpm in 100 ms for the considered shaft inertia of $J = 0.02$ kg m².

B. Reactive Power Injection Flying Start

The proposed flying start method using reactive power injection at positive and negative rated speed is shown in Figs. 12 and 13, respectively. The current reference with a ramp profile is set to the maximum limit satisfying (24), $i_i^* = 4$ A. Unlike the former, once the drive is powered-up at $t = 0$ s, barring a small transient, the braking torque remains negligible and the position error converges to zero. It is worth pointing

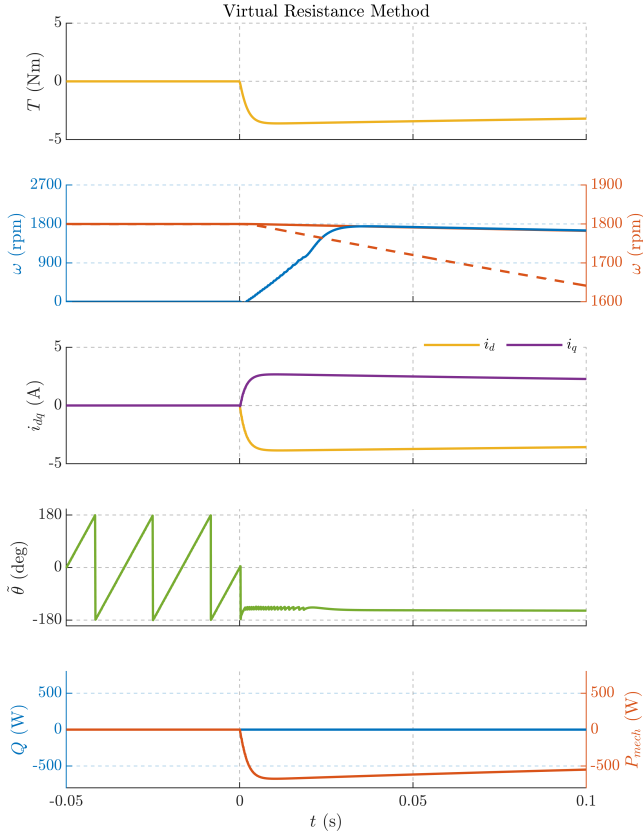


Fig. 11. Analysis of the virtual resistance based flying start method [10] when the drive is powered-up at time $t = 0$ s at the rated rotor speed $\omega = 1800$ rpm and no-load

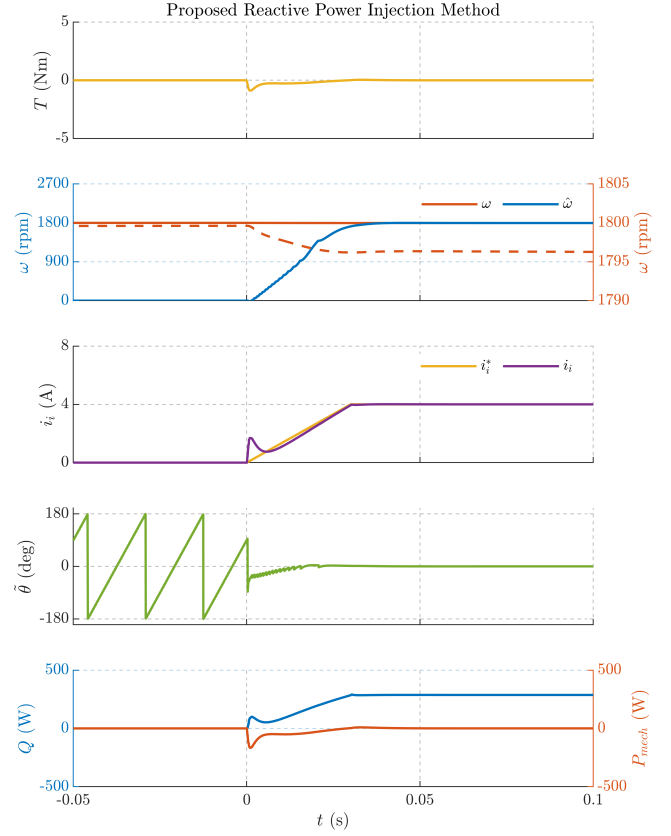


Fig. 12. Analysis of the proposed reactive power injection based flying start method when the drive is powered-up at time $t = 0$ s at the rated rotor speed $\omega = 1800$ rpm and no-load.

out that the rotor remains largely unperturbed and the speed nearly constant (speed reduction < 5 rpm).

C. Validation of Resistance Sensitivity Analysis

The resistance sensitivity is validated at negative low speed region $\omega = -2\pi \cdot 20$ rad/s (0.33 p.u.) for a resistance error of $\hat{R}_s = 2 R_s$ ($\hat{R}_s = -1$ p.u.) as shown in Fig. 14. A steady-state torque $T = -0.2$ Nm and position error $\theta = 10^\circ$ is observed, in agreement with the analytical evaluations in Figs. 8 and 10, respectively. Despite a high resistance error, the proposed method is sufficiently resilient.

V. CONCLUSION

This paper proposed a novel flying start method for sensorless synchronous drives to estimate the initial rotor position and speed with least disturbances on the load. This is achieved by reactive power injection and realized in the stator current reference frame such that the operating point settles on the zero-torque locus. As opposed to the virtual resistance method, the rotor experiences minimal braking torque and the position error converges to zero in steady-state. Guidelines for tuning and gain selection based on the nominal parameters were discussed. Moreover, the proposed method benefits from a simple

structure, resilience to resistance errors and less dependency on the motor parameters.

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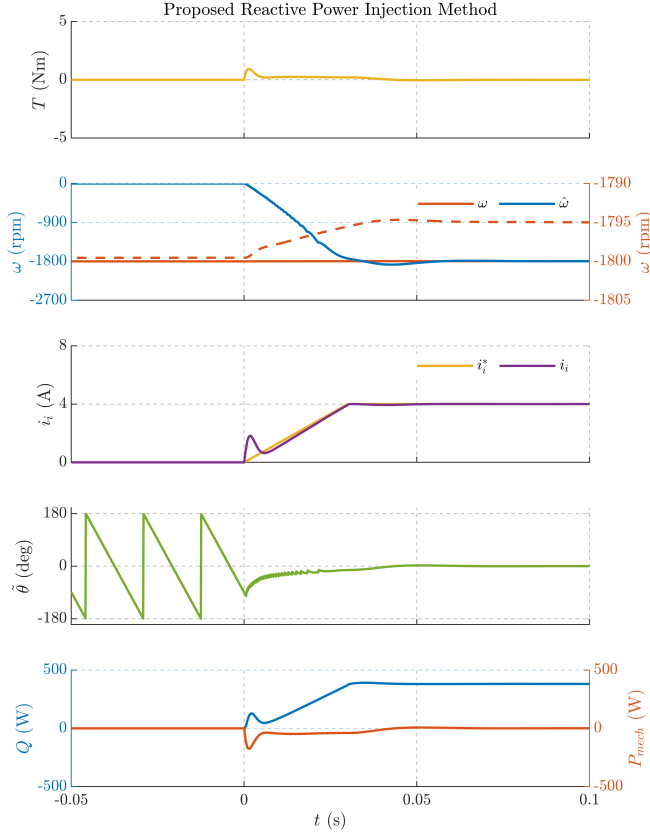


Fig. 13. Analysis of the proposed reactive power injection based flying start method when the drive is powered-up at time $t = 0$ s at the rated rotor speed $\omega = 1800$ rpm and no-load.

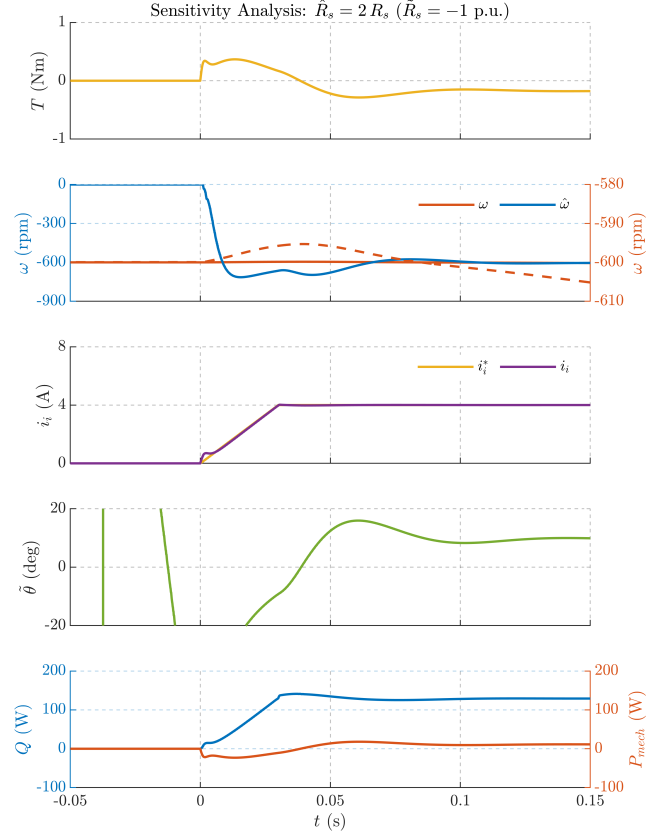


Fig. 14. Analysis of the proposed reactive power injection based flying start method when the drive is powered-up at time $t = 0$ s at the rated rotor speed $\omega = 1800$ rpm and no-load.

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