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Protograph-Based Design for QC Polar Codes

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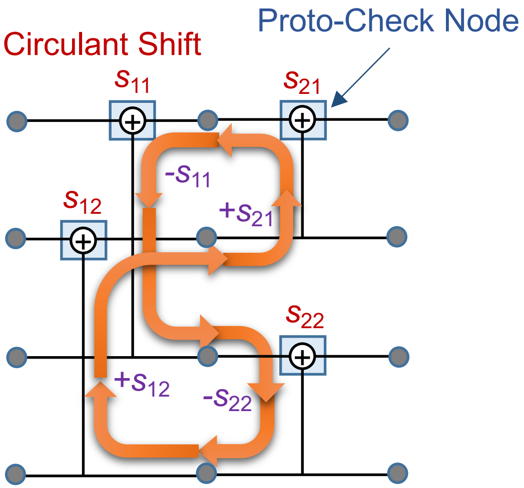
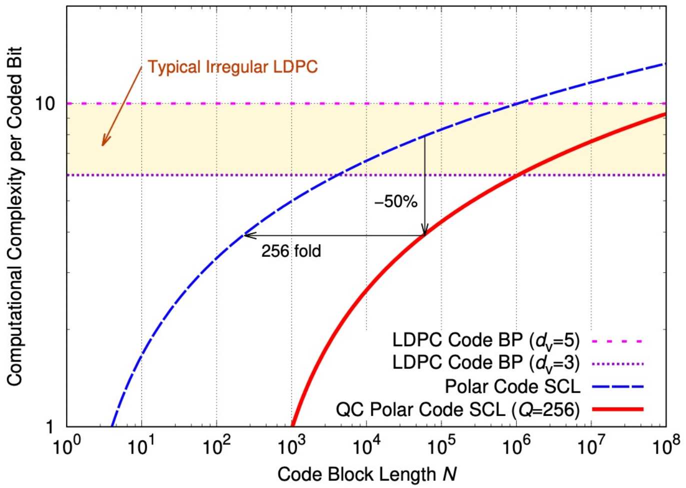
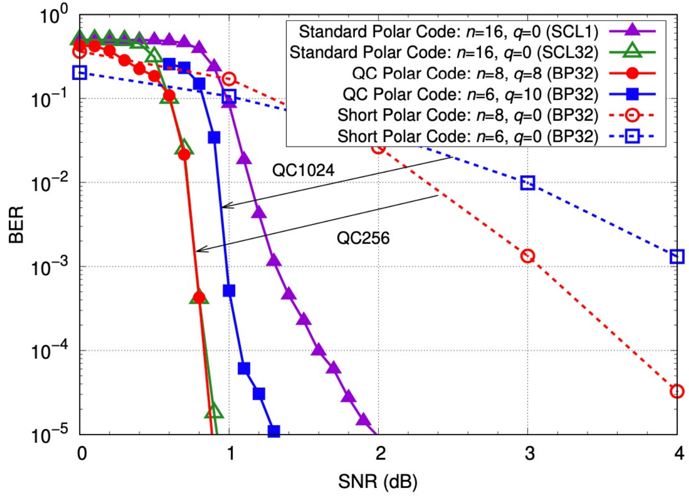
Outline

- Background of polar codes
- Novel protograph polar codes
  - Short-cycle mitigation
  - Frozen-bit location design
  - Fully parallel, high-gain, and low-complexity
- Irregular QC polar codes
- Summary







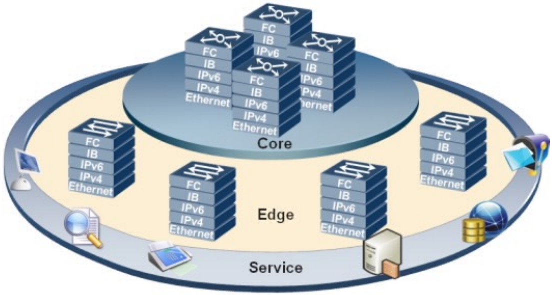
Digital Communications for IoE

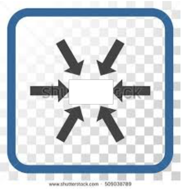
- 5G/6G wireless envisions wide-range applications with the **Internet of everything (IoE)**
  - Environment and energy monitoring
  - Industrial factory and manufacturing networks
  - Building and infrastructure management
  - Medical and healthcare
  - Extended reality (XR)

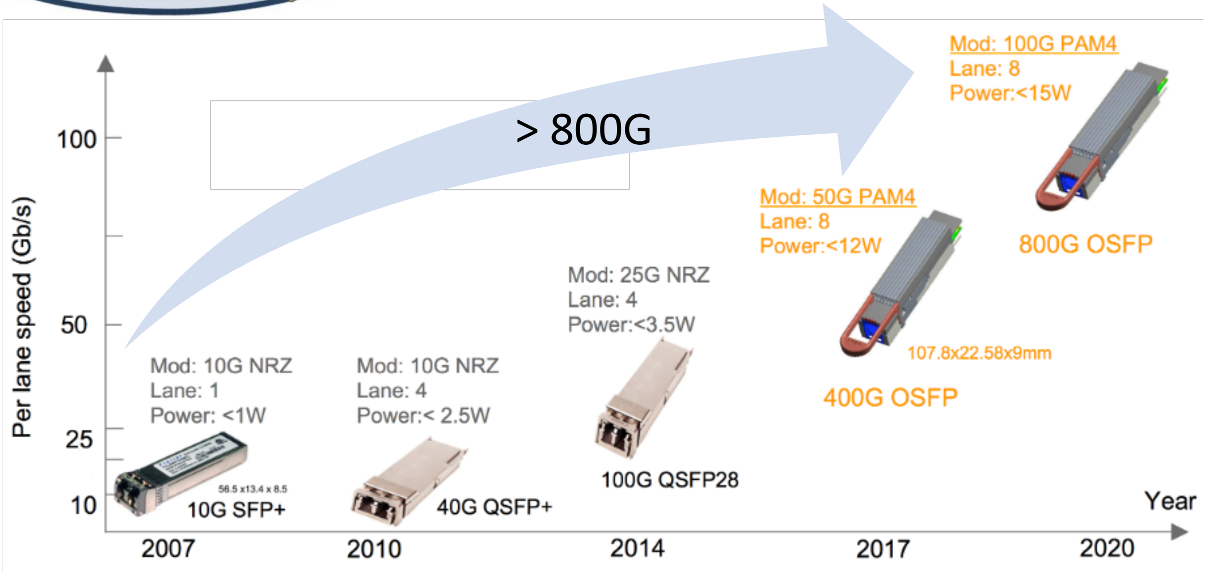
- IoE communications
  - Stringent requirement in reliability, power consumption and latency

Optical Data Center Networks

- Networking over massively large number of servers
- Key demand:
  - Low power
  - Low cost
  - Low latency
  - High density
  - High speed

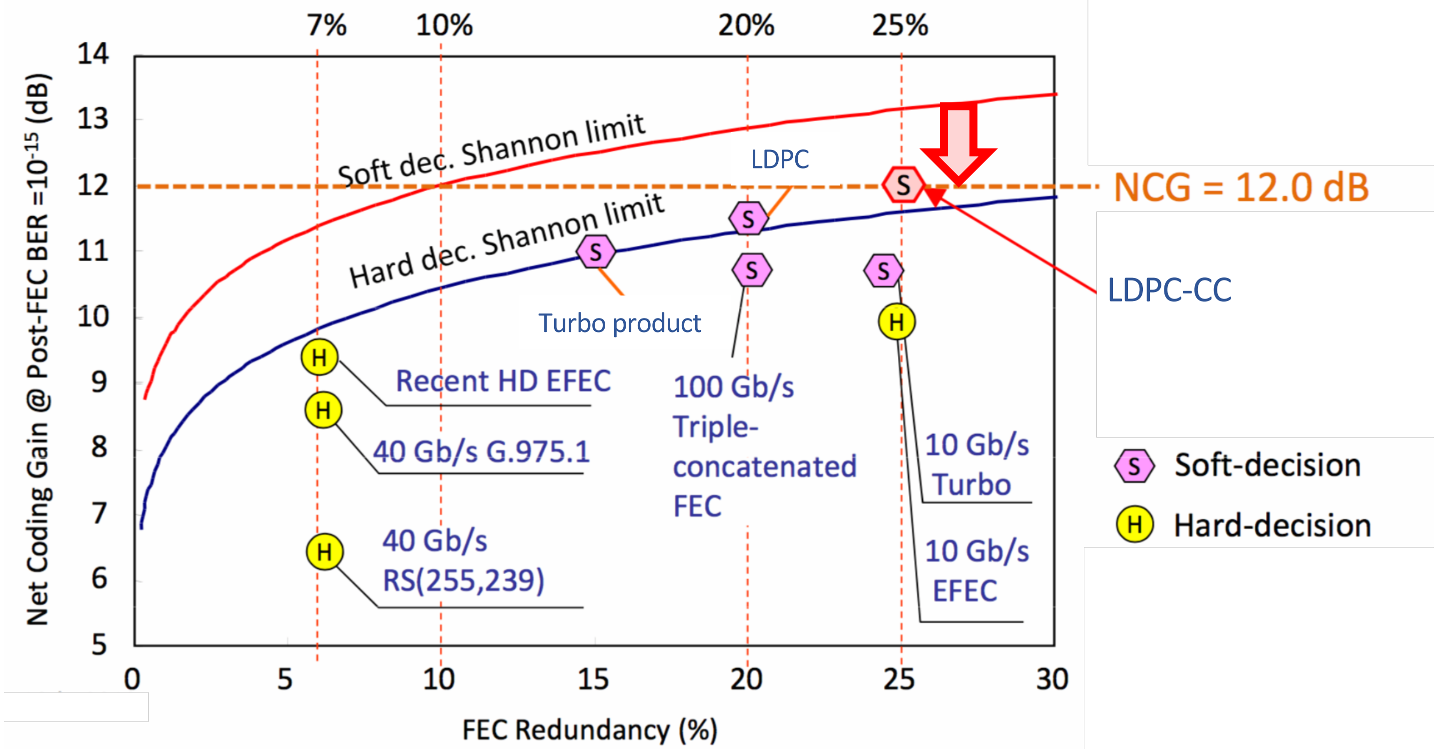






Forward Error Correction (FEC) Codes

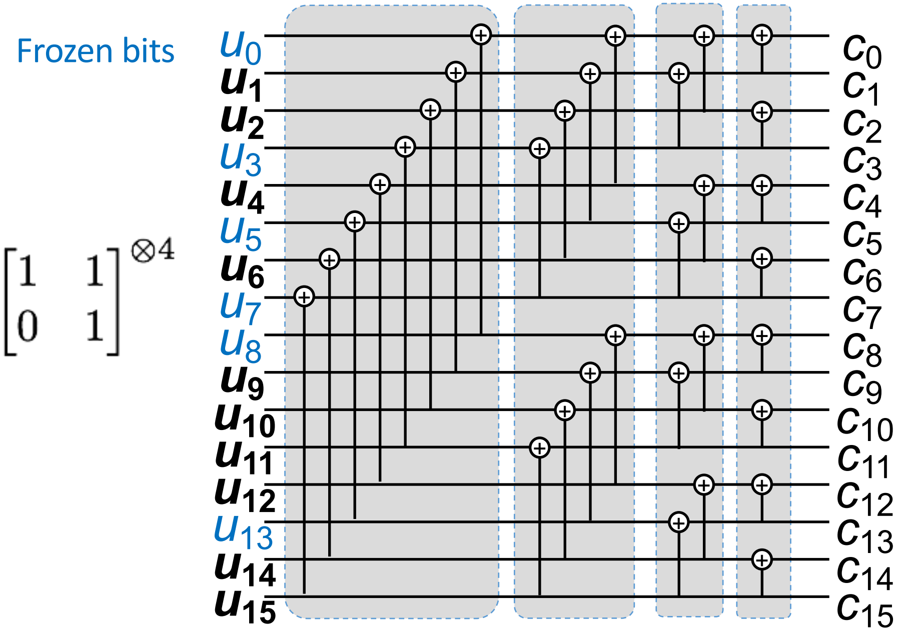
- Hard-decision: Reed-Solomon, BCH, …
- Soft-decision: Turbo, Low-density parity-check (LDPC), …, **Polar codes**



Polar Codes

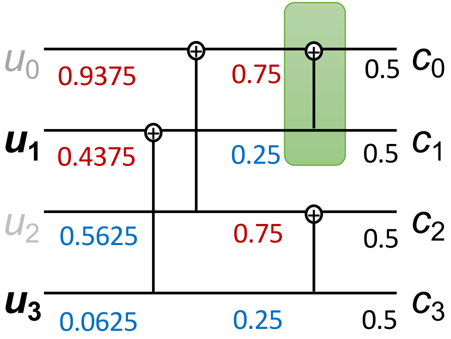
- **Capacity-achieving code** in **arbitrary memoryless channels** [Arikan TIT2009]
- Structured encoding and decoding; Cooley-Tukey-like butterfly architecture
- Flexible in code rates with frozen-bit selection
- **5G new radio standard**

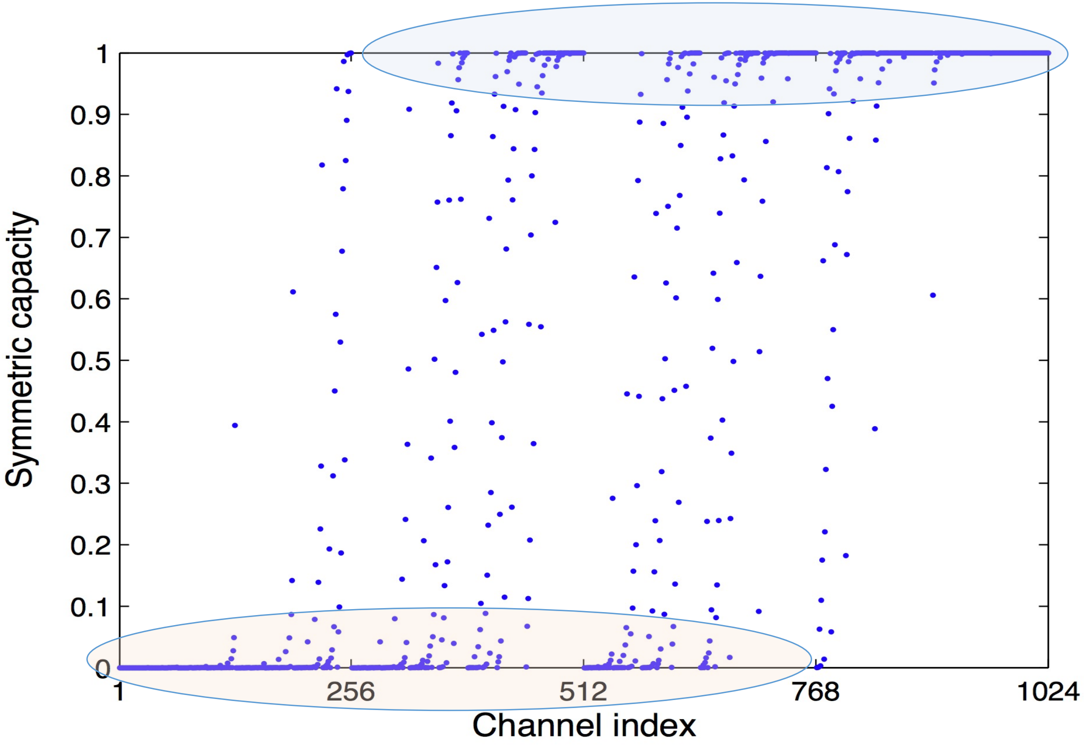
$$\boldsymbol{F} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{\otimes 4}$$



Polarization Phenomenon

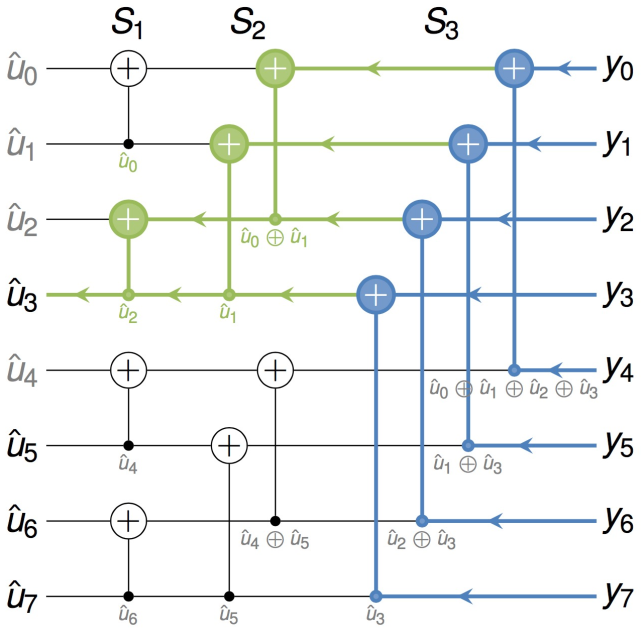
- Polar kernel polarizes messages into `bad’ and `good’ sub-channels
- Proportion of good sub-channels approaches channel capacity





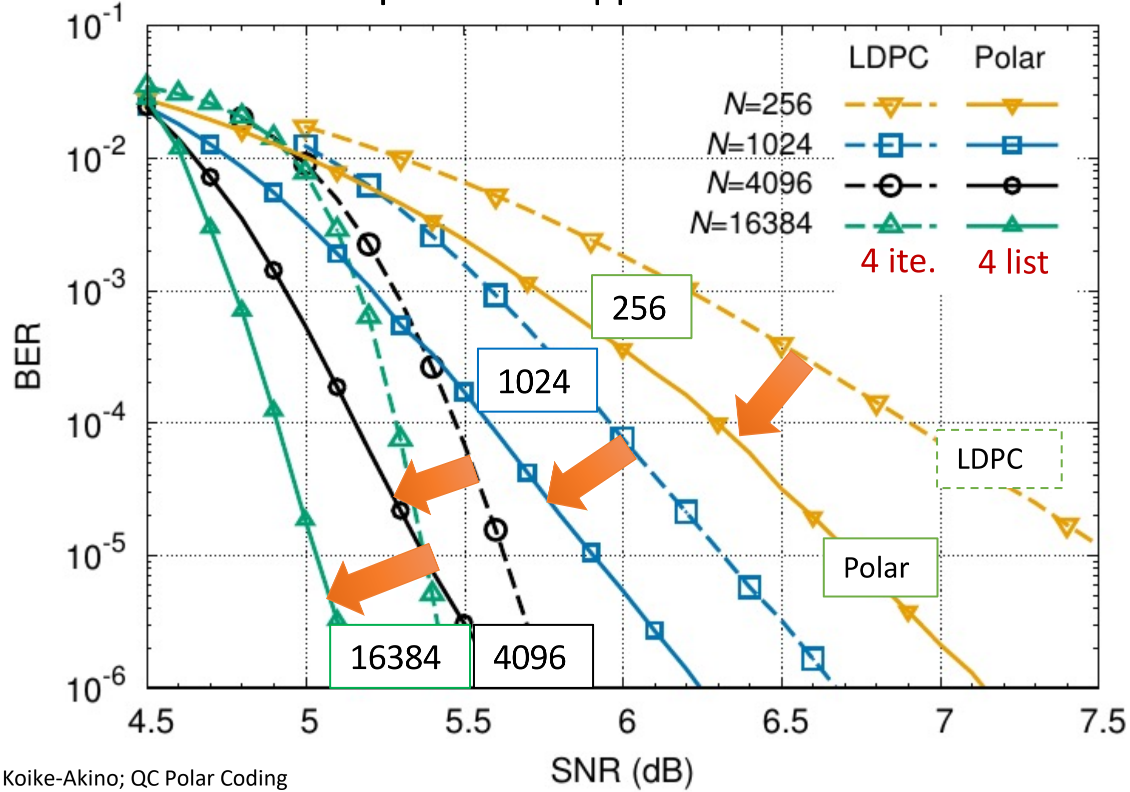
Successive Cancellation (SC) Decoding

- Log-linear decoding complexity: $N \log_2(N)$
- Capacity achieving in long codes
- Significantly improved by **successive cancellation list (SCL)** decoding [Tal-Vardy TIT2015]



Polar vs. LDPC Codes

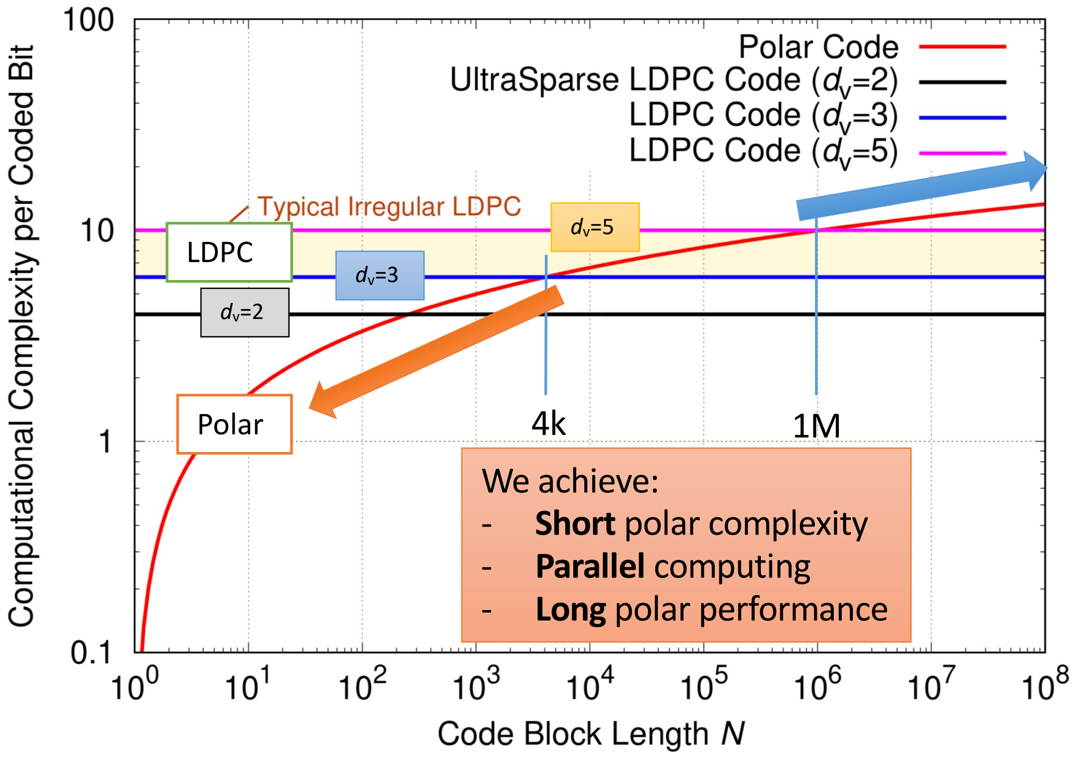
- Polar codes can outperform state-of-the-art LDPC codes for *shorter block* and *lower complexity* regimes
  - Polar codes are suited for short-packet IoE applications



Changes for the Better

Computational Complexity

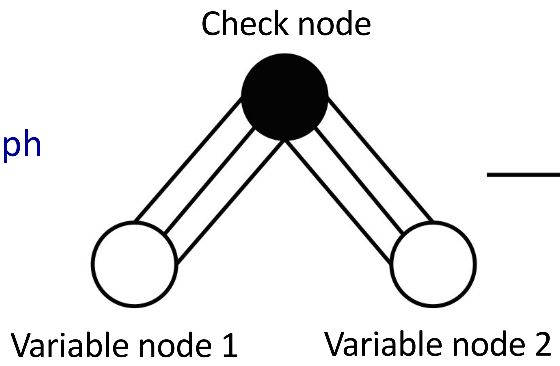
- Polar SCL decoding requires **log-linear** complexity: $L\, N \log_2(N)/2$
- LDPC BP decoding has **linear** complexity: $2\, I\, d_v\, N$



Protograph Codes

- **Protograph**: Compact Tanner graph with a degenerated set of nodes [Thorpe IPN2003]
  - Degree distribution can be determined
  - Edge connection is represented
- Example: **quasi-cyclic** (**QC**) LDPC codes with cyclic-shift permutation
- Parity-check matrix (PCM) is derived by *graph lifting procedure* with **replicate & permutate**

Protograph



$$\mathbf{B} = \begin{bmatrix} 3 & 3 \end{bmatrix}$$

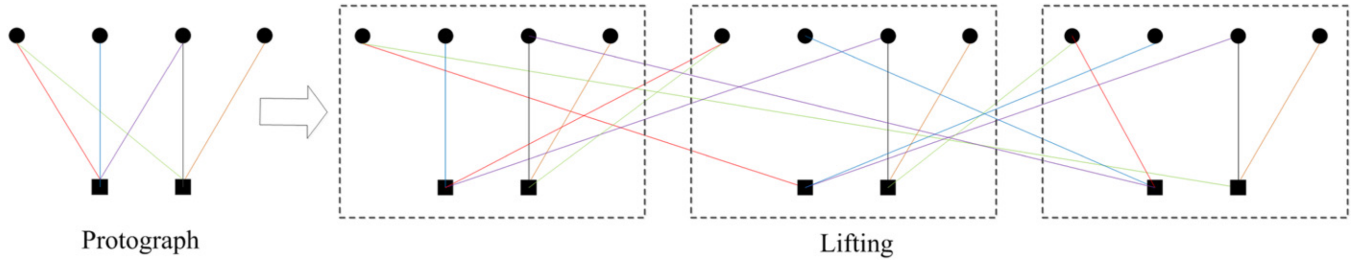
(3,6)-regular LDPC

Non-parallel protograph (QC type)

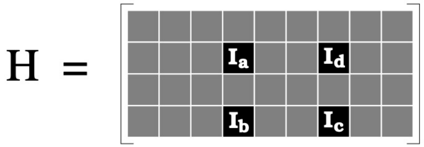
$$\mathbf{B}' = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Graph Lifting and Graph Girth

- Graph lifting: Replicate & permute

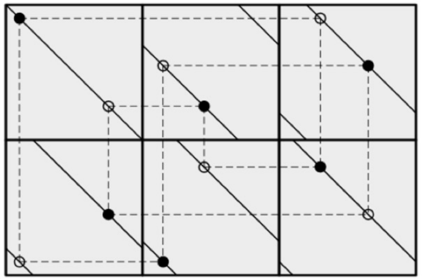


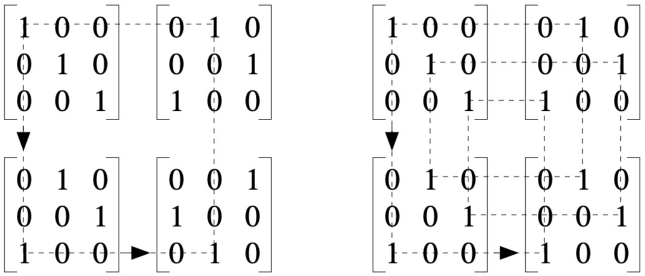
- Protograph LDPC codes: PCM is modified by replacing
  - 1 with *Q*-by-*Q* permutation matrix
  - 0 with *Q*-by-*Q* zeros matrix



- QC LDPC codes: Cyclic-shift permutation
  - Girth <= 12 [Fossorier TIT2004]
  - Girth > 12 [Kim TIT2007]

  "Girth" of a code is the length of the shortest cycle in the code graph





Proposed New Family: Protograph Polar Codes

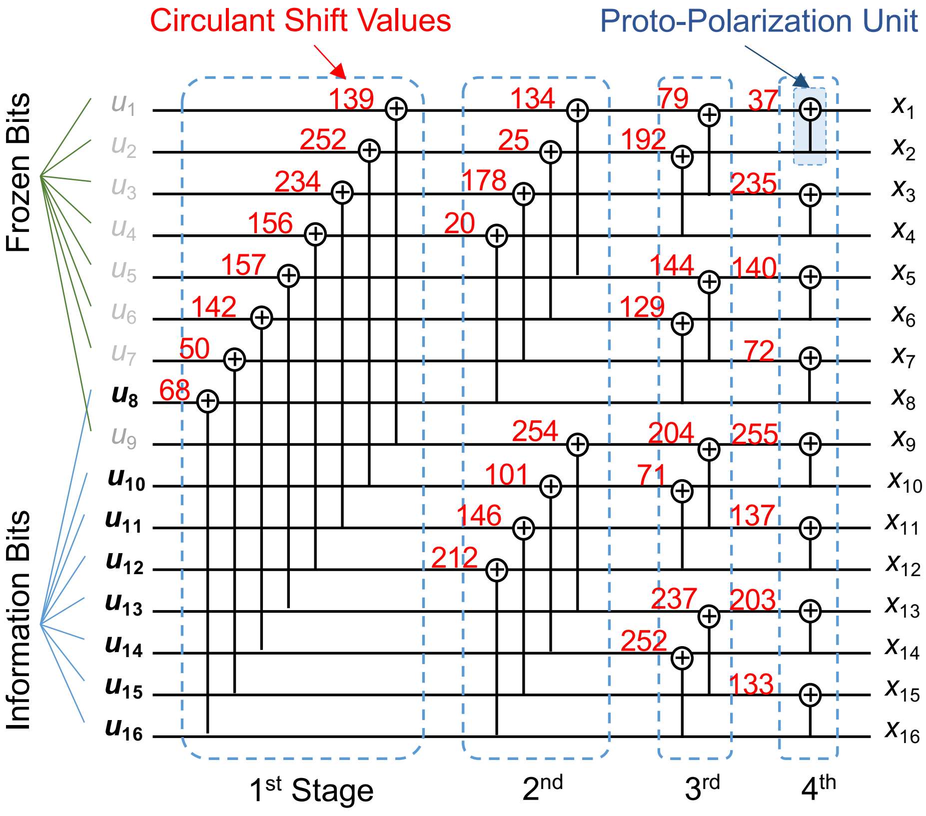
- We propose to introduce protograph concept in polar codes
  - We apply graph lifting to every polarization unit: ***proto-polarization***
- Generator matrix (not PCM) is replaced as Q-by-Q permutation matrices:

$$G^{\otimes 2} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{Lifting}} \begin{bmatrix} \boldsymbol{P}_{1,1} & \boldsymbol{P}_{1,2} & \boldsymbol{P}_{1,3} & \boldsymbol{P}_{1,4} \\ \mathbf{0} & \boldsymbol{P}_{2,2} & \mathbf{0} & \boldsymbol{P}_{2,4} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{P}_{3,3} & \boldsymbol{P}_{3,4} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{P}_{4,4} \end{bmatrix}$$

- QC polar codes with cyclic permutation

QC shift-value base matrix

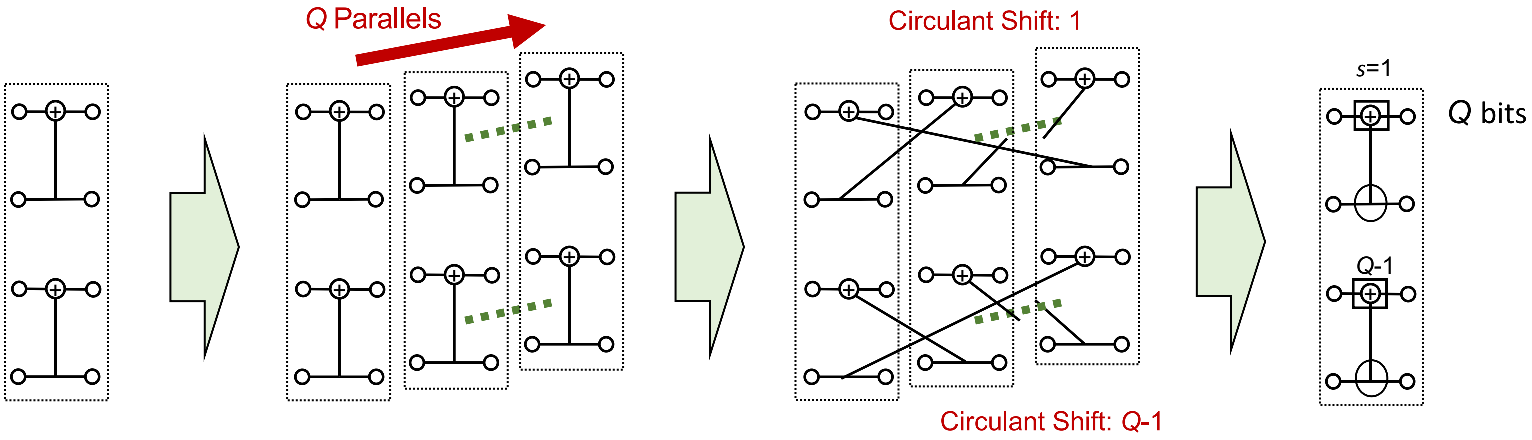
$$\begin{bmatrix} 139 & 252 & 234 & 156 & 157 & 142 & 50 & 68 \\ 134 & 25 & 178 & 20 & 254 & 101 & 146 & 212 \\ 79 & 192 & 144 & 129 & 204 & 71 & 237 & 252 \\ 37 & 235 & 140 & 72 & 255 & 137 & 203 & 133 \end{bmatrix}$$



4-Stage QC Polar Codes ($N=2^4$, $K=2^3$, $Q=2^8$)

Graph Lifting for Proto-Polarization

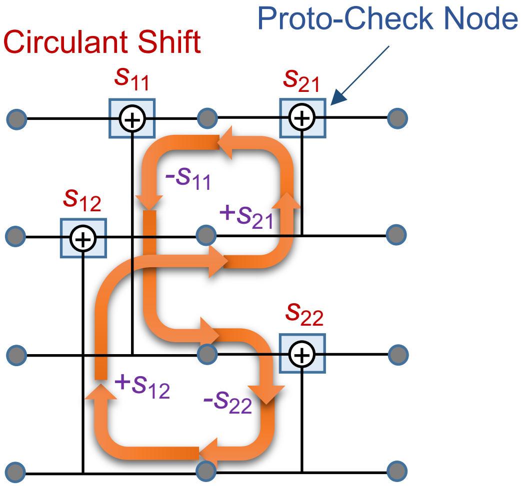
- Graph lifting to every polarization stage: *proto-polarization*
  - Replicate
  - Permute
  - *Q*-bit parallel processing



(a) Polar Units (b) *Q*-fold Replication (c) Permutation (d) Proto-Polar Units

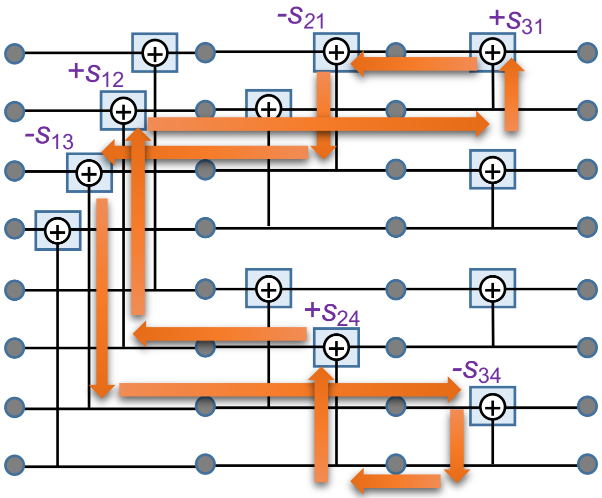
Cycle Mitigation

- Graph lifting can eliminate **short cycles** in polar code graphs
  - Conventional polar codes (Q=1) suffer from cycle-4 loops
    - Hence, belief-propagation (BP) decoding does not work well for standard polar codes
  - Girth maximization method [Wang TIT2013]
  - QC polar codes are viable breakthrough to resolve long-standing issue of loopy polar graphs



(a) Cycle-4 Message Path

$$-s_{1,1} - s_{2,2} + s_{1,2} + s_{2,1} \neq 0 \pmod{Q},$$

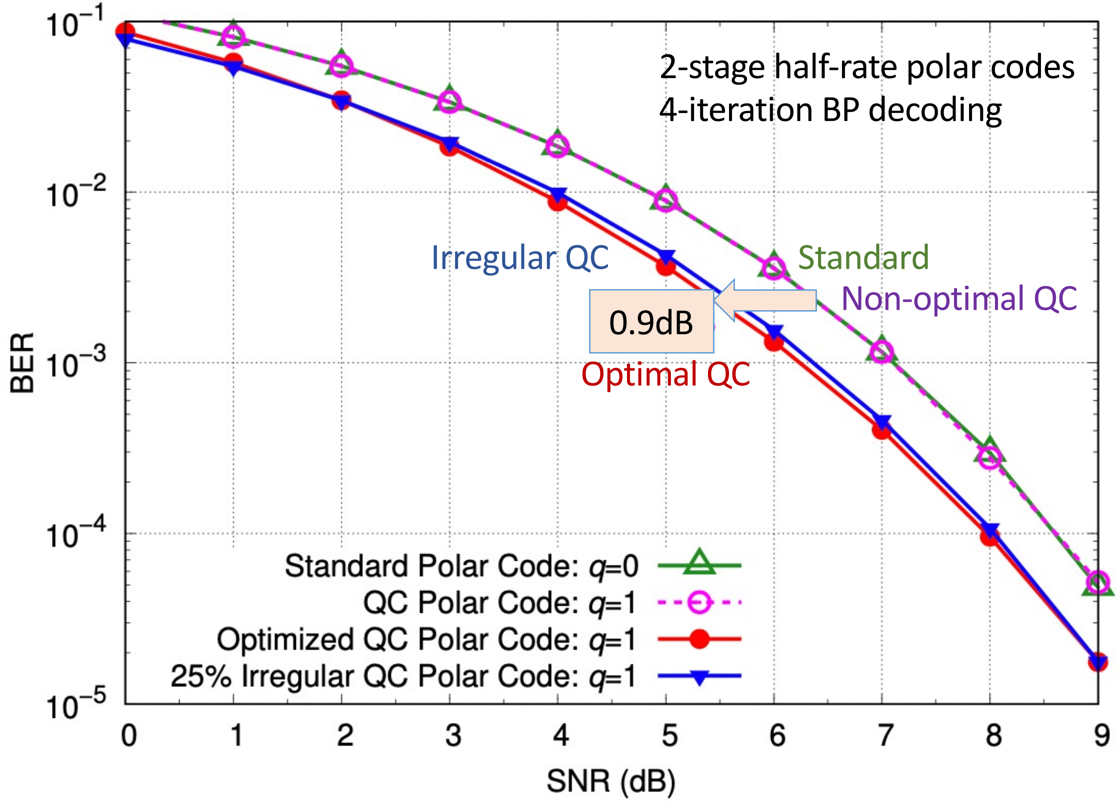


(b) Cycle-6 Message Path

$$-s_{1,3} - s_{3,4} + s_{2,4} + s_{1,2} + s_{3,1} - s_{2,1} \neq 0 \pmod{Q}.$$

2-Stage QC Polar Codes (4, 2, 2^q)

- Girth maximizing shift values can improve performance by **0.9dB**

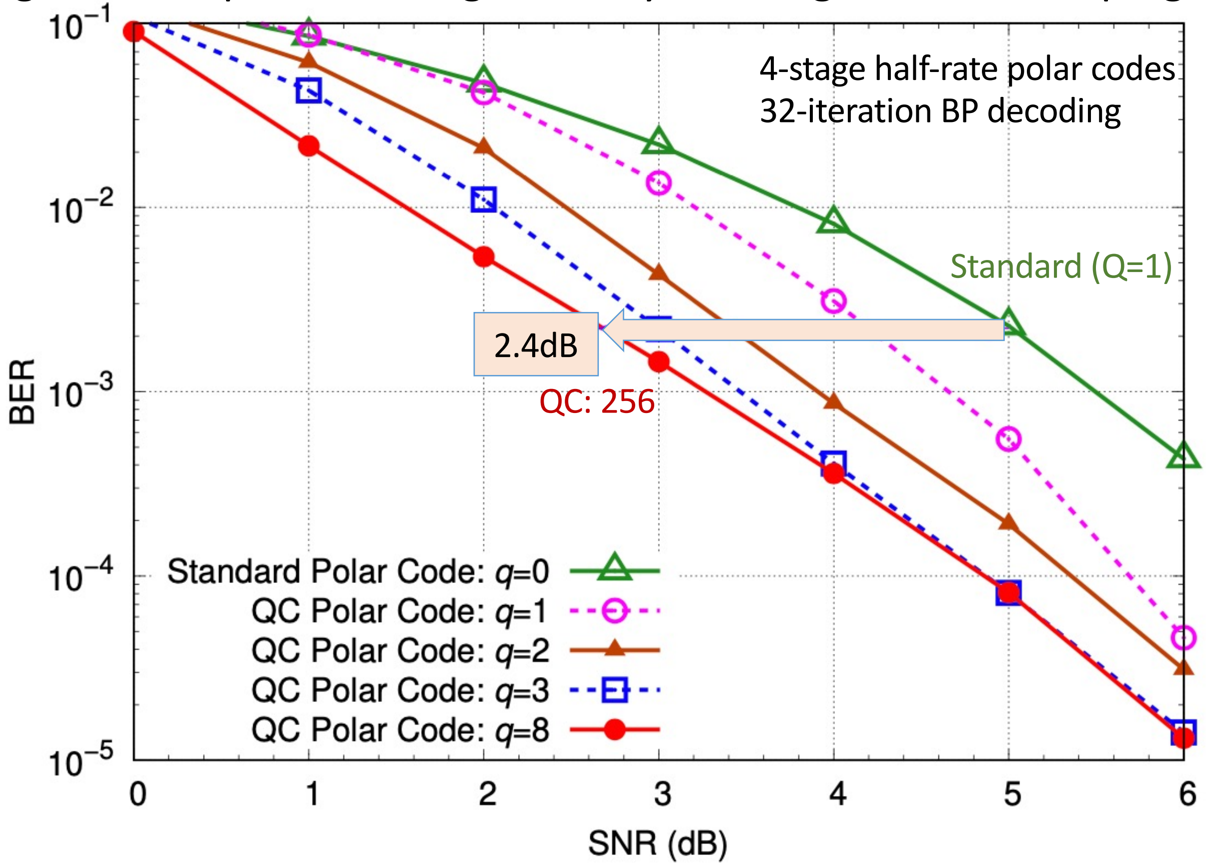


Standard: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, Non-optimal QC: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, Optimal QC: $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, Irregular QC: $\begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix}$

QC shift-value base matrix

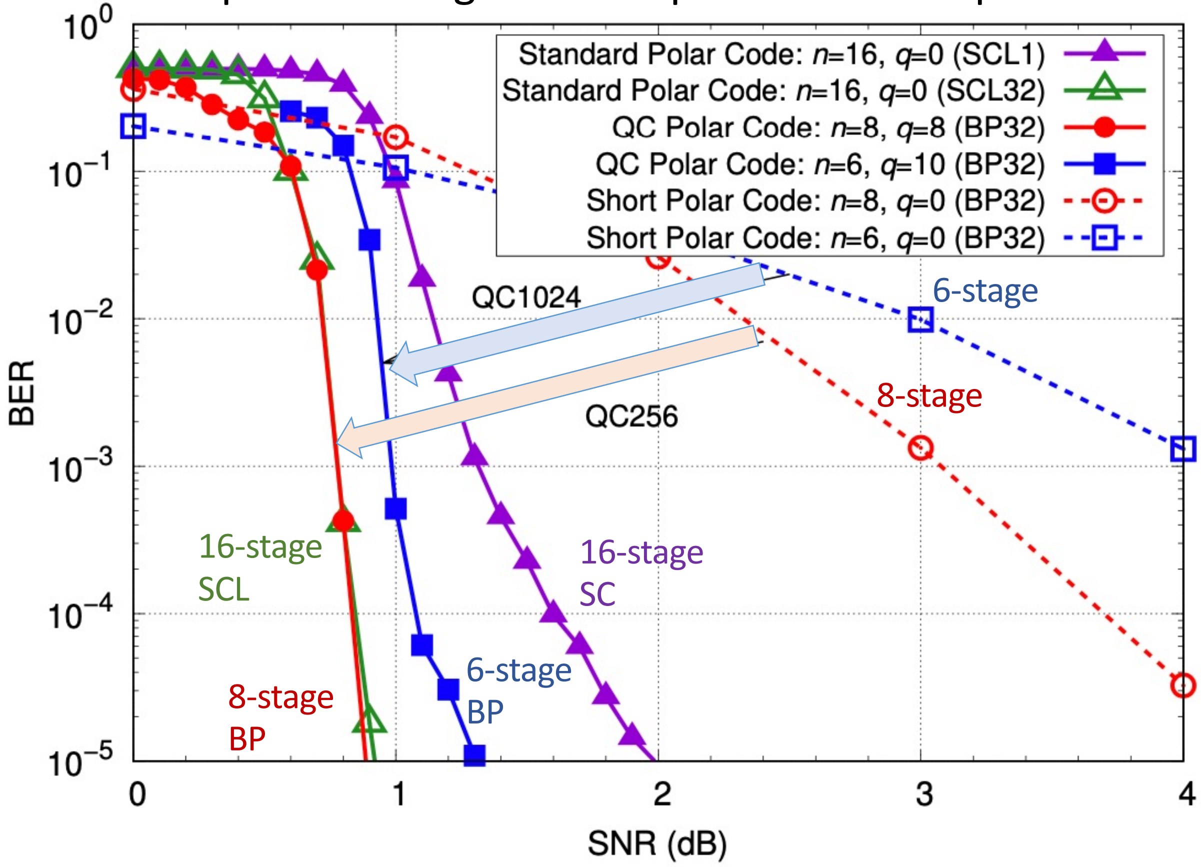
4-Stage QC Polar Codes (16, 8, 2^q)

- Higher lifting factor improve BER significantly; **2.4dB** gain, while keeping same complexity



QC Polar Codes for Block Length of $N=2^{16}$

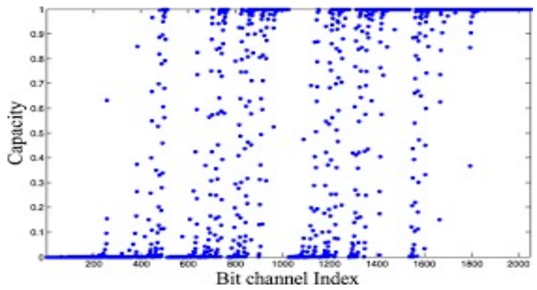
- QC polar codes can outperform long standard polar codes despite shallow polarization

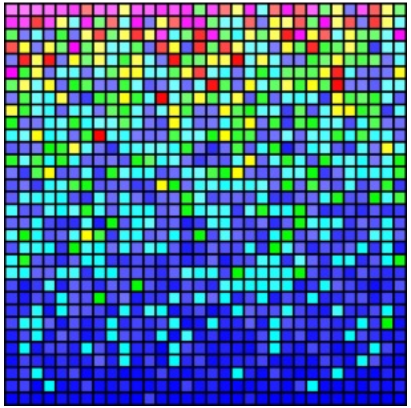


Frozen-Bit Location Design

- Bhattacharyya parameter [Arikan TIT2009]
- Density evolution [Mori-Tanaka COMML2009]
- Quantized density evolution [Tal-Vardy TIT2013]
- Gaussian approximation [Trifonov TCOM2012]
- Beta expansion [He GLOBECOM2017]

SC Decoding





- Genetic algorithm [Elkelesh TCOM2019]
- Deep learning [Ebada Allerton2019]
- Extrinsic information transfer (EXIT) [KoikeAkino ICC2017]
- **Protograph EXIT analysis**

SC & BP Decoding

Protograph EXIT Analysis

- Protograph-based extrinsic information transfer (**P-EXIT**) [Liva GLOBECOM2007]
  - Tracking mutual information updates at all edges
  - Account for edge connection
  - More accurate than EXIT chart
- Extended to Nonbinary P-EXIT [Dolecek TIT2013]

$$\mathbf{B}_1 = \begin{bmatrix} 2 & 1 & 2 & 1 & 1 & 0 \\ 1 & 2 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \mathbf{B}_2 = \begin{bmatrix} 2 & 2 & 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Same degree distribution for LDPC codes …

but $\mathbf{B}_1$ is better than $\mathbf{B}_2$

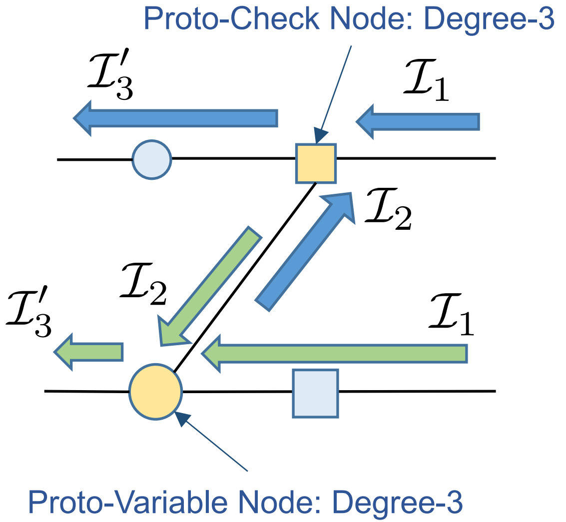
$$\mathbf{B}_1\!: \ \epsilon^* = 0.4531$$
$$\mathbf{B}_2\!: \ \epsilon^* = 0.0192$$

P-EXIT (but conventional EXIT) can analyze accurate threshold

Irregular QC Polar Code Design

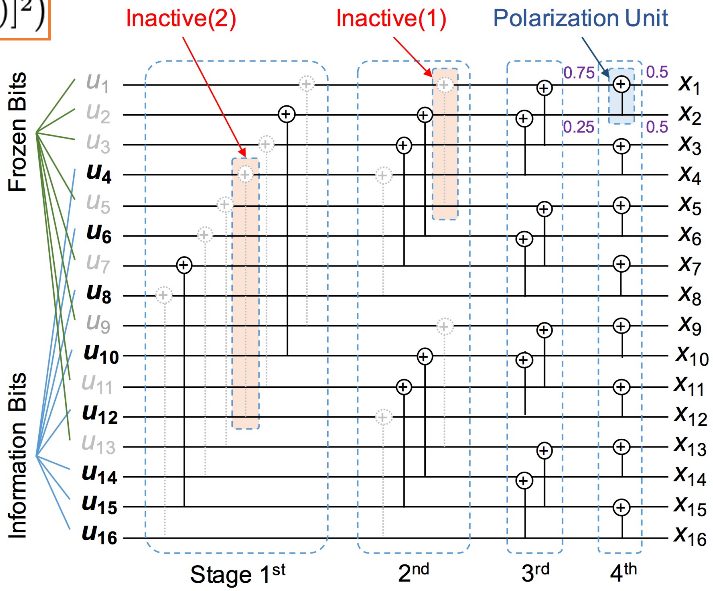
- We extend a greedy design method using **EXIT** analysis [KoikeAkino GLOBECOM 2017]
  - Joint design of **frozen-bit** locations and **inactive** polarization units

$$\mathcal{I}_3' = 1 - J\left(\sqrt{[J^{-1}(1-\mathcal{I}_1)]^2 + [J^{-1}(1-\mathcal{I}_2)]^2}\right)$$



$$\mathcal{I}_3' = J\left(\sqrt{[J^{-1}(\mathcal{I}_1)]^2 + [J^{-1}(\mathcal{I}_2)]^2}\right)$$

$$P_e = \frac{1}{|\mathbb{K}|} \sum_{i \in \mathbb{K}} \mathbb{Q}\left(\frac{1}{2} J^{-1}(\mathcal{I}_i')\right)$$



Algorithm 1 Joint interleaver and irregular polar codes design

Initialize:

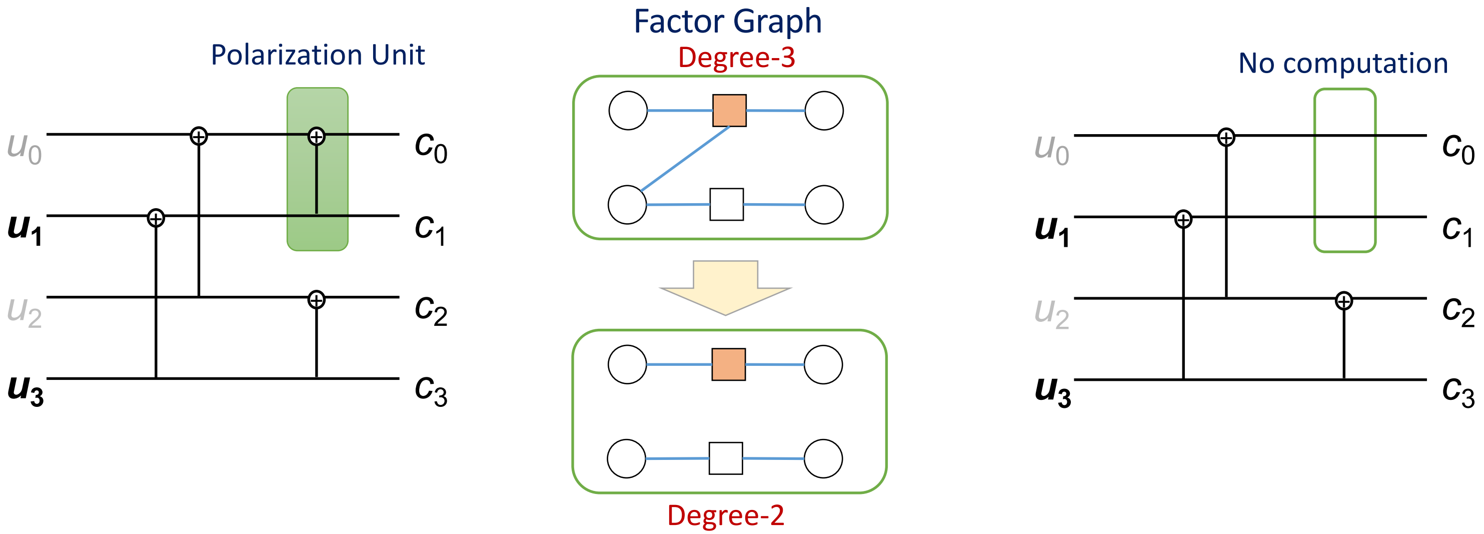
1: $\widetilde{\mathcal{C}} = [\widetilde{C}_1, \widetilde{C}_2, ..., \widetilde{C}_N]$: mutual information of each modulated bit at eigen-mode channels for an ave. SNR of ρ

Start:

2: **for all** interleaver sets Π in consideration **do**
3: perform de-interleaving: $\boldsymbol{\mathcal{I}} = \Pi^{-1}(\widetilde{\mathcal{C}})$
4: activate all polarization units
5: **while** $N_{\mathrm{inact}} \in \{1, 2, \ldots, N_{\mathrm{U}}\}$ **do**
6: **for all** active polarization units **do**
7: inactivate the target polarization unit
8: $\boldsymbol{\mathcal{I}}' = \mathrm{UpdateMI}(\boldsymbol{\mathcal{I}})$ according to (7)
9: select frozen bits $\bar{\mathbb{K}}$ having the $N-k$ smallest $\boldsymbol{\mathcal{I}}'$
10: calculate the upper bound P_e according to (9)
11: reactivate the target polarization unit
12: **end for**
13: inactivate the polarization unit having smallest P_e
14: **end while**
15: **end for**
16: **Return:** best interleaver, frozen bit locations, and inactivated polarization units achieving the smallest P_e

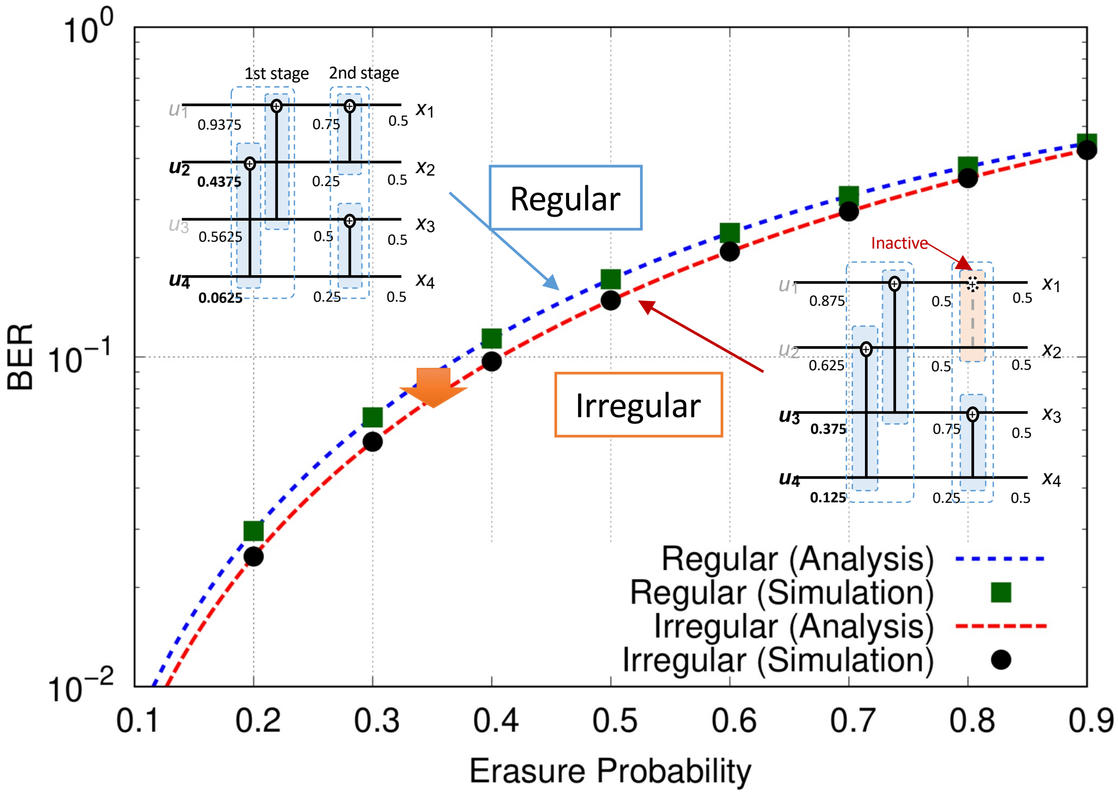
Irregular Polar Codes

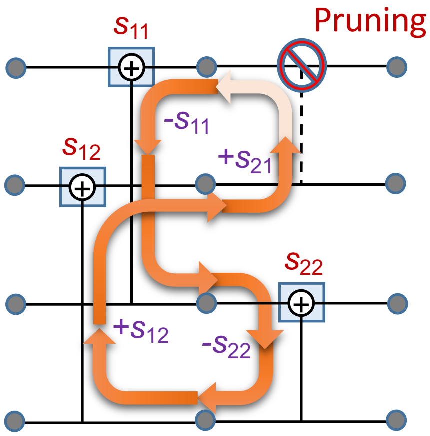
- We proposed to *inactivate* polarization units in an irregular fashion [KoikeAkino ECOC2017]
- We could *reduce*
  - the computational **complexity** for both encoding and decoding; 30%-80%
  - the decoding **latency** of SCL; 25%-95%
  - the bit error rate (**BER**); a marginal gain



Irregular Polarization Gain

- Better BER and lower complexity [KoikeAkino ECOC17]
- Pruning proto-polarization can potentially remove *short cycles*

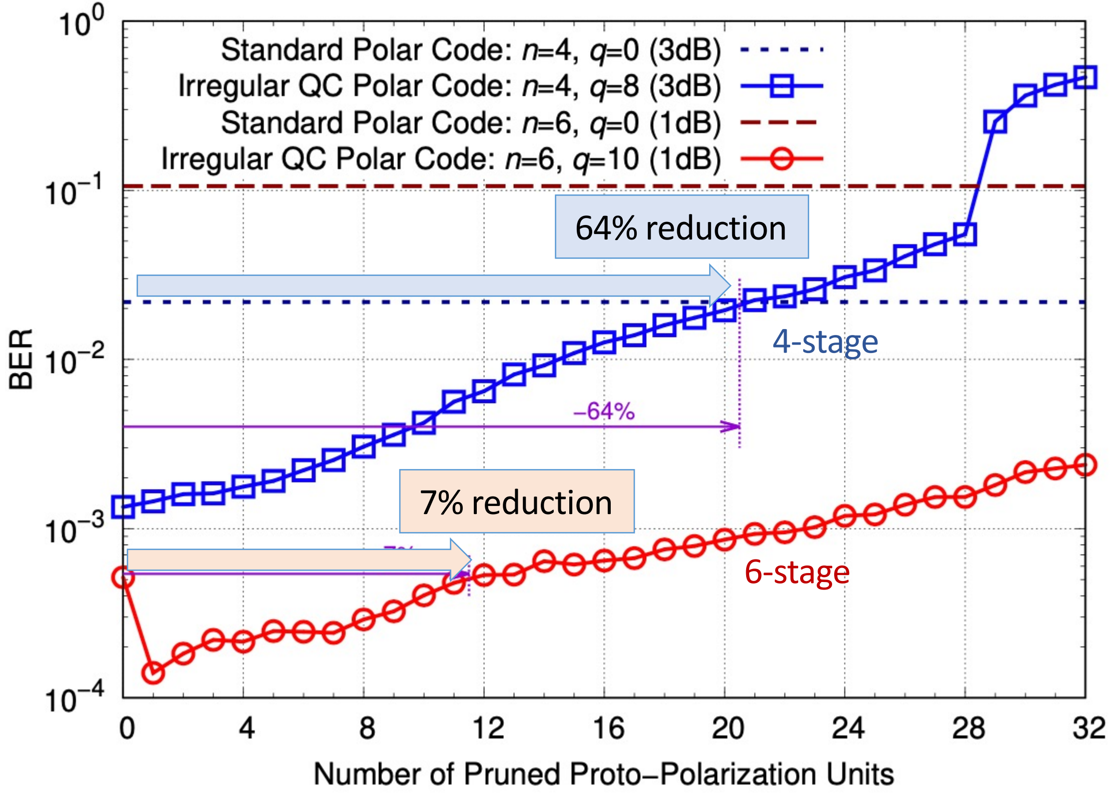




Cycle mitigation by pruning

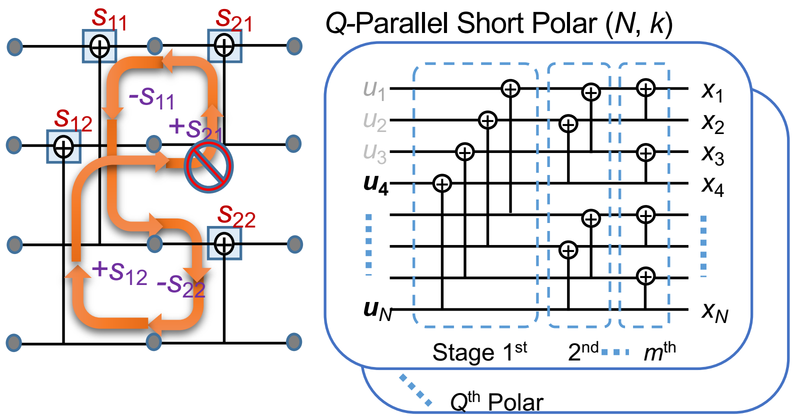
Irregular QC Polar Codes

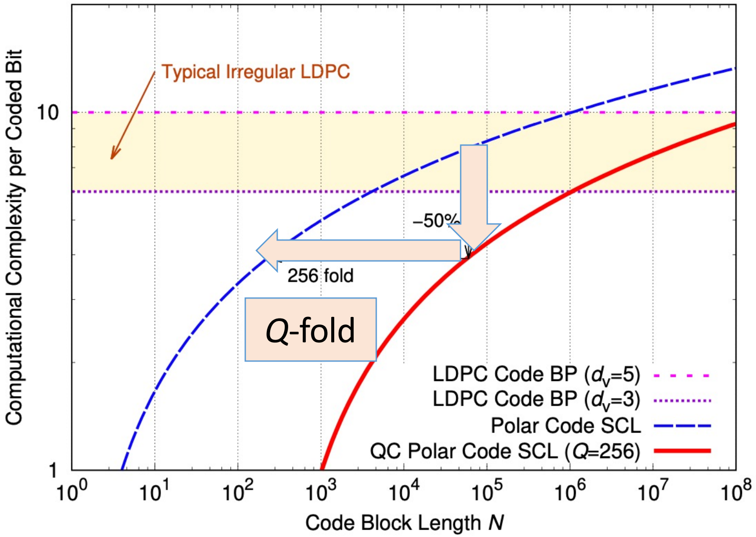
- Irregular QC polar codes outperform standard polar codes even with 64% pruning
  - Irregular codes can be better than regular one; e.g., 6-stage polar with up-to 7% pruning



Advantages of QC Polar Codes

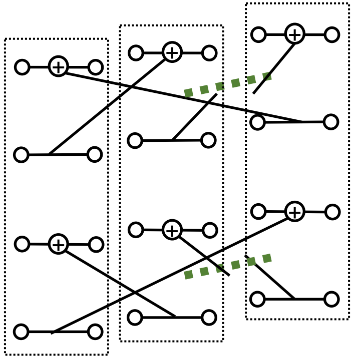
- The **girth** of polar codes can be increased significantly.
- The **BP** decoding can compete with SCL decoding.
- Multiple short polar encoders and decoders are implemented in a fully **parallel** fashion with no additional complexity besides circulant message exchanges.
- It realizes a **low computational** complexity equivalent to *Q*-fold shorter polar codes.
- **Shallow** polarization offers comparable performance to deeper polarization.
- Code design is simpler using shallower polarization.
- There is a higher flexibility in codeword lengths of **non-powers-of-two**, i.e., $N = 2^n Q$.
- **Irregular** polarization is straightforward to apply with the shift value matrix design.
- Well-established techniques such as **P-EXIT** from LDPC codes are applicable.
- It opened a new research field of **polar-type generalized low-density generator matrix (G-LDGM)** codes.

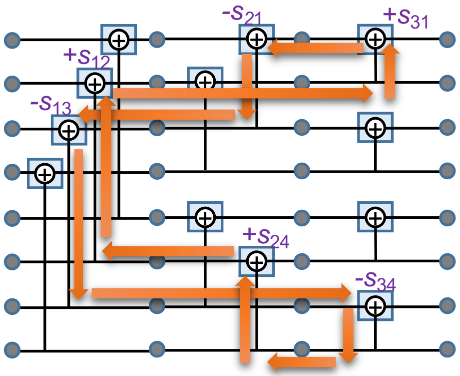




Summary

- We proposed a novel family of polar codes; **protograph-based QC polar codes**
  - Short cycles in factor graphs can be eliminated
  - P-EXIT, used for QC LDPC codes, is applicable for frozen-bit location design
  - Fully parallel encoding and decoding are possible
  - BP decoding can compete with SCL decoding
  - Shallow-stage polarization can compete with deep-stage polarization
  - Irregular QC polar codes can reduce complexity and BER
- We envision further extensions and investigations of **polar-type G-LDGM** codes
  - Protograph polar codes opened many fascinating topics bridging LDPC and polar codes
- Questions?
  - koike@merl.com





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